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APPEAL

I am delighted to introduce this issue of International Journal of Science and Humanities (IJSH) to the students and research community on behalf of Islamiah College (Autonomous), Vaniyambadi, a century old institution serving for the cause of education to socially, economically and educationally weaker sections of the society. The IJSH, is a peer reviewed research journal of interdisciplinary nature that cater the needs of the teaching and research society. The aim of the journal is not only to provide a space for leading research work but also provide a platform for the budding researchers to publish their maiden attempt in the field of science and humanities. The objective of IJSH is to publish up-to-date, high-quality and original research papers alongside relevant and insightful reviews.

The initiative to start this journal was taken by Janab L.M Muneer Ahmed, the Secretary & Correspondent of this College with an aspiration to keep the research vibrant in this campus. Now, the torch is handed over to me from June 2016 onwards to run this journal on non-profitable basis without compromising its aims and objectives. At this juncture, I appeal to all teaching and research communities to concentrate on both teaching and research relevant to society, which are symbolically related as the two faces of the same coin. I also appeal to all reviewers and editors not to compromise with the quality of the input and promote this journal to the next level with excellent output. Finally, I pray Almighty to provide guidance for development and success of this journal. Best wishes and thanks for your contribution to the IJSH.

Mr. L.M. MUNEER AHMED
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IN SILICO ANALYSIS OF SARS-COV-2 SPIKE GLYCOPROTEIN AGAINST FDA APPROVED DRUGS

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Abstract

Coronavirus disease 2019 (COVID-19) is a contagious disease caused by the severe acute respiratory syndrome-coronavirus 2 (SARS-CoV-2). Humans have been infected with the virus, resulting in the continuing Coronavirus pandemic. According to recent research, similarly to SARS-CoV, SARS-CoV-2 utilizes the Spike glycoprotein on the envelope to recognize and bind the human receptor ACE2. This causes the viral and host cell membranes to fuse, allowing the virus to enter the host cell. Despite multiple ongoing clinical investigations, no medicines that specifically target SARS-CoV-2 have been authorized. Repurposing FDA-approved medications could drastically cut the time and expense of developing a vaccine until one becomes accessible. In the present work, FDA approved drugs and their structural analogues in clinical trials are tested for their inhibition towards the COVID 19 protein (6LXT) using in silico drug repurposing strategy. Our data showed that Bictegravir, Atovaquone, Dolutegravir, Piperaquine and Emetine bind the receptor-binding domain of the Spike protein with high affinity and prevent ACE2 interaction.

Keywords: COVID-19, SARS-CoV-2, Drug Repurposing, Drug Development, Molecular Docking.

1. Introduction

COVID-19 is contagious and SARS-CoV-2 virus belongs to Betacoronaviruses genus and has shown to be related to SARS and MERS SARS-CoV-2 is a single-stranded, positive-sense RNA virus with a genetic sequence that is 79.5 percent identical to SARSCoV [1]. Coronaviruses are a large group of viruses that can infect both animals

and humans. The common signs and symptoms of coronavirus comprise cough, difficulty in breathing, pyrexia, highly fatal pneumonia, and more critical forms were kidney infections, confusion, diarrhea, rhinorrhea, vomiting, and nausea. Senior citizens were more likely to be affected by this disease [2].

To combat viral risks, one option is to find effective medications based on therapeutic protein targets. There are mainly five therapeutic protein targets, which are spike protein (S protein), angiotensin converting enzyme 2 (ACE2), main protease (Mpro), papain-like protease (PLpro), and RNA dependent RNA polymerase (RdRp). SARSCoV-2 S protein interacts with the host cell receptor ACE2 to mediate SARS-CoV-2 entry into host cells [3]. The protein S is produced as a precursor that contains almost 1,300 residues; then cleaved to two subunits; 1- a carboxyl (C)-terminal (S2 subunit) region and an amino (N)-terminal (S1 subunit) region. A trimer spike is synthesized and exposed on the viral envelope by assembling three S1/S2 heterodimers. The S1 subunit contains a receptor-binding domain which mediates the viral entry into the host cells through binding to the host receptor. Also, the S2 subunit contains two heptad repeat areas that participate in the fusion process [4].

Angiotensin-converting enzyme 2 (ACE2) is one of the important receptors that can bind to SARS-CoV; while, dipeptidyl peptidase-4 binds to MERS-CoV. ACE2 is an enzyme with 805 residues that is expressed on the surface of the cell membrane of several tissues (lungs, arteries, heart, kidney, and intestine) and interacts with the spike glycoprotein in some coronaviruses, including HCoV-NL63, SARS-CoV, and SARS-CoV-2 [5,6]. The binding affinity of ACE2-spike is crucial for SARS-CoV-2 infection efficiency and completely dependent to the structure and interaction pattern of spike glycoprotein form SARS-CoV-2. Since the binding domain from each structure is available, so it is possible to measure the affinity of the whole complex [7]. 6LXT is a 6 chain structure attaches the virion to the cell membrane by interacting with host receptor, initiating the infection. Binding to human ACE2 receptor and internalization of the virus into the endosomes of the host cell induces conformational changes in the Spike glycoprotein.

New viral components are produced and released following viral RNA replication and translation. Several medications have been reported so far, but none of them are effective against COVID-19. Azithromycin, Bevacizumab, Chloroquine, Hydroxychloroquine, and Lopinavir are some of the most commonly prescribed antibiotics [8,9,10.11,12]. These are in clinical studies, however with the rising public interest in widely available medications, conducting solid clinical trials is allegedly more challenging [13]. A combination of drugs could be more effective; for example, a combination of antitussive noscapine and hydroxychloroquine showed a strong binding affinity to SARSCoV- 2 Mpro [14].

Molecular docking is becoming a more important method in drug development [15]. This is a straightforward and sensible strategy to drug development that benefits from low-cost and efficient screening. The molecular docking technique was utilized to

simulate the interaction between FDA approved antibiotics, antiviral, antifungal and antiparasitic drugs (ligands) and a protein at the atomic level, allowing the analysis of the behavior of small molecules with regard to the target protein. As a result, *in silico* absorption, distribution, metabolism, excretion and toxicity (ADMET) studies were reported to be important in the early stages of drug discovery and development [16]. The earliest ADMET filters involve simple rules of thumb derived from the distribution analysis of physicochemical properties of drugs having or lacking the desired behavior [17]. The "Lipinski's rule of five" is a mnemonic tool used for the rapid assessment of compounds during the drug discovery and optimization process [18]. The analysis of ADMET profiles of the listed ligands is crucial for their clinical as well as the commercial success as potential SARS-CoV-2 drugs [19]. This study, therefore, focuses on an in-silico approach toward assessing and repurposing the effect of FDA approved antimalarial, antiviral, antifungal and antiparasitic drugs in association with the SARS-CoV-2 protein bound to angiotensin converting enzyme 2 (ACE2) (6M0J) to identify potent drug for the treatment of the COVID-19 infection.

2. Material and Methods

2.1. Ligand and Protein Structure

The electron microscopy structure of the SARS-CoV-2 Spike protein (Protein Data Bank [PDB] ID: 6LXT) are obtained from the Research Collaboratory for Structural Bioinformatics (RCSB) Protein Data Bank. BIOVA Discovery Studio software was used to remove water and tiny compounds from crystal structures. Following that, the Kollman and Gasteiger method was used to connect the polar hydrogen atoms and process the atomic charges. Finally, the pdbqt format was used to save the 3D structure of proteins.

2.2. Molecular Docking

The multiple docking of the ligands and proteins were done with Autodock Vina integrated in the PyRx software. PyRx is a Virtual Screening software for computer-based drug discovery that can be used to screen libraries of compounds against potential drug targets. Not all docking softwares can be used for multiple docking as PyRx (https://pyrx.sourceforge.io/). Likewise, the integrated autodock vina is much faster and efficient in optimization and multithreading. It calculates the grid charges internally and set up the docking space. After the preparation of both the ligands and the targets, the docking was run with Vina Wizard. The residues on active site of the target protein were selected to set grid box and vina was run to complete the process of docking.

2.3. Prediction of Lipinski's Rule of Five Properties

Lipinski's rule of five is a rule of thumb that helps with distinguishing between drug-like and non-drug molecules. This was analyzed using scfbio-iitd tool (http://www.scfbioiitd.res.in/software/drugdesign/lipinski.jsp#anchortag).

2.4. ADMET Properties

The purpose of calculating ADMET profiles is to supply, with reasonable accuracy, a preliminary prediction of the *in vivo* behavior of a compound to assess its potential to become a drug. The molecules used in this study will be submitted to the calculation of their absorption, distribution, metabolism, excretion and toxicological properties (ADMET). Also, the physicochemical properties such as molecular hydrogen bond acceptor (HBA), hydrogen bond donor (HBD), molecular weight (MW), topological polar surface area (TPSA), rotatable bond count (RB) and octanol/water partition coefficient (LogP) were calculated using Swiss ADMET web tool.

3. Result and Discussion

Understanding the virus-receptor recognition mechanism that causes COVID-19 infection, pathogenesis, and host range may help researchers create antiviral treatment to battle and cure the worldwide pandemic. Therefore, repurposing drugs available for other diseases would be beneficial as these can be directly tested as anti-SARS-CoV-2 drugs and can be processed for COVID-19 trials. Molecular docking studies were performed to gain insights into the binding mode and crucial molecular interactions of the selected ligands with spike protein. For docking analysis, ten poses per ligand were collected, and possible receptor ligand interactions were assessed using the XP-visualizer. To find the best pose, the interactions, Glide Scores (GScore), and Docking scores of the resulting receptor–ligand complexes were thoroughly analysed. GScore is a scoring function that assesses ligand binding free energy (20). More negative numbers indicate that the drug–protein interaction is more favourable. As a result, compounds with a -6 or lower score were thought to be better candidates for inhibiting SARS-CoV-2 (21).

We have chosen thirty-four popular antifungal, anti-malarial, anti-protozoan, antiviral compounds as a ligand to the COVID-19 spike glycoprotein (6LXT). Out of the tested 34 antifungal, anti-malarial, anti-protozoan, antiviral compounds, docking studies revealed that antiviral compounds have the highest interaction followed by anti-malarial, anti-protozoan, antifungal compounds as shown in Table 1-4.

Among the antiviral drug, Bictegravir (-8.7) and Dolutegravir (8.0), shows the highest binding affinity and Adefovirdipivoxil (-4.5) showed the lowest binding affinity of SARS-CoV-2–ACE2 complex (6LXT). Antimalarial drug such as Piperaquine (-8.0) and Mefloquine (-7.5), shows the highest binding affinity and Chloroquine (-5.2) showed the lowest binding affinity. Docking interaction of antiprotozoal compound such Atovaquone (-8.2) and Emetine (-8.0), shows the highest binding affinity and Eflornithine (-4.5) showed the lowest binding affinity. The antifungal compound with the highest binding energy against ketoconazole and Itraconazole (-7.1), while the lowest binding energy against Voriconazole (-4.0). The results obtained from the antifungal compound showed the least interaction as compared to other compounds. Abd El-Mordy et al. found that phenolic substances such as rutin, myricitrin, mearnsitrin, and quercetin 3-Ob-D-glucoside interact strongly with SARS-

CoV-2 protease, with binding energies of −8.2, −7.1, −7.5 and −7.6, respectively [22]. The results observed from docking the selected ligands to the viral protein expressed good docking scores to bind the active site pocket suggesting those drugs as potent inhibitors to the main spike protein of the virus active site as shown in Fig. 1-4.

The Drug Likeness of the medications is determined by the prediction of the Absorption, Distribution, Metabolism, Excretion, and Toxicity (ADMET) properties, which revealed the pharmacokinetics of the chosen compounds. According to Lipinski's rule of five, it is determined by a complex balance of various molecular properties and structural features such as lipophilicity, electronic distribution, hydrogen bonding characteristics, molecule size and flexibility, and presence of various pharmacophoric features which in turn influence the behavior of a molecule in a living organism. A good drug candidate should not violate more than one of the rules [23]. All the values obtained from the ADMET analysis for our studied compounds are presented in Table 5-8. The results of this study showed that 82% compounds have below ¡500 molecular weight. Water solubility has been deemed vital in research to approximate the absorption of drugs in the body. [24, 25].

Hydrogen bonding analysis is employed to know more insights into the molecular recognition, molecular interactions, and selectivity of the substances being examined within the receptors. The number of hydrogen bond acceptors are donors in this study exhibited good interaction. The PSA is determined using a fragmental method known as topological polar surface area (TPSA), which considers sulphur and phosphorus as polar atoms 25. More than 94% of the drugs showed docking score against SARS-coronavirus, which shadowed physicochemical descriptors of ADME properties. Furthermore, the blood/ brain partition coefficient (log BB) of the top compounds suggested a lower possibility for bridging the blood–brain barrier (BBB). Furthermore, all of the molecules examined have a high GI absorption rate and optimal solubility. All the drugs are non-substrates and non-inhibitors of CYP2C9 and CYP2D6 respectively. All compounds in our study complied with Lipinski's rules of five. More than 85% of examined compounds satisfied Lipinski's rules of five. With regard to Tables 2 and 3, bioavailability and ADMET (Rule of Five, Veber, Ghose, Muegge & Egan) are in the reasonable ranges for the selected drugs in solubility and lipophilicity. Our results show that all of the selected molecules displayed good ADMET properties.

This investigation identified over 100 compounds that require additional research to determine their efficacy against SARS CoV-2 because their binding energies were lower than those of several of the medications now used to treat Covid-19. Highly rated among these ligands are Bictegravir, Atovaquone, Dolutegravir, Piperaquine and Emetine. As a result, the medications projected to be superior than the reference treatments, which are currently used to treat COVID 19, must be validated. As a consequence, more study on the top five drugs, Bictegravir, Atovaquone, Dolutegravir, Piperaquine and Emetine, is expected, and this should be done as soon as possible through in vitro, in vivo, and clinical investigations.

4. Conclusion

COVID 19 is a rapidly spreading infectious respiratory disease that has emerged as a serious threat to global health. Considering the time required to develop a new FDA approved drug, drug repurposing seems the most appealing, safe and straightforward approach. Using a virtual screening strategy and computational chemistry tools, FDA approved medicines and their structural analogues in clinical trials were examined for inhibitory effects towards the COVID 19 protein (6LXT). This study has predicted five drugs Bictegravir, Atovaquone, Dolutegravir, Piperaquine and Emetine which displayed better binding energy and pharmacokinetic properties than the other drugs which are currently investigated for the treatment of COVID-19. Therefore, it is concluded that these five top scoring compounds may act as lead compounds for further experimental validation, clinical trials and for the development of more potent antiviral agents and ascertain their effectiveness against the SARS-CoV-2.

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Table 1: Molecular Docking Analysis of Studied Compounds and some Antifungal Drugs against the Spike Protein (PDB: 6LXT) of SARS-CoV-2

Ligand	Binding Affinity	rmsd/ub	rmsd/lb
Fluconazole	−5.8	0	0
Itraconazole	−7.1	0	0
Sporanox	−6.1	0	0
Voriconazole	−4	0	0
Flucytosine	−5.9	0	0
Ketoconazole	−7.1	0	0

Table 2: Molecular Docking Analysis of Studied Compounds and some Antimalarial Drugs against the Spike Protein (PDB: 6LXT) of SARS-CoV-2

Ligand	Binding Affinity	rmsd/ub	rmsd/lb
Amodiaquine	−6.8	0	0
Mefloquine	−7.5	0	0
Quinine	−7.1	0	0
Primaquine	−5.7	0	0
Halofantrine	−6.9	0	0
Lumefantrine	−6.9	0	0
Chloroquine	−5.2	0	0
Piperaquine	−8	0	0
Sulfadoxine	−6.5	0	0

Table 3: Molecular Docking Analysis of Studied Compounds and some Antiprotozoal Drugs against the Spike Protein (PDB: 6LXT) of SARS-CoV-2

Ligand	Binding Affinity	rmsd/ub	rmsd/lb
Metronidazole	−4.9	0	0
Atovaquone	−8.2	0	0
Benznidazole	−6.5	0	0
Dehydroemetine	−7.9	0	0
Eflornithine	−4.3	0	0
Emetine	−8	0	0
Fenbendazole	−6.9	0	0
Iodoquinol	−5.7	0	0
Nitazoxanide	−6.7	0	0
Nifurtimox	−6.6	0	0

Table 4: Molecular Docking Analysis of Studied Compounds and some Antiviral Drugs against the Spike Protein (PDB: 6LXT) of SARS-CoV-2

Ligand	Binding Affinity	rmsd/ub	rmsd/lb
Rupintrivir	−7.2	0	0
Sofosbuvir	−7.4	0	0
Adefovirdipivoxil	−4.5	0	0
Famciclovir	−5.6	0	0
Tecovirimat	−7.8	0	0
Dolutegravir	−8	0	0
Entecavir	−6.4	0	0
Bictegravir	−8.7	0	0

Table 5: Absorption, Distribution, Metabolism, Excretion and Toxicity (ADMET) Analyses of the selected Antifungal Drugs against the Spike Protein (PDB: 6LXT) of SARS-CoV-2

Molecule	Itraconazole	Sporanox	Voriconazole	Flucytosine	Clotrimazole	Ketoconazole
Formula	C13H12F2N6O	C35H38Cl2N8O4	C35H38Cl2N8O4	C4H4FN3O	C22H17ClN2	C26H28Cl2N4O4
MW	306.27	705.63	705.63	129.09	344.84	531.43
#Heavy atoms	22	49	49	9	25	36
#Aromatic heavy atoms	16	28	28	6	23	17
Fraction Csp3	0.23	0.37	0.37	0	0.05	0.38
#Rotatable bonds	5	11	11	0	4	8
#H-bond acceptors	7	7	7	3	1	5
#H-bond donors	1	0	0	2	0	0
MR	70.71	194.53	194.53	29.22	101.84	144.44
TPSA	81.65	104.7	104.7	71.77	17.82	69.06
iLOGP	0.41	5.26	5.26	0.39	3.07	3.96
XLOGP3	0.35	5.66	5.66	−0.87	5.41	4.34
WLOGP	1.47	4.71	4.71	−0.08	5.38	3.34
MLOGP	1.47	4.21	4.21	−0.37	4.38	2.47
Silicos-IT Log P	0.71	3.7	3.7	0.92	4.98	3.69
Consensus Log P	0.88	4.71	4.71	0	4.64	3.56
ESOL Log S	−2.17	−7.48	−7.48	−0.59	−5.8	−5.69
ESOL Solubility (mg/ml)	2.08E+00	2.35E-05	2.35E-05	3.35E+01	5.43E-04	1.08E-03
ESOL Solubility (mol/l)	6.80E-03	3.33E-08	3.33E-08	2.60E-01	1.57E-06	2.04E-06
ESOL Class	Soluble	Poorly soluble	Poorly soluble	Very soluble	Moderately soluble	Moderately soluble
Ali Log S	−1.63	−7.62	−7.62	−0.16	−5.54	−5.51
Ali Solubility (mg/ml)	7.20E+00	1.68E-05	1.68E-05	9.02E+01	9.96E-04	1.66E-03

Ali Solubility (mol/l)	2.35E-02	2.38E-08	2.38E-08	6.99E-01	2.89E-06	3.13E-06
Ali Class	Very soluble	Poorly soluble	Poorly soluble	Very soluble	Moderately soluble	Moderately soluble
Silicos-IT LogSw	−3.54	−9.24	−9.24	−1.44	−8.59	−7.2
Silicos-IT Solubility (mg/ml)	8.83E-02	4.10E-07	4.10E-07	4.67E+00	8.88E-07	3.37E-05
Silicos-IT Solubility (mol/l)	2.88E-04	5.81E-10	5.81E-10	3.61E-02	2.58E-09	6.34E-08
Silicos-IT class	Soluble	Poorly soluble	Poorly soluble	Soluble	Poorly soluble	Poorly soluble
GI absorption	High	High	High	High	High	High
BBB permeant	No	No	No	No	Yes	Yes
Pgp substrate	No	Yes	Yes	No	Yes	No
CYP1A2 inhibitor	No	Yes	Yes	No	Yes	No
CYP2C19 inhibitor	Yes	Yes	Yes	No	Yes	Yes
CYP2C9 inhibitor	No	Yes	Yes	No	Yes	Yes
CYP2D6 inhibitor	No	Yes	Yes	No	Yes	Yes
CYP3A4 inhibitor	No	Yes	Yes	No	Yes	Yes
log Kp (cm/s)	−7.92	−6.59	−6.59	−7.71	−4.56	−6.46
Lipinski #violations	0	3	3	0	1	1
Ghose #violations	0	3	3	3	0	2
Veber #violations	0	1	1	0	0	0
Egan #violations	0	0	0	0	0	0
Muegge #violations	0	2	2	2	1	0
Bioavailability Score	0.55	0.17	0.17	0.55	0.55	0.55
PAINS #alerts	0	2	2	0	0	1
Brenk #alerts	0	0	0	0	0	0
Leadlikeness #violations	0	3	3	1	1	3
Synthetic Accessibility	2.91	5.77	5.77	1.64	2.7	4.45

Table 6: Absorption, Distribution, Metabolism, Excretion and Toxicity (ADMET) Analyses of the selected Antimalarial Drugs against the Spike Protein (PDB: 6LXT) of SARS-CoV-2

Molecule	**Amodiaquine**	**Mefloquine**	**Quinine**	**Primaquine**	**Halofantrine**	**Lumefantrine**	**Chloroquine**	**Piperaquine**	**Sulfadoxine**	**Atovaquone**
Formula	C20H22ClN3O	C17H16F6N2O	C20H24N2O2	C15H21N3O	C26H30Cl2F3NO	C30H32Cl3NO	C18H26ClN3	C29H32Cl2N6	C12H14N4O4S	C12H14N4O4S
MW	355.86	378.31	324.42	259.35	500.42	528.94	319.87	535.51	310.33	310.33
#Heavy atoms	25	26	24	19	33	35	22	37	21	21
#Aromatic heavy atoms	16	10	10	10	14	18	10	20	12	12
Fraction Csp3	0.25	0.47	0.45	0.4	0.46	0.33	0.5	0.38	0.17	0.17
#Rotatable bonds	6	4	4	6	11	10	8	6	5	5
#H-bond acceptors	3	9	4	3	5	2	2	4	6	6
#H-bond donors	2	2	1	2	1	1	1	0	2	2
MR	105.41	86.51	99.73	79.48	133.57	152.61	97.41	168.72	76.53	76.53
TPSA	48.39	45.15	45.59	60.17	23.47	23.47	28.16	38.74	124.81	124.81
iLOGP	3.65	2.78	3.36	2.77	4.96	5.93	3.95	4.84	1.86	1.86
XLOGP3	5.18	3.62	2.88	2.23	8.61	8.72	4.63	5.64	0.7	0.7
WLOGP	5.03	6.05	2.47	2.59	9.47	8.72	4.62	3.9	1.77	1.77
MLOGP	3.1	3.43	2.23	1.39	6.48	6.6	3.2	3.58	−0.67	−0.67
Silicos-IT Log P	4.12	4.8	3.11	2.55	8.78	9.56	4.32	4.8	−0.2	−0.2
Consensus Log P	4.22	4.13	2.81	2.31	7.66	7.91	4.15	4.55	0.69	0.69
ESOL Log S	−5.39	−4.49	−3.71	−2.85	−7.95	−8.33	−4.55	−6.72	−2.3	−2.3

ESOL Solubility (mg/ml)	1.46E-03	1.23E-02	6.32E-02	3.69E-01	5.55E-06	2.45E-06	9.05E-03	1.03E-04	1.56E+00	1.56E+00
ESOL Solubility (mol/l)	4.10E-06	3.26E-05	1.95E-04	1.42E-03	1.11E-08	4.64E-09	2.83E-05	1.92E-07	5.04E-03	5.04E-03
ESOL Class	Moderately soluble	Moderately soluble	Soluble	Soluble	Poorly soluble	Poorly soluble	Moderately soluble	Poorly soluble	Soluble	Soluble
Ali Log S	−5.94	−4.26	−3.5	−3.13	−8.98	−9.09	−4.95	−6.22	−2.9	−2.9
Ali Solubility (mg/ml)	4.06E-04	2.10E-02	1.03E-01	1.93E-01	5.26E-07	4.27E-07	3.61E-03	3.25E-04	3.92E-01	3.92E-01
Ali Solubility (mol/l)	1.14E-06	5.55E-05	3.18E-04	7.43E-04	1.05E-09	8.08E-10	1.13E-05	6.06E-07	1.26E-03	1.26E-03
Ali Class	Moderately soluble	Moderately soluble	Soluble	Soluble	Poorly soluble	Poorly soluble	Moderately soluble	Poorly soluble	Soluble	Soluble
Silicos-IT LogSw	−7.59	−6.1	−4.31	−5.17	−10.51	−11.79	−6.92	−9.31	−4.03	−4.03
Silicos-IT Solubility (mg/ml)	9.11E-06	3.03E-04	1.60E-02	1.74E-03	1.54E-08	8.64E-10	3.86E-05	2.62E-07	2.91E-02	2.91E-02
Silicos-IT Solubility (mol/l)	2.56E-08	8.02E-07	4.92E-05	6.70E-06	3.08E-11	1.63E-12	1.21E-07	4.90E-10	9.38E-05	9.38E-05
Silicos-IT class	Poorly soluble	Poorly soluble	Moderately soluble	Moderately soluble	Insoluble	Insoluble	Poorly soluble	Poorly soluble	Moderately soluble	Moderately soluble
GI absorption	High	High	High	High	Low	Low	High	High	High	High
BBB permeant	Yes	No	Yes	Yes	No	No	Yes	Yes	No	No
Pgp substrate	No	Yes	No	Yes	Yes	Yes	No	Yes	No	No

CYP1A2 inhibitor	Yes	No	No	Yes	No	No	Yes	No	No	No
CYP2C19 inhibitor	Yes	No	No	No	No	No	No	Yes	No	No
CYP2C9 inhibitor	No	No	No	No	No	No	No	No	No	No
CYP2D6 inhibitor	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No
CYP3A4 inhibitor	Yes	Yes	No	No	Yes	No	Yes	No	No	No
log Kp (cm/s)	−4.79	−6.04	−6.23	−6.3	−3.24	−3.34	−4.96	−5.56	−7.7	−7.7
Lipinski #violations	0	0	0	0	2	2	0	1	0	0
Ghose #violations	0	1	0	0	3	3	0	2	0	0
Veber #violations	0	0	0	0	1	0	0	0	0	0
Egan #violations	0	1	0	0	1	1	0	0	0	0
Muegge #violations	1	0	0	0	1	1	0	1	0	0
Bioavailability Score	0.55	0.55	0.55	0.55	0.17	0.17	0.55	0.55	0.55	0.55
PAINS #alerts	1	0	0	0	0	0	0	0	0	0
Brenk #alerts	1	0	1	0	1	1	0	0	1	1
Leadlikeness #violations	2	2	0	0	3	3	2	2	0	0
Synthetic Accessibility	2.6	3.25	4.34	2.68	3.42	4.52	2.76	3.29	2.99	2.99

Table 7: Absorption, Distribution, Metabolism, Excretion and Toxicity (ADMET) Analyses of the selected Antiprotozoal Drugs against the Spike Protein (PDB: 6LXT) of SARS-CoV-2

Molecule	**Metronidazole**	**Atovaquone**	**Benznidazole**	**Dehydroemetine**	**Eflornithine**	**Emetine**	**Fenbendazole**	**Iodoquinol**	**Nitazoxanide**	**Nifurtimox**
Formula	C6H9N3O3	C22H19ClO3	C12H12N4O3	C29H38N2O4	C6H12F2N2O2	C29H40N2O4	C15H13N3O2S	C9H5I2NO	C12H9N3O5S	C10H13N3O5S
MW	171.15	366.84	260.25	478.62	182.17	480.64	299.35	396.95	307.28	287.29
#Heavy atoms	12	26	19	35	12	35	21	13	21	19
#Aromatic heavy atoms	5	12	11	12	0	12	15	10	11	5
Fraction Csp3	0.5	0.27	0.17	0.52	0.83	0.59	0.07	0	0.08	0.5
#Rotatable bonds	3	2	6	7	5	7	5	0	6	3
#H-bond acceptors	4	3	4	6	6	6	3	2	6	6
#H-bond donors	1	1	1	1	3	1	2	1	1	0
MR	43.25	102.61	69.41	146.58	38.28	147.05	82.45	69.2	76.65	73.88
TPSA	83.87	54.37	92.74	52.19	89.34	52.19	92.31	33.12	142.35	117.08
iLOGP	1.16	2.98	1.15	4.68	0.53	4.67	1.81	2.24	0.9	1.3
XLOGP3	−0.02	5.23	0.91	3.7	−2.92	4.74	3.47	3.92	2.04	1.26
WLOGP	0.09	5.35	0.96	3.6	0.61	3.53	3.7	3.15	2.04	1.34
MLOGP	−0.78	3.28	0.37	2.96	−2.32	3.04	2.85	2.87	0.31	−0.1
Silicos-IT Log P	−1.62	5.19	−0.91	5.37	−0.25	4.98	2.73	3.86	0.49	−1.12
Consensus Log P	−0.23	4.41	0.49	4.06	−0.87	4.19	2.91	3.21	1.16	0.54
ESOL Log S	−1	−5.62	−2.06	−4.93	1.2	−5.6	−4.08	−5.34	−3.02	−2.41

ESOL Solubility (mg/ml)	1.72E+01	8.82E-04	2.27E+00	5.62E-03	2.89E+03	1.21E-03	2.49E-02	1.81E-03	2.92E-01	1.11E+00
ESOL Solubility (mol/l)	1.00E-01	2.41E-06	8.72E-03	1.17E-05	1.59E+01	2.52E-06	8.31E-05	4.57E-06	9.51E-04	3.87E-03
ESOL Class	Very soluble	Moderately soluble	Soluble	Moderately soluble	Highly soluble	Moderately soluble	Moderately soluble	Moderately soluble	Soluble	Soluble
Ali Log S	−1.29	−6.12	−2.44	−4.49	1.6	−5.57	−5.09	−4.31	−4.66	−3.32
Ali Solubility (mg/ml)	8.74E+00	2.78E-04	9.38E-01	1.56E-02	7.30E+03	1.31E-03	2.43E-03	1.92E-02	6.76E-03	1.38E-01
Ali Solubility (mol/l)	5.11E-02	7.58E-07	3.61E-03	3.26E-05	4.01E+01	2.72E-06	8.12E-06	4.85E-05	2.20E-05	4.82E-04
Ali Class	Very soluble	Poorly soluble	Soluble	Moderately soluble	Highly soluble	Moderately soluble	Moderately soluble	Moderately soluble	Moderately soluble	Soluble
Silicos-IT LogSw	−0.41	−6.97	−3.12	−8.13	−0.41	−7.7	−5.66	−4.95	−3.25	−1.61
Silicos-IT Solubility (mg/ml)	6.69E+01	3.97E-05	1.97E-01	3.52E-06	7.15E+01	9.59E-06	6.61E-04	4.45E-03	1.71E-01	7.10E+00
Silicos-IT Solubility (mol/l)	3.91E-01	1.08E-07	7.57E-04	7.36E-09	3.93E-01	2.00E-08	2.21E-06	1.12E-05	5.57E-04	2.47E-02
Silicos-IT class	Soluble	Poorly soluble	Soluble	Poorly soluble	Soluble	Poorly soluble	Moderately soluble	Moderately soluble	Soluble	Soluble
GI absorption	High	High	High	High	High	High	High	High	Low	High
BBB permeant	No	Yes	No	Yes	No	Yes	No	Yes	No	No
Pgp substrate	No	No	No	Yes	No	Yes	No	No	No	No

CYP1A2 inhibitor	No	Yes	No	No	No	No	Yes	Yes	No	No
CYP2C19 inhibitor	No	Yes	No	No	No	No	Yes	Yes	Yes	No
CYP2C9 inhibitor	No	Yes	No	No	No	No	Yes	Yes	No	No
CYP2D6 inhibitor	No	No	No	Yes	No	No	No	No	No	No
CYP3A4 inhibitor	No	Yes	No	Yes	No	No	No	No	No	No
log Kp (cm/s)	−7.36	−4.82	−7.24	−6.59	−9.48	−5.87	−5.66	−5.94	−6.73	−7.16
Lipinski #violations	0	0	0	0	0	0	0	0	0	0
Ghose #violations	0	0	0	2	1	3	0	1	0	0
Veber #violations	0	0	0	0	0	0	0	0	1	0
Egan #violations	0	0	0	0	0	0	0	0	1	0
Muegge #violations	1	1	0	0	2	0	0	0	0	0
Bioavailability Score	0.55	0.85	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55
PAINS #alerts	0	2	0	0	0	0	0	0	0	0
Brenk #alerts	2	1	2	1	0	0	0	1	3	3
Leadlikeness #violations	1	2	0	2	1	2	0	2	0	0
Synthetic Accessibility	2.3	4.07	2.16	5.12	2.17	4.87	2.39	2.04	2.83	3.86

Table 8: Absorption, Distribution, Metabolism, Excretion and Toxicity (ADMET) analyses of the selected Antiviral Drugs against the Spike Protein (PDB: 6LXT) of SARS-CoV-2

Molecule	**Darunavir**	**Amprenavir**	**Sofosbuvir**	**Adefovirdip-ivoxil**	**Famciclovir**	**Tecovirimat**	**Zidovudine**	**Dolutegravir**	**Entecavir**	**Bictegravir**
Formula	C27H37N3O7S	C25H35N3O6S	C22H29FN3O9P	C8H12ClNO	C14H19N5O4	C19H15F3N2O3	C10H13N5O4	C20H19F2N3O5	C12H15N5O3	C21H18F3N3O5
MW	547.66	505.63	529.45	173.64	321.33	376.33	267.24	419.38	277.28	449.38
#Heavy atoms	38	35	36	11	23	27	19	30	20	32
#Aromatic heavy atoms	12	12	12	6	9	6	6	12	9	12
Fraction Csp3	0.52	0.48	0.5	0.38	0.5	0.42	0.6	0.35	0.42	0.38
#Rotatable bonds	13	13	11	1	9	4	3	4	2	4
#H-bond acceptors	8	7	11	2	7	6	7	7	5	8
#H-bond donors	3	3	3	1	1	1	2	2	4	2
MR	142.2	133.62	125.53	47.96	82.02	90.79	61.73	104.48	72.39	107.13
TPSA	148.8	139.57	167.99	33.12	122.22	66.48	134.07	100.87	130.05	100.87
iLOGP	3.2	3.2	3.23	0	2.5	2.28	1.93	2.12	0.39	2.56
XLOGP3	2.94	2.89	0.99	2.53	−0.06	2.61	0.05	2.44	−1.12	2.73
WLOGP	3.46	3.49	1.75	2.46	0.55	3.17	−0.52	1.66	−0.82	2.36
MLOGP	1.18	1.14	0.82	1.14	−0.15	3.91	−1.25	1.05	−1.13	1.24
Silicos-IT Log P	1.46	1.78	0.61	2.09	0.53	1.82	−0.78	1.85	−0.42	2.23
Consensus Log P	2.45	2.5	1.48	1.64	0.68	2.76	−0.11	1.82	−0.62	2.22
ESOL Log S	−4.46	−4.19	−3.27	−2.85	−1.49	−3.72	−1.56	−4.01	−1.05	−4.36

ESOL Solubility (mg/ml)	1.88E-02	3.25E-02	2.86E-01	2.46E-01	1.04E+01	7.20E-02	7.29E+00	4.10E-02	2.45E+01	1.96E-02
ESOL Solubility (mol/l)	3.44E-05	6.44E-05	5.41E-04	1.42E-03	3.24E-02	1.91E-04	2.73E-02	9.79E-05	8.82E-02	4.37E-05
ESOL Class	Moderately soluble	Moderately soluble	Soluble	Soluble	Very soluble	Soluble	Very soluble	Moderately soluble	Very soluble	Moderately soluble
Ali Log S	−5.73	−5.48	−4.11	−2.87	−2.06	−3.66	−2.42	−4.2	−1.12	−4.5
Ali Solubility (mg/ml)	1.03E-03	1.67E-03	4.14E-02	2.33E-01	2.83E+00	8.32E-02	1.02E+00	2.64E-02	2.10E+01	1.41E-02
Ali Solubility (mol/l)	1.88E-06	3.30E-06	7.83E-05	1.34E-03	8.80E-03	2.21E-04	3.81E-03	6.29E-05	7.59E-02	3.14E-05
Ali Class	Moderately soluble	Moderately soluble	Moderately soluble	Soluble	Soluble	Soluble	Soluble	Moderately soluble	Very soluble	Moderately soluble
Silicos-IT LogSw	−5.33	−5.55	−4.09	−2.6	−2.78	−3.53	−1.16	−4.47	−1.23	−4.65
Silicos-IT Solubility (mg/ml)	2.55E-03	1.42E-03	4.32E-02	4.37E-01	5.32E-01	1.11E-01	1.84E+01	1.42E-02	1.65E+01	9.99E-03
Silicos-IT Solubility (mol/l)	4.66E-06	2.81E-06	8.17E-05	2.52E-03	1.66E-03	2.96E-04	6.87E-02	3.39E-05	5.94E-02	2.22E-05
Silicos-IT class	Moderately soluble	Moderately soluble	Moderately soluble	Soluble	Soluble	Soluble	Soluble	Moderately soluble	Soluble	Moderately soluble
GI absorption	Low	Low	Low	High	High	High	High	High	High	High
BBB permeant	No	No	No	Yes	No	Yes	No	No	No	No
Pgp substrate	Yes	Yes	Yes	No	Yes	No	No	Yes	No	No

CYP1A2 inhibitor	No	No	No	No	No	No	No	No	No	No
CYP2C19 inhibitor	No	No	No	No	No	Yes	No	No	No	No
CYP2C9 inhibitor	No	No	No	No	No	No	No	No	No	Yes
CYP2D6 inhibitor	No	Yes	No	No	No	No	No	No	No	Yes
CYP3A4 inhibitor	Yes	Yes	Yes	No	No	Yes	No	No	No	Yes
log Kp (cm/s)	−7.55	−7.33	−8.83	−5.56	−8.3	−6.74	−7.89	−7.13	−8.79	−7.1
Lipinski #violations	1	1	2	0	0	0	0	0	0	0
Ghose #violations	3	2	1	0	0	0	1	0	1	0
Veber #violations	2	1	2	0	0	0	0	0	0	0
Egan #violations	1	1	1	0	0	0	1	0	0	0
Muegge #violations	0	0	2	1	0	0	0	0	0	0
Bioavailability Score	0.55	0.55	0.17	0.55	0.55	0.55	0.56	0.55	0.55	0.55
PAINS #alerts	0	0	0	0	0	0	1	0	0	0
Brenk #alerts	1	1	1	0	1	2	3	0	1	0
Leadlikeness #violations	2	2	2	1	1	1	0	1	0	1
Synthetic Accessibility	5.67	4.98	6.02	1.56	2.43	4.17	3.93	4.16	3.53	4.87

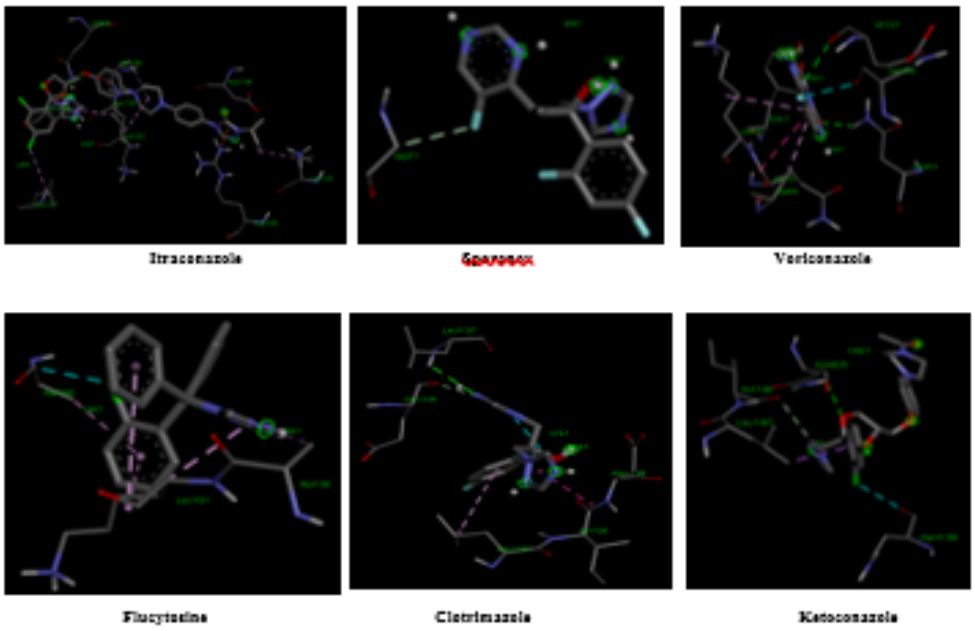

Figure 1: The Binding Interaction of Antifungal Drugs in the Active Site of 6LXT

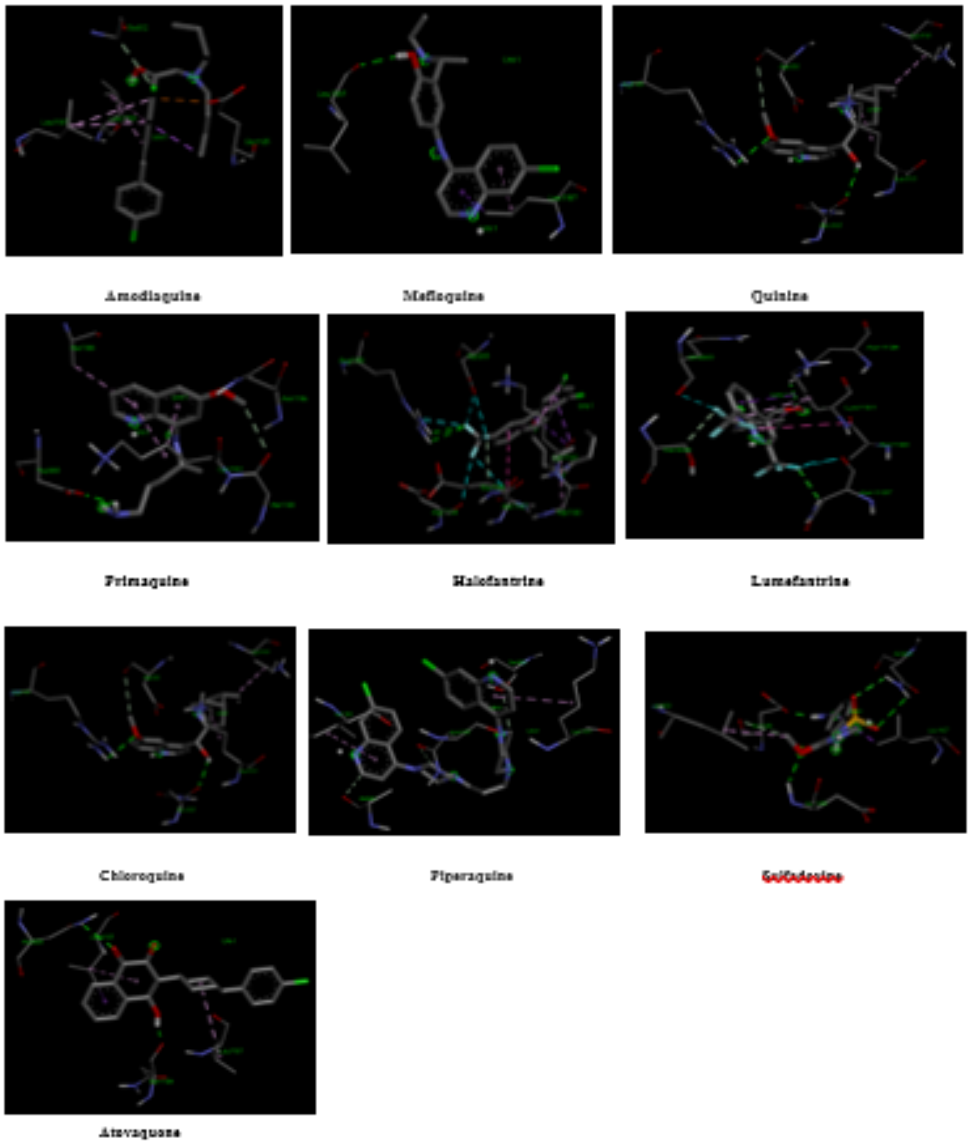

Figure 2: The Binding Interaction of Antimalarial Drugs in the Active Site of 6LXT

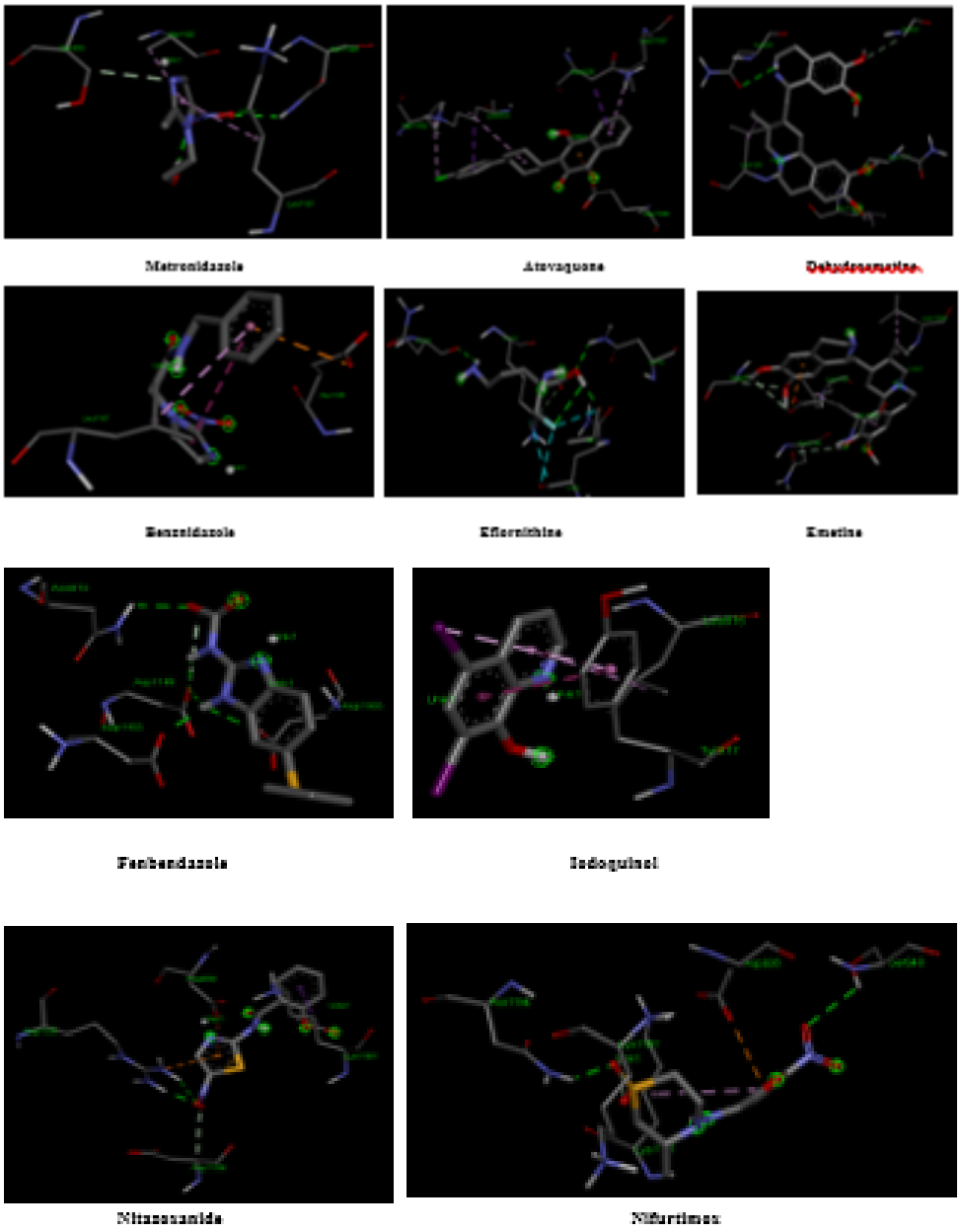

Figure 3: The Binding Interaction of Antiprotozoal Drugs in the Active Site of 6LXT

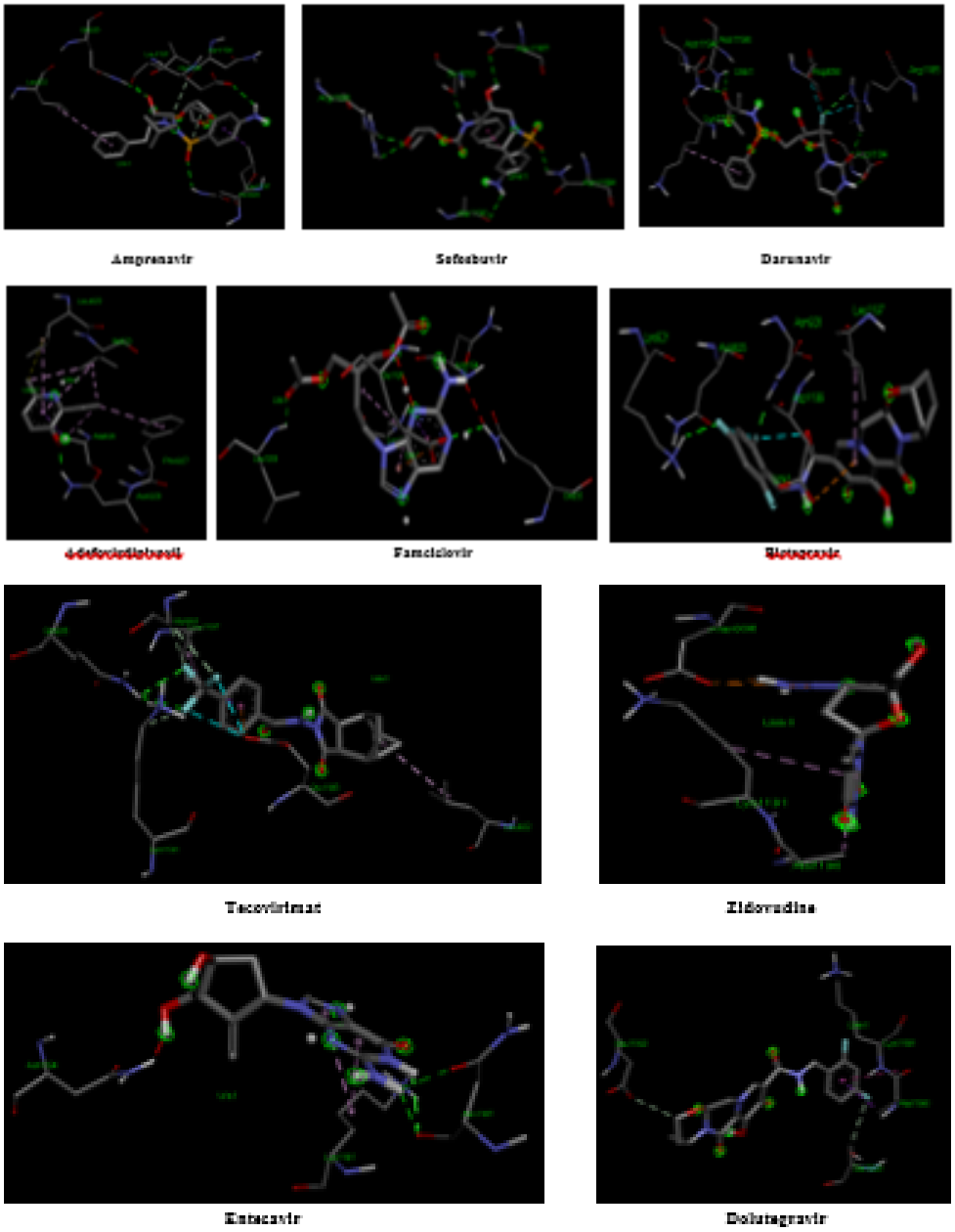

Figure 4: The Binding Interaction of Antiviral Drugs in the Active Site of 6LXT

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IDENTIFICATION OF ASCOMYCOTA FUNGI USING ITS REGION AS MOLECULAR MARKER

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Abstract

In this study Ascomycota fungi was isolated from the soil collected from cucumber field. The micromorphological characters of the isolated fungus were disorganizing with overlapping characters and identification was difficult. Hence, nuclear ribosomal internal transcribed spacer (ITS) region, the ideal molecular marker, accepted as DNA barcode for fungal identification was sequenced and compared with the existing GenBank, NCBI, database to identify the species. These sequences were deposited in GenBank, NCBI under the accession number MW940968 and MW945404. Based on the homology and phylogenetic analysis the isolated fungus was identified as *Aspergillus terrus*.

Keywords: Internal Transcribed Spacer (ITS) Region, DNA Barcode, Ascomycota Fungi, Aspergillus Terrus.

1. Introduction

Soil is an important region in the earth's crust, which gives shelter to many organisms from microbes to plants and animals. Diseases that arise from soil-borne pathogens are major problem in the reduction of cucumber yield (Wu et al., 2006). Fungi are an exceptionally flexible class of organisms constituted mainly of saprophytes, which flourish on dead biological matter. The identification of fungi depends mainly on their external and biological characters. Conversely, the distinctive characteristics of fungi make snags in identification and cataloging based on morphology. Accordingly, lone well-trained specialists are able to properly identify fungi species based on its morphology (Samson et al, 2010).

Currently, numerous molecular methods have been developed over the years for the which includes techniques of modern technologies like fluorescent in situ hybridization (FISH), denaturing gradient gel electrophoresis (DGGE), terminal length restriction

fragment polymorphism (T-RFLP), DNA array hybridization and DNA sequencing, pulsed field gel electrophoresis (PFGE) etc. The method based on DNA sequencing is the most widely used ones (Ali et al, 2010). This novel concept for the hasty and exact documentation of an unknown fungal specimen using ITS as molecular marker is called DNA barcoding. Hence in the present study two fungi isolated from soil were identified through sequencing the ITS region.

2. Materials and Methods

2.1. Soil Collection and Serial Dilution

Soil samples were collected from the agricultural field of Tirupatthur district in Tamil Nadu. Soil was collected in a pre-sterilized beaker using a pre sterilized spatula and transferred to laboratory. 1 gm of the soil sample was taken in 10 ml of Ringer's solution and were serially diluted to 10^{-10} dilutions.

2.2. Fungal Culture

0.1 ml of the serially diluted sample was spread on petridishes containing potato dextrose agar (PDA) under aseptic condition at 28°C and allowed to grow for 7 days at 28°C in dark.

2.3. Morphological Examinations

Colony morphology of the 7 days grown fungus on PDA was examined with a Leica DM14000B (Germany) microscope equipped with a camera.

2.4. DNA Extraction

Fungal strain grown for 7 days in potato dextrose agar (PDA) at 28°C in dark was used for DNA extraction. DNA extraction was performed using Hi PurA^{TM} Fungal DNA Mini Kit (Himedia) following the manufacturer's protocol. The extracted DNA was subjected to concentration and purity using Nanodrop Lite UV-Vis Spectrophotometer.

2.5. Amplification, Sequencing and Phylogenetic Analysis

Amplification of ITS region of the ribosomal RNA gene cluster was performed using the method of White et al, 1990. Sequencing of the amplicons was performed in a 3500 Genetic Analyzer (Thermo Fisher Scientific, Waltham, USA) in the sequencing Centre. The obtained sequence was aligned using Bioedit tool. And phylogenetic analysis was performed using MEGA 5 Software. The good quality sequence was deposited in the GenBank, NCBI under the accession number given in the result section.

3. Results and Discussion

3.1. Morphological Identification

Aspergillus terrus

```
Eukaryota; Fungi
Dikarya; Ascomycota
Pezizomycotina; Eurotiomycetes;
Eurotiomycetidae; Eurotiales; Aspergillaceae
             Aspergillus; Aspergillus
Genus; Aspergillus
Species; terrus
```

The morphological characteristics of *A. terreus* were distinctive, with its external appearance dark to coffee colonies, pyriform vesicles, biseriate conidial heads and conidial were in columnar arrangement. *A. terreus* produced a good sporulation on PDA. Following the Keys of Klich (2002) the microscopic and macroscopic characteristics were observed and studied which were in accordance with the descriptions of *A. terreus*.

3.2. Molecular Identification

Table 1: Homology Searches Result

Study Sequence	Most similar Sequence	E-Value	Query Coverage	% identity	Accession Number
ICBTS1	*Aspergillus terrus*	0.0	100%	100%	MT558939
ICBTSS4	*Aspergillus terrus*	0.0	100%	100%	MT558939

Table 2: Genetic Distance Result

Fungal sequences	Positions	
	1	2
Aspergillus terrus ICBTSS4		
Aspergillus terrus ICBTS1	0.009	

Four fungal strains were isolated from the soil collected from cucumber field. The micromorphological characters were studied (Fig. 1 & Fig. 2) and compared with

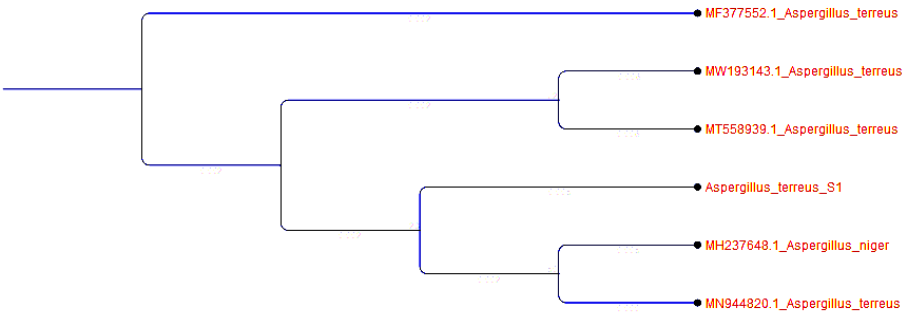

Figure 1: Phylogenetic Analysis of Study Sample S1

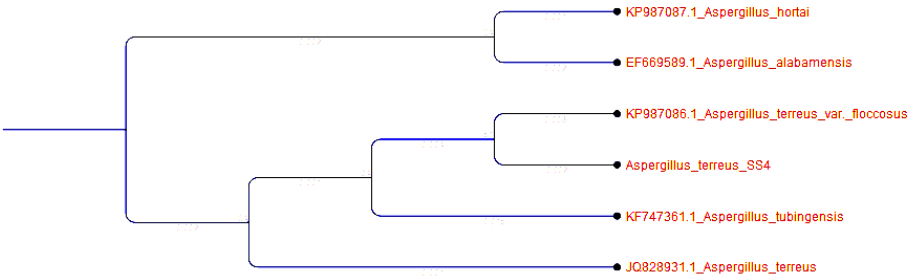

Figure 2: Phylogenetic Analysis of Study Sample SS4

the standard fungal taxonomical keys for identification. Morphological identification of *A. terreus* isolates were based on colony appearance and its colour, like coffee with arrangement of biseriate conidial heads in columnar shape. Despite *A. terreus* isolated from different substrates but its identification was based on its macroscopic and microscopic characteristics (Afzal et al. 2013) which produced similar morphological characteristics. Identification of *A. terreus* isolates through molecular markers substantiated with identification of other *A. terreus* isolates using morphological thereby exhibiting 100% similarity with *A. terreus* NRRL255 a reference strain. Phylogenetic study revealed all the isolates were clustered in the one clade. Further the efficiency of molecular marker can be proven with the phylogenetic studies of other researchers (Varga et al, 2005 and Samson et al, 2011) where their findings showed *A. terreus* into a distinct clade, alienated from other Aspergillus species.

The fungus labelled as S1 was identified to be *Aspergillus terrus*, after combined analysis of morphological characters and ITS sequence similarity results. Sample SS4 displayed differing characters in the external (phenotypical) appearance, but on morphological examination and ITS sequencing it matched *Aspergillus terrus*, confirming its exact identity. Both these sequences were deposited in GenBank, NCBI under the accession numbers, MW940968 and MW945404.

4. Conclusion

Several molecular methods have been developed in recent times for the identification of microbes including fungi. The most effective method proven was DNA barcoding due to its accuracy and swiftness. In case of fungi, the nuclear ribosomal Internal transcribed (ITS) region has been the most widely sequenced to identify fungi at the species level. Hence in the current study the ITS region proved as an effective molecular marker in the identification of the soil – isolated fungi.

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THEORETICAL STUDY OF IMIDAZOLE AND ITS DERIVATIVES FOR ENHANCED REACTIVITY ANALYSIS

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Abstract

Miconazolewas first synthesized is 1969, Miconazole ($C_{18}H_{14}Cl_4N_2O$) is an azole derivative (imidazole) and is an antifungal drug used in the treatment of superficial infections. It is also widely used to treat mucosal yeast infections and candidiasis [1]. The mechanism of inhibition activity has three main aspects: through inhibition of CY-P450, inhibition of peroxidase and catalase and Inhibition of 14-α-lansterol demethylation. Our study is based on Semi Empirical method where derivatives of Miconazole (e.g. Methyl, nitro, ethyl, propyl, etc.) are optimised and reactivity evaluated. Correlation Studies between Miconazole and their derivatives with molecular descriptors proved to be useful in enhancing the drug activity and reducing a few common side effects.

Keywords: Miconazole, MOPAC, Antifungal Activity, Semi Empirical Methods.

1. Introduction

1.1. Methods of Computational Chemistry

The quantum chemistry, classical mechanics, statistical physics and thermodynamics are the basic of the computational chemistry [2], the most important numerical techniques are ab-initio, semi empirical methods [3] and molecular dynamics.

(a) ab-initio [4] (Schrodinger equation and fundamental constants).

(b) Semi empirical (based on approximation calculations).

(c) Molecular mechanics (based on pre-determined force fields).

2. Theories behind Computational Chemistry

2.1. Schrodinger Equation

This equation is derived by Austrian physicist '*Erwin Schrodinger*' in the standard interpretation of quantum mechanics the wave function [5] (ϕ) it can be given to a physical system. Schrodinger equation can be mathematically transformed into matrix mechanics.

2.2. Koopmans' Theorem

This theorem is named after '*Tjalling Koopmans*: According to Koopmans' theorem [6], the first ionization energy of molecular system is equal to the negative of the orbital energy of the highest occupied molecular orbital (HOMO). The first ionization energies calculated on this way for small molecule are used to predict some parameters such as ground state & excited state ions, electron affinities (DFT-KS), MO-interactions (HOMO/LUMO), FMO (Frontier molecular orbital theory), which is useful to study the theoretical computational aspect of reactivity and stability.

2.3. Semi Empricial Methods

Semi-empirical quantum chemistry approximation methods are based on the Hartree-Fock formalism, but make many approximations and obtain some parameters from empirical data. They play vital role in computational chemistry for treating huge molecules. Within the framework of HartreeFock calculations, semi-empirical methods are parameterized, that is their results are fitted by a set of parameters, normally in such a way as to produce results that best agree with experimental data, but sometimes to agree with **ab initio** results. Semi-empirical methods are also called as empirical methods. For pi electron systems which was introduced by Erich Huckel. For extended electron systems was introduced by Roald Hoffmann. Semi-empirical calculations [8] are much faster than their *ab initio* counterparts, mostly due to the use of the zero differential overlap approximation. As a result, semi empirical methods are very fast, applicable to large molecules, and may give accurate results when applied to molecules that are similar to the molecules used for parameterization.

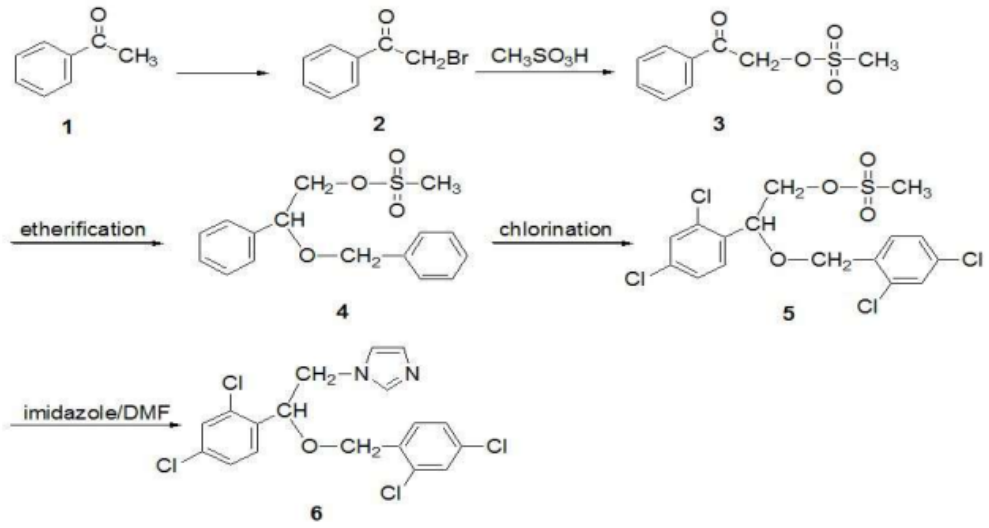
3. History of Miconazole (Imidazole Derivative)

Miconazole is an azole antifungal used to treat a variety of conditions, in which Empirical formula is a $C_{18}H_{14}Cl_4N_2O$ (molecular weight - 416.28), including those caused by *Candida* overgrowth. Which is Unique among the azoles derivatives, Miconazole act to through three main mechanisms. First one is CYP450[9] 14-α-lanosterol demethylase enzyme; second one, inhibits fungal peroxidise [8] and catalase and the Third mechanism of action lanosterol demethylation [10] inhibition.

3.1. Expriemental Method

Syntheisi of Miconazole

Miconazole was prepared from Acetophenone by Molina Caprile [11]: Miconazole (imidazole derivative) synthesized from Phenyl methyl ketone which is treated with bromo substituted benzene which is gave methyl sulfonate its further treated into etherification, chlorination as well as imidazole/DMF, finally 62% of product obtained (Miconazole).



4. Methodology and Computational Details

4.1. Hardware

The programs were run on a CPU supported by -INTEL® CORE(TM) 2DUO , E8400 processor clocking @ 3.00 GHZ, 300 MHz, 2 crore(s), 2 Logical processors(s), running on Microsoft windows 10 Enterprise with 32 bit (x86-based pc) support over a RAM of 8.00 GB. The graphics were visualized using the embedded graphical processor in the mother board.

4.2. Software

4.2.1 Chemsketch [12]

The molecule was drawn on ChemSketchand was subjected to 2D optimization. It was saved in '.sk2' and 'mol' format to be later retrieved to other GUI programs. The fragments were incrementally changed according to desired substituents. All

the derivatives were drawn using chemSketch which also provided IUPAC naming and numbering of the desired derivatives. ChemSketch provided few more physical properties of the molecule like such as Empirical formula, molecular weight, molar volume, Log P and molar density and polarizability.

4.2.2 Winmostar [13]

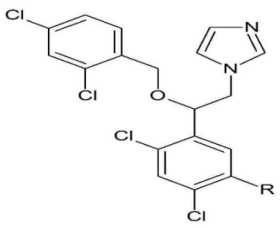
Winmostar is graphical user interface software which is used to analyze the result of Semi empirical Quantum chemical Calculations and 3D-visualization of the results. Winmostar uses Z-matrix format (internal coordinate) for molecular modelling. The program allows batch calling of MOPAC2016 and execution of the program. CNDO/S calculation were executed using the archive file (arc) generated by MOPAC. Output files were visualized in the 3D window.

4.2.3 MOPAC2016 [14]

MOPAC (Molecular Orbital Package) is a Semi-empirical quantum chemistry program based on by Dr. James J.P. Stewart Dewar and Thiel's NDDO approximation. MOPAC2016 is MOPAC2012, MOPAC2009 plus the PM7 and PM7-TS methods. The keywords used were AM1, RM1, VECTORS, BOND, SYMMETRY, PRECISE, ESP, NOINTER, and the output generated was visualized using Winmostar. CNDO/S command was later invoked from the OUT file. CNDO/S command provided UV-Vis spectrum of the molecule '.out' file provided molecular descriptors were theoretically.

5. Result and Discussion

General Structure of Miconazole (Fig-1)



This work was achieved by molecular modelling of the drug molecule with various substituents at important positions based on chemical intuition.These are the molecular fragments which have been found to influence the reactivity of the entire molecule.Chosen Molecular descriptors which represent reactivity and stability have been evaluated. For example, the Hardness of the molecule, a descriptor, represents resistivity to change and likewise the softness of the molecule, represents the reactive nature of the molecule.

Table 1: (R= Substituents & Empirical formula)

S.NO	KEYWORDS	SUBSITUENTS (R=)	EMPIRICAL FORMULA
1	MZS_1	$CONH_2$	$C_{19}N_3H_{15}O_2Cl_4$
2	MZS_2	CO	$C_{19}N_2H_{14}O_2Cl_4$
3	MZS_3	NO_2	$C_{18}N_3H_{13}O_3Cl_4$
4	MZS_4	CN	$C_{19}N_3H_{13}OCl_4$
5	MZS_5	OH	$C_{18}N_2H_{14}O_2Cl_4$
6	MZS_6	NH_3	$C_{18}N_3H_{16}OCl_4$
7	MZS_7	COOH	$C_{19}N_2H_{14}O_3Cl_4$
8	MZS_8	NH_2	$C_{18}N_3H_{15}OCl_4$
9	MZS_9	CHO	$C_{19}N_2H_{14}O_2Cl_4$
10	MZS_{10}	$COCH_3$	$C_{20}N_2H_{16}O_2Cl_4$
11	MZS_{11}	NH_2OH	$C_{18}N_3H_{16}O_2Cl_4$
12	MZS_{12}	IMI	$C_{21}N_4H_{16}OCl_4$
13	MZS_{13}	CH_3	$C_{19}N_2H_{16}OCl_4$
14	MZS_{14}	CH_2F	$C_{19}N_2H_{15}OCl_4F$
15	MZS_{15}	SO_2	$C_{18}N_2H_{13}SO_3Cl_4$
16	MZS_{16}	SCN	$C_{19}N_3H_{13}SOCl_4$
17	MZS_{17}	NCS	$C_{19}N_3H_{13}SOCl_4$
18	MZS_{18}	ORG	$C_{18}N_2H_{14}OCl_4$
19	MZS_{19}	SO_2NH_2	$C_{18}N_3H_{15}SO_3Cl_4$
20	MZS_{20}	CF_3	$C_{19}N_2H_{13}F_3OCl_4$
21	MZS_{21}	CH_3COCl	$C_{20}N_2H_{15}O_2Cl_5$

*IMI-imidazole,*ORG- miconazole *MZS-Miconazole substituent

From **Frontier Molecular Orbital** [15] (FMO) applying Koopmans' theorem [16], the Ionization potential (IP) Chemical potential (μ) and Electron affinity (EA) can be related to Homo and Lumo energies and the values have been calculated theoretically.

By this way it has been found that the Hardness value for the original compound is 8.516. Nucleophilic substituents like NH_2, OH, $COCH_3$ and SO_2NH_2, SCN decrease the Hardness of the molecule. If, at particular place, reactivity of the molecule is needed, then these derivatives may be prepared and tested for biological activity. On the other hand, substituents like CO, $CONH_2$, NH_2OH, CH_2F and NCS represent very low Hardness value compared to the original compound and it can be also noted that the isomer of SCN changes the Hardness value to a greater extent. Therefore, derivatives can be carefully chosen based the isomeric structure too. Derivatives with and NH_2OH and NH_2 are found to be very reactive.

It can also be seen that LUMO values are lower for compounds having higher Hardness values. A particular derivative, with R= CH_2F represents the molecule with lowest unoccupied molecular orbital (LUMO) value (-0.943eV) But, it possesses

relatively lower Highest occupied molecular orbital (HOMO) values also, which mean the electron available for excitation into the reactivity level are easily available. The highest HOMO is values shown by SO_2 substituent (-10.850 eV) and this suggests that the molecule is highly reactive because the 'Band Gap' is very low. Relative to this HOMO value is the band gap which is very lower and it suggested that the electron can be ejected to Rydberg level [17] (R_H) where they can be brought into reactivity quickly. Most of the times Hardness valuesare quite opposite to the Softness values.

Anti-fungal drugs require very good reactivity pattern to affect the pathogen and disturb its proliferation. Therefore, SO_2 derivative at R may be prepared and tested for antifungal activity in the lab. The next derivative happens to be CO. It has a very high HOMO value (11.802eV). But the band gap is very large compared to SO_2, therefore, electronic excitation to higher levels becomes relatively difficult and this molecule can be avoided. Derivatives of the same types like $CONH_2$ may be considered for next level test because NH_2 can provide non-bonding electron promotion to $\pi-$orbitals of the molecule by means of resonance or electronic effects. Once into $\pi-$level the electron may be promoted to the Rydberg Level (R_H) for reactions as the molecule has relatively lower band gap. The $CONH_2$ derivatives can also been prepared in the lab and tested for anti-fungal activity.

6. Tables and Graphs

Table 2: Showing Substituents and their Ionization Potential (Increasing Order)

S.NO	KEYWORDS	SUBSITUENTS (R=)	IONIZATION POTENTIAL (eV)
1	MZS_1	$CONH_2$	4.739
2	MZS_6	NH_3	6.047
3	MZS_{11}	NH_2OH	6.681
4	MZS_{14}	CH_2F	7.588
5	MZS_8	NH_2	8.811
6	MZS_{17}	NCS	8.903
7	MZS_{16}	SCN	8.983
8	MZS_2	CO	9.031
9	MZS_{13}	CH_3	9.173
10	MZS_5	OH	9.191
11	MZS_{18}	ORG	9.193
12	MZS_{10}	$COCH_3$	9.241
13	MZS_9	CHO	9.244
14	MZS_{19}	SO_2NH_2	9.275
15	MZS_7	COOH	9.276

16	MZS_{21}	CH_3COCl	9.283
17	MZS_{12}	IMI	9.286
18	MZS_4	CN	9.311
19	MZS_{20}	CF_3	9.319
20	MZS_3	NO_2	9.383
21	MZS_{15}	SO_2	9.875

*IMI-imidazole,*ORG- miconazole *MZS-Miconazole substituent

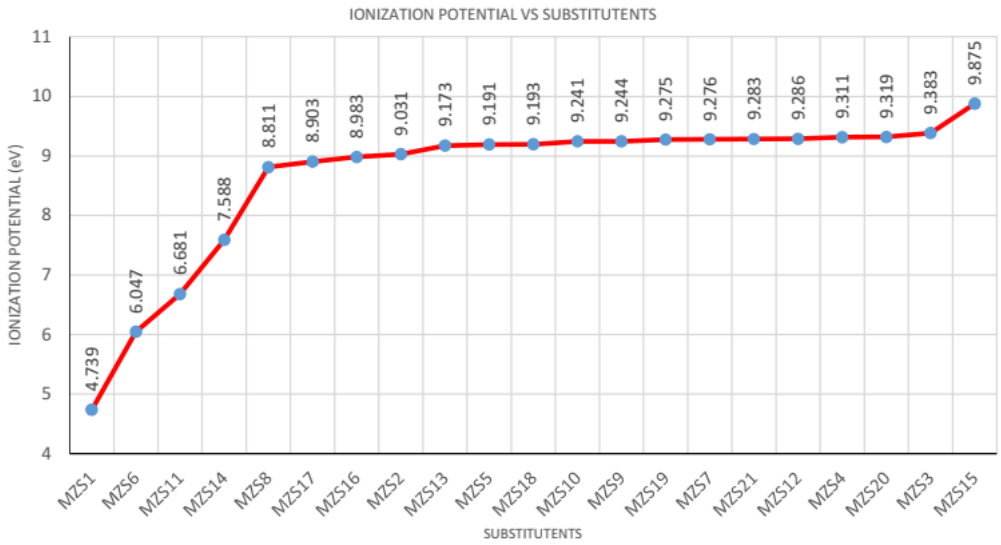


Figure 1: Graph showing Variation Ionization Potential on various Substituents

Table 3: Showing Substituents and their Chemical Potential Values (Increasing Order)

S.NO	KEYWORDS	SUBSITUENTS (R=)	HOMO (eV)	LUMO (eV)	CHEMICAL POTENTIAL (eV)
1	MZS_{15}	SO_2	−10.850	−7.865	9.358
2	MZS_2	CO	−11.082	−7.194	−9.138
3	MZS_{11}	NH_2OH	−10.906	−4.942	−7.924
4	MZS_6	NH_3	−10.849	−4.776	−7.813
5	MZS_3	NO_2	−9.327	−2.947	−6.137
6	MZS_9	CHO	−9.218	−2.097	−5.657
7	MZS_7	COOH	−9.255	−1.918	−5.587
8	MZS_4	CN	−9.273	−1.825	−5.549

9	MZS_{20}	CF_3	−9.258	−1.580	−5.419
10	MZS_1	$CONH_2$	−8.250	−2.423	−5.337
11	MZS_{21}	CH_3COCl	−9.255	−1.379	−5.317
12	MZS_{12}	IMI	−9.164	−1.422	−5.293
13	MZS_{10}	$COCH_3$	−9.204	−1.299	−5.251
14	MZS_{19}	SO_2NH_2	−9.131	−1.250	−5.191
15	MZS_{17}	NCS	−8.786	−1.573	−5.180
16	MZS_5	OH	−9.181	−1.176	−5.178
17	MZS_{16}	SCN	−9.069	−1.250	−5.159
18	MZS_{13}	CH_3	−9.147	−1.051	−5.099
19	MZS_8	NH_2	−9.103	−1.055	−5.079
20	MZS_{14}	CH_2F	−7.867	−0.943	−4.405
21	MZS_{18}	ORG	−9.072	7.961	−0.555

*IMI-imidazole,*ORG- miconazole *MZS-Miconazole substituent

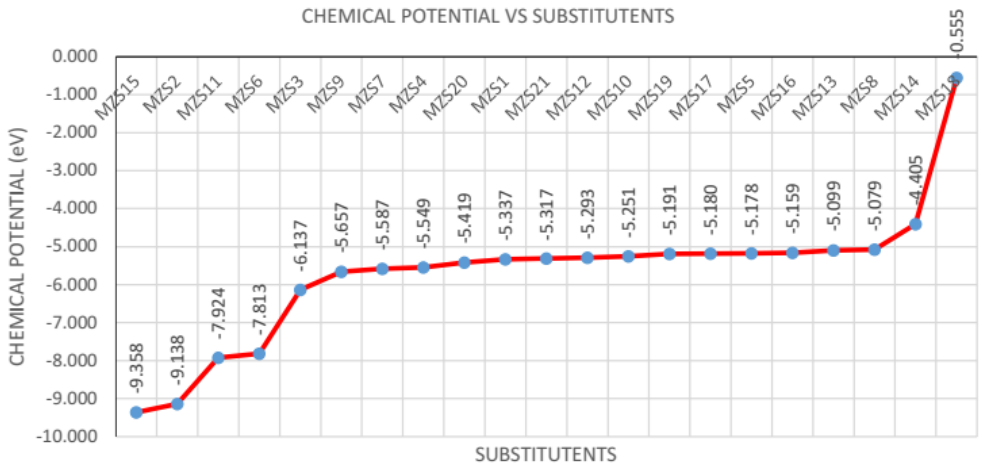


Figure 2: Graph showing Variation on Chemical Potential for various Substituents

Table 4: Showing Substituents and their Hardness Vvalues (Increasing Order)

S.NO	KEYWORDS	SUBSITUENTS (R=)	HOMO (eV)	LUMO (eV)	HARDNESS
1	MZS_{15}	SO_2	−10.850	−7.865	1.493
2	MZS_2	CO	−11.082	−7.194	1.944
3	MZS_1	$CONH_2$	−8.250	−2.423	2.914
4	MZS_{11}	NH_2OH	−10.906	−4.942	2.982

5	MZS_6	NH_3	−10.849	−4.776	3.037
6	MZS_3	NO_2	−9.327	−2.947	3.190
7	MZS_{14}	CH_2F	−7.867	−0.943	3.462
8	MZS_9	CHO	−9.218	−2.097	3.561
9	MZS_{17}	NCS	−8.786	−1.573	3.607
10	MZS_7	COOH	−9.255	−1.918	3.668
11	MZS_4	CN	−9.273	−1.825	3.724
12	MZS_{20}	CF_3	−9.258	−1.580	3.839
13	MZS_{12}	IMI	−9.164	−1.422	3.871
14	MZS_{16}	SCN	−9.069	−1.250	3.909
15	MZS_{21}	CH_3COCl	−9.255	−1.379	3.938
16	MZS_{19}	SO_2NH_2	−9.131	−1.250	3.941
17	MZS_{10}	$COCH_3$	−9.204	−1.299	3.952
18	MZS_5	OH	−9.181	−1.176	4.003
19	MZS_8	NH_2	−9.103	−1.055	4.024
20	MZS_{13}	CH_3	−9.147	−1.051	4.048
21	MZS_{18}	ORG	−9.072	7.961	8.516

*IMI-imidazole,*ORG- miconazole *MZS-Miconazole substituent

Table 5: Showing Substituents and their Softness Values (Increasing Order)

S.NO	KEYWORDS	SUBSITUENTS (R=)	HOMO (eV)	LUMO (eV)	SOFTNESS
1	MZS_{18}	ORG	−9.072	7.961	0.059
2	MZS_8	NH_2	−9.103	−1.055	0.124
3	MZS_{13}	CH_3	−9.147	−1.051	0.124
4	MZS_5	OH	−9.181	−1.176	0.125
5	MZS_{10}	$COCH_3$	−9.204	−1.299	0.127
6	MZS_{19}	SO_2NH_2	−9.131	−1.250	0.127
7	MZS_{21}	CH_3COCl	−9.255	−1.379	0.127
8	MZS_{16}	SCN	−9.069	−1.250	0.128
9	MZS_{12}	IMI	−9.164	−1.422	0.129
10	MZS_{20}	CF_3	−9.258	−1.580	0.130
11	MZS_4	CN	−9.273	−1.825	0.134
12	MZS_7	COOH	−9.255	−1.918	0.136
13	MZS_{17}	NCS	−8.786	−1.573	0.139
14	MZS_9	CHO	−9.218	−2.097	0.140
15	MZS_{14}	CH_2F	−7.867	−0.943	0.144
16	MZS_3	NO_2	−9.327	−2.947	0.157

17	MZS_6	NH_3	−10.849	−4.776	0.165
18	MZS_{11}	NH_2OH	−10.906	−4.942	0.168
19	MZS_1	$CONH_2$	−8.250	−2.423	0.172
20	MZS_2	CO	−11.082	−7.194	0.257
21	MZS_{15}	SO_2	−10.850	−7.865	0.335

*IMI-imidazole,*ORG- miconazole *MZS-Miconazole substituent

7. Conclusion

We have evaluated from the present work that pharmacodynamics study would prove to be very useful in drug designing and modelling. The substituents rich with non-bonding electron such as R=NH_2, OH, $COCH_3$, SO_2NH_2 and SCN show very good reactivity pattern. The reactivity can be considered as with that of receptor molecules. These substituents can be used for studies such as docking with receptors and other studies involved in drug designing in future. The substituents like R= CO, NH_2, NH_2OH, CH_2F, NCS show very little reactivity. Therefore, these derivatives can be avoided in future studies concerned with drug design and reactivity.

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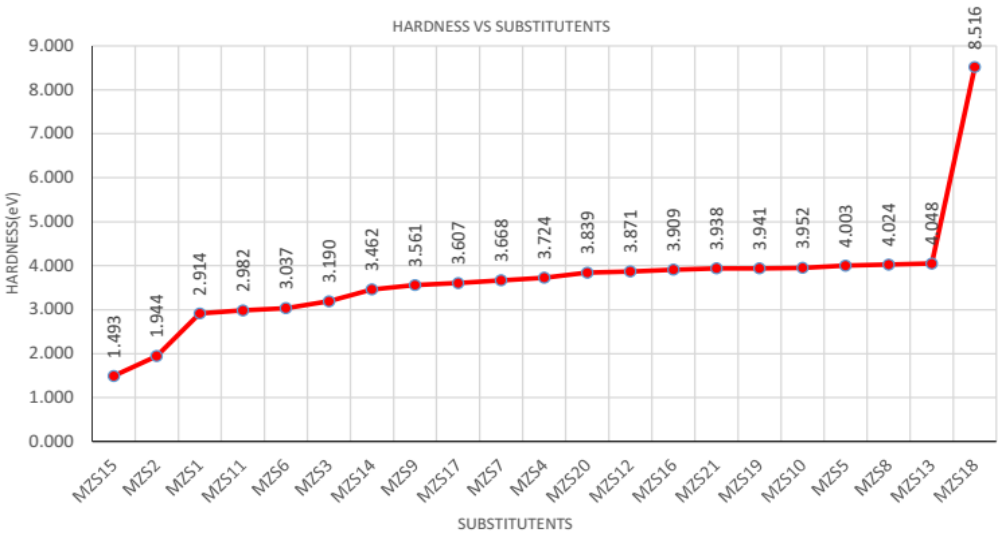

Figure 3: Graph showing Variation on Hardness Values on Substituents

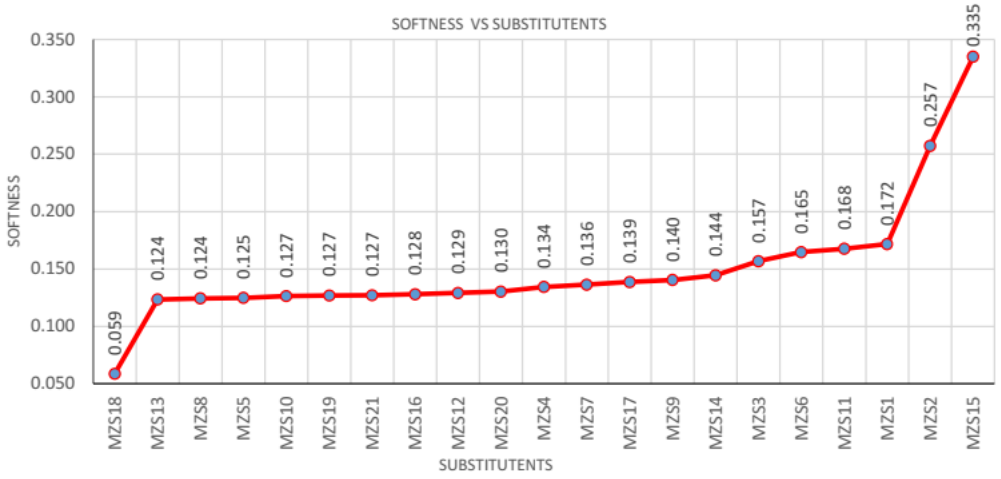

Figure 4: Graph showing Variation of Softness Values on Substituents

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OXIDATION REACTION OF PENTAAMMINE COBALT (III) COMPLEXES OF $\alpha-$HYDROXY ACIDS BY PYRAZINIUM CHLOROCHROMATE (PyzCC) IN SURFACE ACTIVE SOLUTION

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Abstract

The Kinetic oxidation of PyzCC unavailable with Co(III) complexes of α-hydroxy acid and in perchloric acid medium in the presence of surface active solution. To observe increase in the concentration of the surfactant, increase on rate is taken in account. The kinetic and stoichiometric result have been accounted by a suitable mechanism.

Keywords: Pentaammine cobalt(III) complexes, Cetyl trimethyl ammonium bromide, $\alpha-$hydroxy acids, pyrazinium chlorochromate(PyzCC).

1. Introduction

For the oxidation of organic compound pyrazinium chlorochromate (PyzCC) is found to be mild reagent, oxidation kinetics of primary, secondary and allylic alcohols, substituted benzyl alcohols, aliphatic aldehydes, aromatic aldehydes, heterocyclic aldehydes, cyclohexanone β-diketones, aliphatic and aromatic carboxylic acids, styrenes, substituted cinnamates and 2-naphthols were carried out by various authors. In the oxidation of diols where a single hydroxyl group was oxidized to give hydroxy carbonyl compound as product in this oxidation the mildness of PyzCC was demonstrated effectively. A mechanism involving the C-C bond cleavage leading to the formation of aldehydes as the product was proposed. The formation of ketoacids by the C-H cleavage of the α-hydroxy acids was reported with many chromium(VI) oxidants. In the case of complexes for the oxidation with C-H cleavage the hydroxyl group of the bound ligand is free leading to the formation of keto complexes. With decarboxylation and synchronous C-C cleavage accompanied by an induced electron transfer to the Co(III) in complex lead to formation of aldehydes as a product and one electron transfer to the oxidant. The contest between the mechanisms become interesting and produced definite result when mild oxidizing agents like PyzCC were kinetically followed.

2. Experimental

2.1. Pyrazinim Chlorochromate

Methods for preparation of pyrazinium chlorochromate(PyzCC) as follows. Chromium trioxide (10g, 0.1mol) was dissolved in water (15ml) add cooled to 0°C. To this solution hydrochloric acid (17ml, 48%) was added slowly with vigorous stirring the pyrazinium (13ml, 0.1mol) was added dropwise during 10 minutes. The reaction mixture was allowed to cool for 2 -3 hours and filtered. The resulting yellow orange needles were dried and recrystallized. To check the purity of reagent iodometric method is used.

2.2. Carbanatopentaammine Cobalt(III) Nitrate

A solution of 300g of Cobalt (III) nitrate hexahydrate (1.03 mol) in 500ml of water was thoroughly mixed with a solution of 450g of ammonium carbonate (4.68 mol) in 450 ml of water and 750 ml of concentrated aqueous ammonia (Sp.Gr. 0.90, 28% NH_3). A stream of air bubbled slowly through the mixture for 24 hours. After the mixture has been cooled in ice salt bath overnight, the product was collected on a filter, washed with not more than 50 ml of ice-cold water, followed by alcohol and ether and dried 50°C (yield 180g, 64%). This crude material was purified by recrystallization from water. The 180g of solid was dissolved with stirring in 550 ml of water at 90°C. The solution was filtered and filtrate was cooled in ice-bath. The crystals were collected, washed with 50 ml of ice-cold water followed by alcohol and ether and dried at 50°C.

2.3. Pentaammine Cobalt(III) Complexes of α-Hydroxy Acids

The monomeric cobalt (III) complexes of lactic, mandelic and glycolic acids were prepared as their perchlorates following the procedure of Fan and Gould 10 mol of ligand was dissolved in 20 ml of methanol taken in a 50 ml R.B flask and a pellet (0.50 to 1.00g) of sodium hydroxide was added about 0.40g of finely powdered carboanato pentaammine cobalt (III) nitrate was added and the mixture was refluxed at 70°C for 2 hours. It was then cooled under ice for 30 minutes. About 3 ml of 10% perchloric acid was added drop wise while shaking the mixture and it was cooled again under ice for 1 hour. The Cobalt (III) complex precipitated as perchlorate and was filtered through a sintered glass crucible, washed well with ethanol followed by diethyl ether, dried and preserved in a desiccator.

2.4. Product Analysis

After the completion of reactions by induced electron transfer in pentaammine cobalt (III) complexes of α-hydroxy acids in presence of $HClO_4$ Co(II) was estimated. The reaction mixture was diluted to 10 times with conc. HCl after nine half-lives with allowing the evolution of chlorine to cease and then measuring the absorbance of chloro complex Co(II) as 692 nm [$\triangle$=560 dm^3 mol^{-1} cm^{-1}]. It was found that 35% decrease

in absorbance in Co(III) complexes of α-hydroxy acids evidencing the induced electron transfer. The absorbance measurement made with corresponding amount of pyrazinium chlorochromate (PyzCC) only diluted with conc. HBr 10 times was a blank correction.

Table 1: Stoichiometric Data for PyzCC Oxidation of Co(III) Bound and Unbound α-Hydroxy Acid in the Presence of CTAB. CTAB = 2.00 $\times 10^{-4}$ mol dm^{-3}, $HClO_4$ = 1.00 mol dm^{-3}, Temperature = $31 \pm 0.2^{\circ}$**C**

10^{-3} [Compound] mol dm^{-3}	10^2[PyzCC] Initial mol dm^{-3}	10^2[PyzCC] Final mol dm^{-3}	$\triangle 10^3$[PyzCC] mol dm-3	[Compound] : $\triangle$[PyzCC]
Lactic acid				
4.00	2.00	1.74	2.63	1.00:0.65
5.00	2.00	1.67	3.24	1.00:0.64
6.00	3.00	2.60	3.96	1.00:0.66
Mandelic acid				
4.00	2.00	1.73	2.68	1.00:0.67
5.00	2.00	1.66	3.20	1.00:0.64
6.00	3.00	1.62	3.90	1.00:0.65
Glycolic acid				
4.00	2.00	1.72	2.52	1.00:0.63
5.00	2.00	1.68	3.25	1.00:0.65
6.00	3.00	2.60	3.96	1.00:0.66
Co(III)-Lactato				
4.00	2.00	1.86	1.32	1.00:0.33
5.00	2.00	1.84	1.60	1.00:0.32
6.00	2.00	2.80	1.98	1.00:0.33
Co(III)-Mandelato				
4.00	2.00	1.86	1.36	1.00:0.34
5.00	2.00	1.82	1.75	1.00:0.35
6.00	2.00	2.79	2.04	1.00:0.34
Co(III)-Glycolato				
4.00	2.00	1.86	1.32	1.00:0.33
5.00	2.00	1.83	1.70	1.00:0.34
6.00	2.00	2.78	2.10	1.00:0.35

3. Result and Discussion

The Kinetic data for the oxidation of α-hydroxy acids and their complexes by PyzCC in the presence of perchloric acid is summarised in the Table. The reaction show a total second order dependence on the substrate catalyzed by perchloric acid and the oxidant. There was a smaller difference in rate of the reaction between lactic glycolic acid and mandelic acid among the α-hydroxy acid which was not observed with other chromium halochromates. An explanation for the higher reactivity of lactic acid involving C-H cleavage was given based on the higher acidity of C-H proton. In the present investigation, the absence of such an observation lead to C-C cleavage. Due to C-C cleavage for the formation of aldehyde is confirmed by product analysis and not keto acid as a product. IBC undergoes two electron transfer and the overall reaction between PyzCC and α-hydroxy acid could be written as,

$$\text{Rate} = k\,[\alpha - \text{hydroxy acid}]\,[\text{IBC}]$$

Table 2: PyzCC = 2.00×10^{-3} **mol dm**$^{-3}$, **HClO$_4$ = 1.00 mol dm**$^{-3}$, **Temperature =** $31 \pm 0.2^{\circ}$**C**

[α-Hydroxy acid]	10^4 k$_1$ s^{-1}	10^2 k$_2$ dm^3 mol^{-1}
Lactic acid		
1.00	0.885	0.885
2.00	1.770	0.885
4.00	3.560	0.890
6.00	5.328	0.888
8.00	7.180	0.897
Mandelic acid		
1.00	0.660	0.660
2.00	1.322	0.661
4.00	2.680	0.670
6.00	4.062	0.677
8.00	5.382	0.672
Glycolic acid		
1.00	0.524	0.524
2.00	1.050	0.525
4.00	2.104	0.526
6.00	3.160	0.527
8.00	4.202	0.525

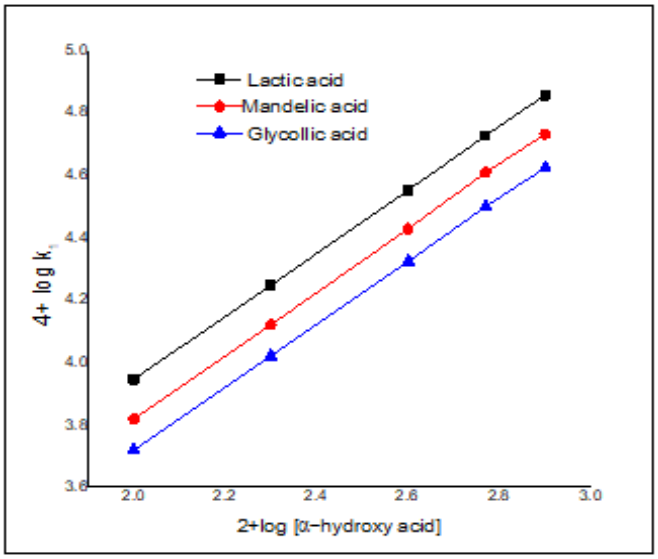

Figure 1: Dependence of Rate on [α-Hydroxy Acid]

Table 3: Lactic acid = 2.00×10^{-2} **mol dm**$^{-3}$, **PyzCC =** 2.00×10^{-3} **mol dm**$^{-3}$, **Micelle =** 1.00×10^{-4} **mol dm**$^{-3}$, **HClO4 =** 1.00 **mol dm**$^{-3}$, **Temperature =** $31 \pm 0.2^{\circ}$**C**

[α-Hydroxy acid]	$10^4 k_1$ s^{-1}	10^2 k_2 dm^3 mol^{-1}
Lactic acid		
1.00	1.070	1.070
2.00	2.160	1.080
4.00	4.340	1.085
6.00	6.488	1.081
8.00	8.682	1.085
Mandelic acid		
1.00	0.864	0.864
2.00	1.736	0.868
4.00	3.464	0.866
6.00	5.204	0.867
8.00	6.929	0.866
Glycolic acid		
1.00	0.676	0.676
2.00	1.356	0.678
4.00	2.718	0.679
6.00	4.076	0.679
8.00	5.428	0.678

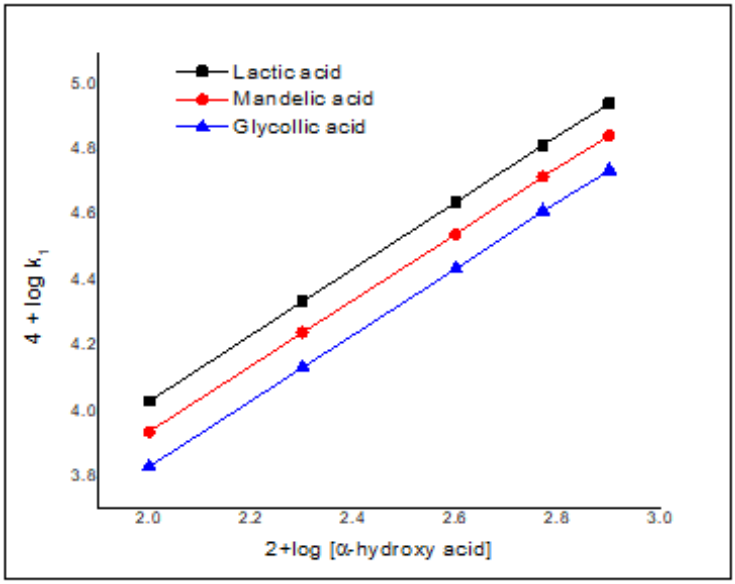

Figure 2: Dependence of Rate on [α-Hydroxy Acid]

4. Mechanism

The reaction exhibit first order dependence on the concentration of Cr(VI). The reaction shows Arrhenius dependence and thermodynamics parameters evaluated are in the expected order for a bimolecular reactions. Among the three complexes of Co(III) lactate, Co(III) mandelato, Co(III)glycolato, the Co(III) lactate reacts faster than Co(III) mandelato and Co(III)glycolato complex. Among Co(III) bound and unbound ligand, Co(III) bound α-hydroxy acid complex gets oxidizing at a higher rate than the unbound ligand and there is 100% reduction at Co(III) centre. The rate of PyzCC induced electron transfer in Co(III) complexes of α-hydroxy acid increases in the presence of CTAB much more than without CTAB. 1.00 mole of Co(III) complex reacts with nearly 0.35 mol of PyzCC yielding 100% Co(II) and 100% carbonyl compound similarly 1.00 mol of α-hydroxy acids react with nearly 0.65 mol of PyzCC yielding 100% of carbonyl product and CO_2.

5. Conclusion

Earlier studies on induced electron transfer reaction involved mainly one equivalent oxidant Ce(III) and pentaammine cobalt(III) complexes $[(NH_3)_5CoIII\text{-}L]^{2+}$ with bound ligands featuring conjugated fragments. Generation of a radical at the bound organic ligand by the one equivalent oxidant help in achieving the ultimate reduction at cobalt(III) center. Due to the partitioning of reaction path the percentage of cobalt(III) formation will differ.

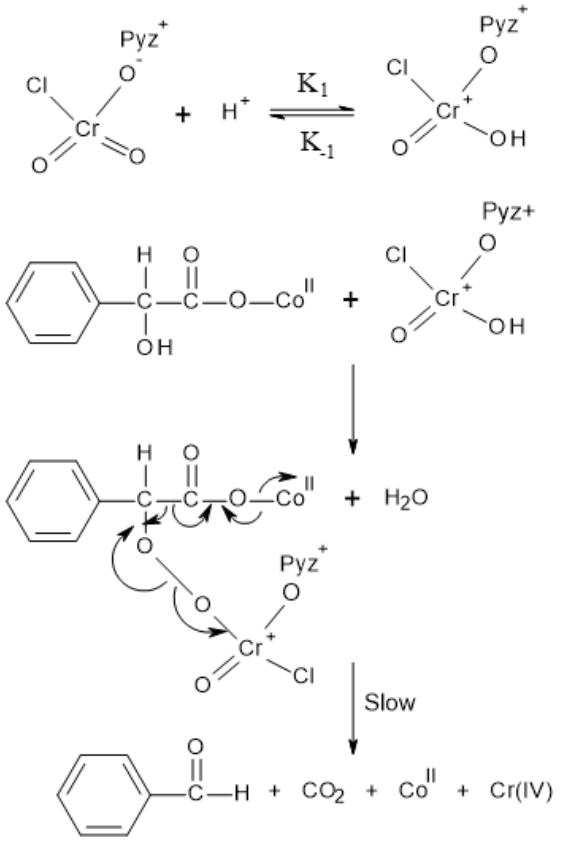

Figure 3: Mechanism of PyzCC Oxidation of Pentaamminecobalt(III) Complexes

An induced electron transfer reaction has been attempted presently with Pyrazinium chloro chromate and pentaammine cobalt(III) complexes of α-hydroxy acids in the presence of CTAB medium and also in the absence of CTAB medium. The reaction exhibits second order kinetics and in the case of mandelato complex the amount of cobalt(III) reduced corresponding to nearly 20% of initial concentration and the amount of cobalt(II), aldehyde and CO_2 formed 20% while nearly 70% of its converted to keto acid cobalt(III) complex. There will be increase in rate when concentration detergent as increases. Added CTAB enhance the rate of oxidation of reaction much more than without micellar medium. Similar trend has been observed in lactato and glycolato cobalt(III) complexes.

The rate of oxidation PyzCC of pentaammine cobalt(III) complexes of α-hydroxy acids in micellar medium is first order kinetics each in cobalt (III) and chromate induced electron transfer in $[(NH_3)_5CoIII\text{-}L]^{2+}$ complexes of α-hydroxy acids readily yields 100% of cobalt(II) with nearly 100% of C-C bond cleavage products suggesting that it behaves mainly as one equivalent oxidation in micellar medium with unbound ligand also it behaves only as C-C cleavage agent rather than C-H cleaving agent.

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A STUDY ON GAME THEORY IN NEUTROSOPHIC ENVIRONMENT

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Abstract

Objective of this paper, a new method is developed to solve the game matrix in neutrosophic environment and evaluation matrix is constructed followed by Score functions have been defined and applied Max-Min principle in the score function to find the value of the game and numerical example has been given in support of the solution method.

Keywords: Neutrosophic Environment, Game Matrix, Evaluation Matrix, Score Functions, Max-Min Principle.

1. Introduction

In every competitive situation, it is often required to take the decision where there are two or more opposite parties with conflicting of interests and the action of one depends upon the action which is taken by the opponent. A variety of competitive situation is seen in real life society like, in political campaign, elections, advertisement, marketing, etc. Game theory is a mathematical way out for describing the strategic interactions among multiple players who select several strategies from the set of admissible strategies. In 1944, Von Neumann and Oscar Morgenstern [1] introduced game theory in their most pioneer work "Theory of Games and Economic Behavior". Since then many diverse kinds of mathematical games have been defined and different types of solution methodologies have been proposed. The participants in the game are called the players. During the past, it is assumed that all the information about game is known precisely by players. But in traditional game theory, the precise information about the game is more difficult to collect due to the lack of information about the exact values of certain parameters and uncertain measuring of several situations by players. To overcome these types of situation, the problem can be formulated using the concept of uncertainty theory and the domain of payoffs are considered from uncertain environment like fuzzy, interval, stochastic, fuzzy-stochastic environment etc. In such

cases fuzzy set theory is a vital tool to handle such situation. Fuzzy set Zadeh (1965) introduced an effective way to model data uncertainty by defining membership function in the range of [0,1]. the fuzzy set has demonstrated a good performance in handling data uncertainty by using membership function, they failed to handle non-membership degree and indeterminacy membership degree.

Intuitionistic fuzzy set (IFs) introduced by Atanassov (1986) is an extension of fuzzy set in order to overcome the lack of knowledge about the non-membership degree. It is characterized by a membership degree and a non-membership degree functions. TOPSIS has been extended to solve MCGDM problems with intuitionistic fuzzy data. Pramanik (2011) studied the teacher selection in intuitionistic fuzzy environment. Intuitionistic fuzzy TOPSIS has been used for the employee performance appraisal by Yinghui (2015). However, the intuitionistic fuzzy set can handle the membership degree and the non-membership degree, it can't handle problems involving indeterminate and inconsistent information.

Neutrosophic set (NS) first introduced by Smarandache (1999) in order to handle the problems with indeterminate and inconsistent information. NS is a generalization of crisp sets, fuzzy sets, intuitionistic fuzzy sets, interval-valued fuzzy sets, and interval-valued intuitionistic fuzzy sets. NS is characterized by three membership functions, Truth membership function (T), Indeterminacy membership function (I), and Falsity membership function (F). NS is difficult to apply in real problems, so the single-valued neutrosophic set was introduced by Wang (2010) to be applied to real scientific and engineering situations. Biswas (2016) extended TOPSIS method to solve multi-attribute group decision making problem under SVN environment. MCGDM problems can include more than one participant, each of them needs to take the best decision, so competition may appear between these participants. Game theory is a powerful tool which used to handle the competition situations between two or more participants. A game is a formal description of a strategic situation.

This paper is organized as follows: Section. 2 briefly introduces some basic preliminaries related with neutrosophic sets. In Section. 3 Principles of game theory are included in Section. 4. The proposed approach for solving the problem with uncertainty and competition is introduced in Section. 5. A numerical example is illustrated in Section.6. Finally, conclusions and future work are pointed out at the end of this paper.

2. Preliminaries

2.1. Neutrosophic Set

Let U be the space of points (or objects) with generic element x. A neutrosophic set A in U is characterized by a truth membership function T_A, and indeterminacy function I_A and a falsity membership function F_A, where T_A, I_A and F_A are real standard or non- standard subsets of $]^-0, 1^+[$, i.e., $\sup T_A : X \rightarrow]^-0, 1^+[$, $\sup F_A : X \rightarrow]^-0, 1^+[$, $\sup I_A : X \rightarrow]^-0, 1^+[$.

A neutrosophic set A upon U as an object is defined as $\frac{x}{T_A(x), I_A(x), F_A(x)} = \left\{ \frac{x}{T_A, I_A, F_A} : x \in U \right\}$, where $T_A(x), I_A(x)$ and $F_A(x)$ are subintervals or union of subintervals of $[0, 1]$.

2.2. Algebraic Operations with Neutrosophic Set

For two neutrosophic sets A and B,

(a) Complement of A

$$A' = \left\{ \frac{x}{T, I, F} \mid T = 1 - T_A, I = 1 - I_A, F = 1 - F_A \right\}.$$

(b) Intersection of A and B

$$A \cap B = \left\{ \frac{x}{T, I, F} \mid T = T_A T_B, I = I_A I_B, F = F_A F_B \right\}.$$

(c) Union of A and B

$$A \cup B = \left\{ \frac{x}{T, I, F} \mid T = T_A + T_B - T_A T_B, I = I_A + I_B - I_A I_B, F = F_A + F_B - F_A F_B \right\}.$$

(d) Cartesian Product of A and B

$$A \times B = \left\{ \left(\frac{x}{T_A, I_A, F_A}, \frac{y}{T_B, I_B, F_B} \right) \mid \frac{x}{T_{A,}, I_A, F_A} \in A, \frac{y}{T_B, I_B, F_B} \in B \right\}.$$

(e) A is a subset of B

$$A \subseteq B, \forall \frac{x}{T_A, I_A, F_A} \in A \text{ and } \frac{y}{T_B, I_B, F_B} \in B, T_A \leqslant T_B, I_A \geqslant I_B, F_A \geqslant F_B.$$

(f) Difference of A and B

$$A \backslash B = \left\{ \frac{x}{T, I, F} \mid T = T_A - T_A T_B, I = I_A - I_A I_B, F = F_A - F_A F_B \right\}.$$

3. Neutrosophic Matrix Game

Let A_i $(i = 1, 2, 3, \ldots, n)$ and B_j $(j = 1, 2, 3, \ldots, m)$ be pure strategies for players A and B, respectively. If player A adopts the pure strategy A_i (i.e., the row) and player B adopts pure strategy B_j (i.e., the column), then the pay-off for players is expressed with the Neutrosophic number ($T_A(x)$, $I_A(x)$ and $F_A(x)$).

3.1. Pure Strategy

Pure strategy is a decision making rule in which one particular course of action is selected. For fuzzy games the min - max principle is described by Nishizaki [2]. The course of the fuzzy game is determined by the desire of to maximize his gain and that of restrict his loss to a minimum.

3.1.1 Definition (Saddle Point):

The concept of saddle point in classical form is introduced by Neumann [12]. The position of the pay-off matrix will be called a saddle point, if and only if, We call the position of entry a saddle point, the entry itself, the value of the game (denoted by) and the pair of pure strategies leading to it are optimal pure strategies.

3.2. Solution Procedure of Neutrosophic Game:

To solve Neutrosophic game, first we have calculated the Evaluation Matrix for each alternative. Using the elements of Evaluation Matrix for alternatives Score function (S_{ij}) matrix has been calculated. By applying Max-Min principle method is described by Nishizaki [2] in the Score function matrix (S_{ij}), then find the value of the game.

4. Algorithm for Neutrosophic Game:

Step 1. Construct the Neutrosophic game matrix $D = (C_{ij})_{m\times n}$.

Step 2. Determine the Evaluation Matrix of the job J_i as $E(J_i) = \left[T^l_{j_i}, T^u_{j_i}\right]$, where

$$T^l_{j_i}, T^u_{j_i} = \begin{bmatrix} \min\left(\dfrac{T_{J_{ij}} + I_{J_{ij}}}{2}\right) & \left(\dfrac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right) \\ \max\left(\dfrac{T_{J_{ij}} + I_{J_{ij}}}{2}\right) & \left(\dfrac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right) \end{bmatrix}$$

Step 3. Compute the Score function $S(J_{ij})$ of an alternative

$$S(J_{ij}) = 2\left(T^u_{J_{ij}} - T^l_{J_{ij}}\right) = 2\begin{bmatrix} \max\left(\dfrac{T_{J_{ij}} + I_{J_{ij}}}{2}\right) & \left(\dfrac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right) \\ \min\left(\dfrac{T_{J_{ij}} + I_{J_{ij}}}{2}\right) & \left(\dfrac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right) \end{bmatrix},$$

where $0 \leqslant S(J_{ij}) \leqslant 1$.

Step 4. By applying Max-Min principle method in the Score function matrix (S_{ij}),

Step 5. Find the value of the game.

5. Numerical Example

Let us consider a Neutrosophic game, the matrix contains neutrosophic elements denoting pure strategies for players A and B respectively.

$$\begin{matrix} B_1 & B_2 & B_3 \end{matrix}$$
$$\begin{bmatrix} [0.75, 0.39, 0.1] & [0.8, 0.6, 0.5] & [0.4, 0.8, 0.45] \\ [0.6, 0.5, 0.25] & [0.75, 0.9, 0.05] & [0.68, 0.46, 0.2] \\ [0.8, 0.4, 0.2] & [0.45, 0.1, 0.5] & [0.1, 0.5, 1.0] \\ [0.4, 0.6, 0.3] & [0.5, 0.4, 0.8] & [0.5, 0.6, 0.9] \end{bmatrix}$$

Solution:
Evaluate $E(J_i)$ as the evaluation function of the job J_i as

$$E(J_i) = \left[T^l_{J_i}, T^u_{J_i}\right]$$

where

$$\left[T^l_{j_i} T^u_{j_i}\right] = \begin{bmatrix} \min\left(\dfrac{T_{J_{ij}} + I_{J_{ij}}}{2}\right) & \left(\dfrac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right) \\ \max\left(\dfrac{T_{J_{ij}} + I_{J_{ij}}}{2}\right) & \left(\dfrac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right) \end{bmatrix}.$$

Therefore elements of the Evaluation matrix for alternatives.

$$\left[T^l_{j_i} T^u_{j_i}\right] = \begin{bmatrix} [0.57, 0.645] & [0.7, 0.725] & [0.6, 0.675] \\ [0.55, 0.625] & [0.825, 0.925] & [0.57, 0.63] \\ [0.6, 0.6] & [0.275, 0.3] & [0.25, 0.75] \\ [0.5, 0.65] & [0.3, 0.45] & [0.35, 0.55] \end{bmatrix}.$$

Compute the Score function $S(J_{ij})$ of an alternative

$$S(J_{ij}) = 2\left(T^u_{J_{ij}} - T^l_{J_{ij}}\right) = 2\begin{bmatrix} \max\left(\dfrac{T_{J_{ij}} + I_{J_{ij}}}{2}\right) & \left(\dfrac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right) \\ \min\left(\dfrac{T_{J_{ij}} + I_{J_{ij}}}{2}\right) & \left(\dfrac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right) \end{bmatrix},$$

where $0 \leqslant S(J_{ij}) \leqslant 1$.

Therefore elements of Score function matrix will be as follows-

$$S(J_{ij}) = \begin{bmatrix} 0.15 & 0.05 & 0.15 \\ 0.15 & 0.2 & 0.12 \\ 0.0 & 0.05 & 1.0 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}.$$

Solving $S(J_{ij})$ by Max-Min principle,

	B_1	B_2	B_3	(Row Min)
A_1	0.15	0.05	0.15	0.05
A_2	0.15	0.2	0.12	0.12
A_3	0.0	0.05	1.0	0.0
A_4	0.3	0.3	0.4	0.3
(Column Max)	0.3	0.3	0.4	

Min (Column Max) = Max (Column Min) = 0.3.
The Saddle points are (A_4, B_1) and (A_4, B_2).
The value of the game is 0.3.

6. Conclusion

In this paper, a matrix game has been considered with pay-off elements as Neutrosophic set. There is a scope to try different type defuzzication like ranking method to find the score function. Our new approach gives a strategic solution and value of the game as an Neutrosophic set. The example given, establishes the theory on strong ground. It has strong impact on modern socio economic structure where conflicting interests exist.

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A SOLUTION TO ASSIGNMENT PROBLEMS WITH NEUTROSOPHIC COSTS

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Abstract

In this study, a single valued neutrosophic number has been presented in a new direction so that a decision maker has a scope of flexibility to choose different numbers in their study. Its structural characteristics are also studied here. Then this kind of number has been converted into a numeric value by means of a parameter (whose value is pre-assigned) to practice in real fields and using this, a ranking function is defined to compare two or more single valued neutrosophic numbers. In continuation, an assignment problem and its solution methodology have been developed in neutrosophic environment. Two real problems are solved to demonstrate the proposed method.

Keywords: Single Valued Neutrosophic Number, Ranking Function, Assignment Problem.

1. Introduction

Decision makers will have to go through an uncertain atmosphere in some cases whenever taking a decision in real world problems with incomplete and imprecise information since our daily life activities are being complicated from day to day. Classical set theory is not suitable to handle that situation as it only indicates whether an element either belongs or not to a set. Although the probability theory was an age old tool but the theory of fuzzy sets [15], the theory of intuitionistic fuzzy sets [2] brought a nice opportunity in that concern. Fuzzy set theory provides the degree of belongingness of an element whereas intuitionistic fuzzy set theory gives the degree of belongingness as well as the degree of not belongingness of an element. Both the theories are widely practiced in vague and uncertain atmosphere from their initiation.

To deal with uncertainty more precisely, Smarandache [11, 12] generalised the intuitionistic fuzzy set to neutrosophic set (NS). In neutrosophic logic, each object

is characterized by a triplet $(T,\ I,\ F)$ where T, I, F respectively refer the truth-membership value, the indeterminacy-membership value and the falsity-membership value. Intuitionistic fuzzy set theory can not provide the indeterminate information of an object. In NS theory $T, I, F \in]^{-}0, 1^{+}[$ and they are independent each. Accordingly, the limitation '$0 \leqslant$ membership value + non-membership value $\leqslant 1$' in intuitionistic fuzzy set theory was replaced by '$-0 \leqslant \sup T + \sup I + \sup F \leqslant 3^{+}$' in NS theory. In order to practice the NS theory in real field, Wang et al. [13] brought the concept of single valued neutrosophic set where $T, I, F \in [0, 1]$ only.

The notion of ranking of fuzzy numbers and intuitionistic fuzzy numbers is being widely practiced in decision making and optimization theory over a last few decades. Chen [4] proposed a fuzzy assignment model and proved some related theorems. Lin and Wen [7] solved an assignment problem with fuzzy interval number costs. Deli and Subas [5] have presented a ranking method of neutrosophic number and applied it to multi-attributive decision making problems. Some different kinds of ranking technique [6, 8, 10, 14] in decision making are reported here.

In the present paper the concept of single valued neutrosophic number (SVN-number) has been introduced in a different mode along with the study of its structural characteristics. Then a model of assignment problem with its solution methodology have been developed in neutrosophic environment. The proposed work are also demonstrated by two real problems. Organisation of this paper is as follows.

Some preliminary useful definitions are placed in Section 2. The concept of SVN-number has been introduced in a different way in Section 3. In Section 4, a model of assignment problem in neutrosophic environment has been presented with proper demonstration by two real problems. Finally, the conclusion of the present work has been drawn in Section 5.

2. Preliminaries

We recall some necessary definitions and results to make out the main thought.

Definition 2.1 (11)**.** *An NS B over the universe U is characterized by a triplet $(T_B,\ I_B,\ F_B)$ respectively called truth-membership function, indeterminacy-membership function and falsity-membership function where T_B, I_B, F_B are real standard or nonstandard subsets of $]^{-}0, 1^{+}[$ i.e., $T_B,\ I_B,\ F_B\ :\ U \rightarrow]^{-}0, 1^{+}[$. Here $1^{+} = 1 + \epsilon$, where 1 is its standard part and ϵ is its non-standard part. Similarly $^{-}0 = 0 - \epsilon$, where 0 is its standard part and ϵ is its non-standard part. Thus the NS B is defined as : $B = \{\langle u,\ T_B(u),\ I_B(u),\ F_B(u)\rangle : u \in U\}$ with $^{-}0 \leqslant \sup T_B(u) + \sup I_B(u) + \sup F_B(u) \leqslant 3^{+}$.*

Definition 2.2 (13)**.** *In an NS B over U, if the components $T_B(u), I(u)$ and $F(u)$ are all real standard elements of $[0, 1]$ for $u \in U$, then it is called a single valued*

neutrosophic set. Thus it is defined as : $B = \{\langle u, T_B(u), I_B(u), F_B(u)\rangle : u \in U\}$ *with* $T_B(u), I_B(u), F_B(u) \in [0,1]$ *and* $0 \leqslant \sup T_B(u) + \sup I_B(u) + \sup F_B(u) \leqslant 3$.

Definition 2.3 (3)**.** *Let B be an NS over the universal set U. The* $(\alpha,\ \beta,\ \gamma)$*-cut of U is denoted by* $B_{(\alpha,\beta,\gamma)}$ *and is defined as :* $B_{(\alpha,\beta,\gamma)} = \{u \in U\ :\ T_B(u) \geqslant \alpha,\ I_B(u) \leqslant \beta,\ F_B(u) \leqslant \gamma\}$ *where* $\alpha,\beta,\gamma \in [0,1]$ *with* $0 \leqslant \alpha+\beta+\gamma \leqslant 3$. *Clearly, it is a crisp subset U.*

Definition 2.4 (1)**.** *A fuzzy number A consists of a pair* $(A_L,\ A_R)$ *and satisfies the followings:*

(i) A_L *is left continuous function and bounded monotone increasing.*

(ii) A_R *is right continuous function and bounded monotone decreasing.*

(iii) $A_L(r) \leqslant A_R(r), r \in [0,1]$.

A trapezoidal fuzzy number B is expressed as $(a,\ b,\ \alpha,\ \beta)$ where $[a,\ b]$ is interval defuzzifier and $\alpha(>0), \beta(>0)$ are respectively called the left fuzziness, right fuzziness. The support of B is $(a-\alpha,\ b+\beta)$ and its membership function is:

$$B(x) = \begin{cases} \dfrac{1}{\alpha}(x-a+\alpha), & x \in [a-\alpha,\ a], \\ 1, & x \in [a,\ b], \\ \dfrac{1}{\beta}(b-x+\beta), & x \in [b,\ b+\beta], \\ 0, & otherwise. \end{cases}$$

In parametric form $B_L(r) = a-\alpha+\alpha r$, $B_R(r) = b+\beta-\beta r$.

For arbitrary trapezoidal fuzzy numbers $A = (A_L,\ A_R), B = (B_L,\ B_R)$ and scalar $k > 0$, the addition and scalar multiplication are defined by:

$$(A+B)_L(r) = A_L(r) + B_L(r),$$
$$(A+B)_R(r) = A_R(r) + B_R(r)$$

and

$$(kA)_L(r) = kA_L(r),$$
$$(kA)_R(r) = kA_R(r).$$

3. Single Valued Neutrosophic Number

In this section, the concept of SVN-number is presented in a new direction along with the study of its characteristics. Then a ranking function has been constructed in order to compare the SVN-numbers.

Definition 3.1. *An SVN-number* $\tilde{a} = \{[a_1,\ b_1,\ \sigma_1,\ \eta_1],\ [a_2,\ b_2,\ \sigma_2,\ \eta_2],\ [a_3,\ b_3,\ \sigma_3,\ \eta_3]\}$ *is one kind of NS defined over the set of real numbers* $\mathbb{R}$ *where* $\sigma_i(>0)$, $\eta_i(>0)$ *are respectively the left spreads, the right spreads and* $[a_i,\ b_i]$ *are the modal intervals of truth-membership, indeterminacy-membership and the falsity-membership functions for* $i = 1, 2, 3$, *respectively in a. The truth-membership, indeterminacy-membership and the falsity- membership functions are defined as follows:*

$$T_{\tilde{a}}(x) = \begin{cases} \dfrac{1}{\sigma_1}(x - a_1 + \sigma_1), & a_1 - \sigma_1 \leqslant x \leqslant a_1, \\ 1, & x \in [a_1,\ b_1], \\ \dfrac{1}{\eta_1}(b_1 - x + \eta_1), & b_1 \leqslant x \leqslant b_1 + \eta_1, \\ 0, & \textit{otherwise}. \end{cases}$$

$$I_{\tilde{a}}(x) = \begin{cases} \dfrac{1}{\sigma_2}(a_2 - x), & a_2 - \sigma_2 \leqslant x \leqslant a_2, \\ 0, & x \in [a_2,\ b_2], \\ \dfrac{1}{\eta_2}(x - b_2), & b_2 \leqslant x \leqslant b_2 + \eta_2, \\ 1, & \textit{otherwise}. \end{cases}$$

$$F_{\tilde{a}}(x) = \begin{cases} \dfrac{1}{\sigma_3}(a_3 - x), & a_3 - \sigma_3 \leqslant x \leqslant a_3, \\ 0, & x \in [a_3,\ b_3], \\ \dfrac{1}{\eta_3}(x - b_3), & b_3 \leqslant x \leqslant b_3 + \eta_3, \\ 1, & \textit{otherwise}. \end{cases}$$

In parametric form, an SVN-number $\tilde{a}$ consists of three pairs $(T^l_{\tilde{a}},\ T^u_{\tilde{a}}), (I^l_{\tilde{a}},\ I^u_{\tilde{a}}), (F^l_{\tilde{a}},\ F^u_{\tilde{a}})$ of functions $T^l_{\tilde{a}}(r),\ T^u_{\tilde{a}}(r),\ I^l_{\tilde{a}}(r),\ I^u_{\tilde{a}}(r),\ F^l_{\tilde{a}}(r),\ F^u_{\tilde{a}}(r)$, $r \in [0, 1]$ and satisfies the following requirements.

(i) $T^l_{\tilde{a}},\ I^u_{\tilde{a}},\ F^u_{\tilde{a}}$ are continuous function and bounded monotone increasing.

(ii) $T^u_{\tilde{a}}, I^l_{\tilde{a}}, F^l_{\tilde{a}}$ are continuous function and bounded monotone decreasing.

(iii) $T^l_{\tilde{a}}(r) \leqslant T^u_{\tilde{a}}(r), I^l_{\tilde{a}}(r) \geqslant I^u_{\tilde{a}}(r), F^l_{\tilde{a}}(r) \geqslant F^u_{\tilde{a}}(r)$

with $T^l_{\tilde{a}}(r) = a_1 - \sigma_1 + \sigma_1 r$, $T^u_{\tilde{a}}(r) = b_1 + \eta_1 - \eta_1 r$; $I^l_{\tilde{a}}(r) = a_2 - \sigma_2 r$, $I^u_{\tilde{a}}(r) = b_2 + \eta_2 r$ and $F^l_{\tilde{a}}(r) = a_3 - \sigma_3 r$, $F^u_{\tilde{a}}(r) = b_3 + \eta_3 r$.

Definition 3.2. *An SVN-number $\tilde{a}$ is called an SVTN-number if three modal intervals in a are equal. Thus $\tilde{C} = \langle [a_0,\ b_0,\ \sigma_1,\ \eta_1],\ [a_0,\ b_0,\ \sigma_2,\ \eta_2],\ [a_0,\ b_0,\ \sigma_3,\ \eta_3] \rangle$ is an SVTN-number.*

Let $\tilde{a} = \langle [a,\ a',\ \sigma_1,\ \eta_1],\ [a,\ a',\ \sigma_2,\ \eta_2],\ [a,\ a',\ \sigma_3,\ \eta_3] \rangle$ and $\tilde{b} = \langle [b,\ b',\ \xi_1,\ \delta_1],\ [b,\ b',\ \xi_2,\ \delta_2],\ [b,\ b',\ \xi_3,\ \delta_3] \rangle$ be two SVTN-numbers. Then for any real number x,

(i) Image of $\tilde{a}$:

$$-\tilde{a} = \langle [-a',\ -a,\ \eta_1,\ \sigma_1],\ [-a',\ -a,\ \eta_2,\ \sigma_2],\ [-a',\ -a,\ \eta_3,\ \sigma_3] \rangle.$$

(ii) Addition :

$$\begin{aligned} \tilde{a} + \tilde{b} \ = \ & \langle [a+b,\ a'+b',\ \sigma_1+\xi_1,\ \eta_1+\delta_1],\ [a+b,\ a'+b',\ \sigma_2+\xi_2,\ \eta_2+\delta_2], \\ & [a+b,\ a'+b',\ \sigma_3+\xi_3,\ \eta_3+\delta_3] \rangle. \end{aligned}$$

(iii) Scalar multiplication:

$$x\tilde{a} = \begin{cases} \langle ([xa,\ xa',\ x\sigma_1,\ x\eta_1], [xa,\ xa',\ x\sigma_2,\ x\eta_2], [xa,\ xa',\ x\sigma_3,\ x\eta_3] \rangle, \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for } x > 0 \\ \langle [xa',\ xa,\ -x\eta_1,\ -x\sigma_1],\ [xa',\ xa,\ -x\eta_2,\ -x\sigma_2],\ [xa',\ xa,\ -x\eta_3,\ -x\sigma_3] \rangle, \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for } x < 0. \end{cases}$$

Definition 3.3. *An SVTN-number $\tilde{a}$ is called an SVTrN-number if the modal interval in $\tilde{a}$ is reduced to a modal point i.e., if the end points of the modal interval are equal. Thus $\tilde{a} = \langle [a_0,\ \sigma_1,\ \eta_1][a_0,\ \sigma_2,\ \eta_2],\ [a_0,\ \sigma_3,\ \eta_3] \rangle$ is an SVTrN-number.*

Definition 3.4. *1. The zero SVTN-number is denoted by $\tilde{0}$ and is defined as :*

$$\tilde{0} = \langle [0,0,0,0], [0,0,0,0], [0,0,0,0] \rangle.$$

2. The zero SVTrN-number is denoted by $\tilde{0}$ and is defined as :

$$\tilde{0} = \langle [0,0,0],\ [0,0,0],\ [0,0,0] \rangle.$$

Definition 3.5. *The values of each component corresponding an SVN-number $\tilde{a} = \langle [a_1,\ b_1,\ \sigma_1,\ \eta_1], [a_2,\ b_2,\ \sigma_2,\ \eta_2], [a_3,\ b_3,\ \sigma_3,\ \eta_3] \rangle$ is calculated here using cut set.*

1. *Any $\alpha-$cut set of the SVN-number $\tilde{a}$ for truth membership function is denoted by $\tilde{a}_\alpha$ and is given by a closed interval as:*

$$\tilde{a}_\alpha = [a_1 - \sigma_1 + \sigma_1\alpha,\ b_1 + \eta_1 - \eta_1\alpha] \quad for \quad \alpha \in [0,1].$$

The value of $\tilde{a}$ for α-cut set is denoted by $v_T(\tilde{a})$ and is calculated as:

$$\begin{aligned} v_T(\tilde{a}) &= \int_0^1 [(a_1 - \sigma_1 + \sigma_1\alpha) + (b_1 + \eta_1 - \eta_1\alpha)]f(\alpha)d\alpha \\ &= \frac{1}{6}(3a_1 + 3b_1 - \sigma_1 + \eta_1), \end{aligned}$$

taking $f(\alpha) = \alpha$, where $f(\alpha) \in [0,1], f(0) = 0$ and $f(\alpha)$ is monotone increasing for $\alpha \in [0,1]$.

2. *Any β-cut set of the SVN-number $\tilde{a}$ for indeterminacy membership function is denoted by $\tilde{a}^\beta$ and is given by a closed interval as :*

$$\tilde{a}^\beta = [a_2 - \sigma_2\beta,\ b_2 + \eta_2\beta] \quad for \quad \beta \in [0,1].$$

The value of $\tilde{a}$ for β-cut set is denoted by $V_I(\tilde{a})$ and is calculated as :

$$\begin{aligned} V_I(\tilde{a}) &= \int_0^1 [(a_2 - \sigma_2\beta) + (b_2 + \eta_2\beta)]g(\beta)d\beta \\ &= \frac{1}{6}(3a_2 + 3b_2 - \sigma_2 + \eta_2), \end{aligned}$$

taking $g(\beta) = 1 - \beta$, where $g(\beta) \in [0,1], g(1) = 0$ and $g(\beta)$ is monotone decreasing for $\beta \in [0,1]$.

3. *Any $\gamma-$cut set of the SVN-number $\tilde{a}$ for falsity membership function is denoted by $\gamma_{\tilde{a}}$ and is given by a closed interval as :*

$$\gamma_{\tilde{a}} = [a_3 - \sigma_3\gamma,\ b_3 + \eta_3\gamma] \quad for \quad \gamma \in [0,1].$$

The value of $\tilde{a}$ for γ-cut set is denoted by $v_F(a)$ and is calculated as:

$$\begin{aligned} V_F(\tilde{a}) &= \int_0^1 [(a_3 - \sigma_3\gamma) + (b_3 + \eta_3\gamma)]h(\gamma)d\gamma \\ &= \frac{1}{6}(3a_3 + 3b_3 - \sigma_3 + \eta_3), \end{aligned}$$

taking $h(\gamma) = 1 - \gamma$, where $h(\gamma) \in [0,1]$, $h(1) = 0$ and $h(\gamma)$ is monotone decreasing for $\gamma \in [0,1]$.

Definition 3.6. *For $\theta \in [0,1]$, the θ-weighted value of an SVN-number $\tilde{b}$ is denoted by $V_\theta(\tilde{b})$ and is defined as :*

$$V_\theta(\tilde{b}) = \theta^n V_T(\tilde{b}) + (1-\theta^n)V_I(\tilde{b}) + (1-\theta^n)V_F(\tilde{b}),$$

n being any natural number.

Thus, the θ-weighted value for the SVN-number a stated in Definition 3.5 is:

$$V_\theta(\tilde{a}) = \frac{1}{6}[(3a_1+3b_1-\sigma_1+\eta_1)\theta^n+(3a_2+3b_2-\sigma_2+\eta_2)(1-\theta^n)+(3a_3+3b_3-\sigma_3+\eta_3)(1-\theta^n).$$

Proposition 3.1. *The θ-weighted value obeys the following disciplines for two SVN-numbers $\tilde{a}, \tilde{b}$.*

(i) $V_\theta(\tilde{a} \pm \tilde{b}) = V_\theta(\tilde{a}) \pm V_\theta(\tilde{b})$.

(ii) $V_\theta(\mu\tilde{a}) = \mu V_\theta(\tilde{a})$, *$\mu$ being any real number.*

(iii) $V_\theta(\tilde{a} - \tilde{a}) = V_\theta(\tilde{0})$.

(iv) $V_\theta(\tilde{a})$ *is monotone increasing or decreasing or constant according as* $V_T(\tilde{a}) > V_I(\tilde{a}) + V_F(\tilde{a})$ *or* $V_T(\tilde{a}) < V_I(\tilde{a}) + V_F(\tilde{a})$ *or* $V_T(\tilde{a}) = V_I(\tilde{a}) + V_F(\tilde{a})$ *respectively.*

Proof. We shall prove (iv) only. Remaining can be easily verified by taking two arbitrary SVN-numbers. Here,

$$V_\theta(\tilde{a}) = \theta^n V_T(\tilde{a}) + (1-\theta^n)V_I(\tilde{a}) + (1-\theta^n)V_F(\tilde{a})$$
$$\frac{dV_\theta(\tilde{a})}{d\theta} = n\theta^{n-1}[V_T(\tilde{a}) - (V_I(\tilde{a}) + V_F(\tilde{a}))]$$

Since $\theta \in [0,1]$, so $\frac{dV_\theta(\tilde{a})}{d\theta} >, < 0$ when $[V_T(\tilde{a}) - (v_I(\tilde{a}) + V_F(\tilde{a}))] >, < 0$ respectively. For $\theta \in (0,1]$, we have $\frac{dV_\theta(\tilde{a})}{d\theta} = 0$ when $[V_T(\tilde{a}) - (V_I(\tilde{a}) + V_F(\tilde{a}))] = 0$. This clears the fact. □

Definition 3.7. *Let SVN($\mathbf{R}$) be the set of all SVN-numbers defined over $\mathbf{R}$. For $\theta \in [0,1]$, a mapping $\mathfrak{R}_\theta : SVN(\mathbf{R}) \to \mathbf{R}$ is called a ranking function and it is defined as: $\mathfrak{R}_\theta(\tilde{a}) = V_\theta(a)$ for $a \in SVN(\mathbf{R})$.*

An useful method for comparing of SVN-numbers is by practice of ranking function. For $\tilde{a}, \tilde{b} \in SVN(\mathbf{R})$, their order is defined as follows :

$$V_\theta(\tilde{a}) > V_\theta \Leftrightarrow \tilde{a} > \mathfrak{R}_\theta$$
$$V_\theta(\tilde{a}) < V_\theta \Leftrightarrow \tilde{a} < \mathfrak{R}_\theta$$
$$V_\theta(\tilde{a}) = V_\theta \Leftrightarrow \tilde{a} = \mathfrak{R}_\theta\, \tilde{b}.$$

Proposition 3.2. *Consider two SVTN-numbers* $\tilde{c} = \langle[x,\ y,\ \sigma_1,\ \eta_1],\ [x,\ y,\ \sigma_2,\ \eta_2],\ [x,\ y,\ \sigma_3,\ \eta_3]\}$ *and* $\tilde{d} = \langle[p,\ q,\ \omega_1,\ \xi_1], [p,\ q,\ \omega_2,\ \xi_2], [p,\ q,\ \omega_3,\ \xi_3]\}$ *with* $[x,\ y] = [p,\ q]$. *Then* $\tilde{c} > \mathfrak{R}_\theta \tilde{d}$ *iff the followings hold.*

(i) $(\eta_1 + \omega_1) > (\sigma_1 + \xi_1)$ *for* $\theta = 1$.

(ii) $(\eta_2 + \eta_3) + (\omega_2 + \omega_3) > (\sigma_2 + \sigma_3) + (\xi_2 + \xi_3)$ *for* $\theta = 0$.

Proof. (i)

$$
\begin{aligned}
&\eta_1 + \omega_1 > \sigma_1 + \xi_1\\
\Leftrightarrow &(\eta_1 - \sigma_1) > (\xi_1 - \omega_1)\\
\Leftrightarrow &(\eta_1 - \sigma_1)\theta^n > (\xi_1 - \omega_1)\theta^n \text{ (as } \theta = 1)\\
\Leftrightarrow &(\eta_1 - \sigma_1)\theta^n + \langle(\eta_2 - \sigma_2) + (\eta_3 - \sigma_3)\}(1 - \theta^n)\\
&> (\xi_1 - \omega_1)\theta^n + \{(\xi_2 - \omega_2) + (\xi_3 - \omega_3)\}(1 - \theta^n) \text{ (as } \theta = 1)\\
\Leftrightarrow &\frac{1}{6}[(3x + 3y + \eta_1 - \sigma_1)\theta^n + \{(3x + 3y + \eta_2 - \sigma_2) + (3x + 3y + \eta_3 - \sigma_3)\}(1 - \theta^n)]\\
&> \frac{1}{6}[(3p + 3q + \xi_1 - \omega_1)\theta^n + \{(3p + 3q + \xi_2 - \omega_2) + (3p + 3q + \xi_3 - \omega_3)\}(1 - \theta^n)]\\
&\text{(as } [x,\ y] = [p,\ q])\\
\Leftrightarrow &V_\theta(\tilde{c}) > V_\theta(\tilde{d}) \Leftrightarrow \tilde{c} > \mathfrak{R}_\theta \tilde{d} \text{ with } [x,\ y] = [p,\ q].
\end{aligned}
$$

(ii)

$$
\begin{aligned}
&(\eta_2 + \eta_3) + (\omega_2 + \omega_3) > (\sigma_2 + \sigma_3) + (\xi_2 + \xi_3)\\
\Leftrightarrow &\{(\eta_2 - \sigma_2) + (\eta_3 - \sigma_3)\}(1 - \theta^n) > \{(\xi_2 - \omega_2) + (\xi_3 - \omega_3)\}(1 - \theta^n) \text{ (as } \theta = 0)\\
\Leftrightarrow &(\eta_1 - \sigma_1)\theta^n + \{(\eta_2 - \sigma_2) + (\eta_3 - \sigma_3)\}(1 - \theta^n)\\
&\qquad > (\xi_1 - \omega_1)\theta^n + \{(\xi_2 - \omega_2) + (\xi_3 - \omega_3)\}(1 - \theta^n) \text{ (as } \theta = 0)\\
\Leftrightarrow &\frac{1}{6}[(3x + 3y + \eta_1 - \sigma_1)\theta^n + \{(3x + 3y + \eta_2 - \sigma_2) + (3x + 3y + \eta_3 - \sigma_3)\}(1 - \theta^n)]\\
&> \frac{1}{6}[(3p + 3q + \xi_1 - \omega_1)\theta^n + \{(3p + 3q + \xi_2 - \omega_2) + (3p + 3q + \xi_3 - \omega_3)\}(1 - \theta^n)]\\
&\text{(as } [x,\ y] = [p,\ q])\\
\Leftrightarrow &V_\theta(\tilde{c}) > V_\theta(\tilde{d}) \Leftrightarrow \tilde{c} > \mathfrak{R}_\theta \tilde{d} \text{ with } [x,\ y] = [p,\ q].
\end{aligned}
$$

Similar conclusion can be drawn in case of SVTrN-numbers. □

4. Assignment Problem with Neutrosophic Costs

An assignment problem in crisp sense can be defined by an $n \times n$ cost matrix of real numbers as given in Table 1 which assigns men to jobs, jobs to machines etc. It is also assumed that one person can perform one job at a time and thus all the jobs will be assigned to all available persons and so on in other cases. The problem is optimal if it minimizes the total cost or minimizes the total time or maximizes the profit of performing all the jobs. where c_{ij} is the cost of assigning the j^{th} job to the i^{th} person. Mathematically, the problem can be put as :

Table 1: Tabular Form of Assignment Problem.

		JOBS					
		J_1	J_2	J_3	$\cdots$	$\cdots$	J_n
PERSONS	P_1	c_{11}	c_{12}	c_{13}	$\cdots$	$\cdots$	c_{1n}
	P_2	c_{21}	c_{22}	c_{23}	$\cdots$	$\cdots$	c_{2n}
	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
	P_n	c_{n1}	c_{n2}	c_{n3}	$\cdots$	$\cdots$	c_{nn}

Determine $x_{ij} \geqslant 0, i, j = 1, 2, \cdots, n$ which

optimize $z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$

such that

$$\sum_{j=1}^{n} x_{ij} = 1, 1 \leqslant i \leqslant n$$

and

$$\sum_{i=1}^{n} x_{ij} = 1, 1 \leqslant j \leqslant n$$

with

$$x_{ij} = \begin{cases} 1, & \text{if the } j^{\text{th}} \text{ job is assigned to the } i^{\text{th}} \textit{ person} \\ 0, & \text{otherwise.} \end{cases}$$

Now if we consider the costs c_{ij} as SVN-numbers (we write $\tilde{a}_{ij}$), then the total cost $\tilde{z} = \mathfrak{R}_\theta \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{a}_{ij} x_{ij}$ becomes an SVN-number. Then, we can not apply the crisp concept directly to optimize it. We shall adopt the following technique for that.

(Solution approach of assignment problem with neutrosophic costs)

We now develop a method to find a solution of this class of problem. In the present study, the costs in assignment problem are taken as SVN-numbers. We shall first find

the θ-weighted value function of each neutrosophic cost and thus we shall get a crisp cost matrix from a neutrosophic cost matrix for a pre-assigned θ. Now applying the computational procedure in crisp sense, we get optimal solutions of the assignment problem. Consequently the optimal value of the original problem can be calculated as:

$$\tilde{z} = \mathfrak{R}_\theta \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{a}_{ij} x_{ij}$$

which gives a numeric value using θ-weighted value function for that θ.

Definition 4.1. *An SVN-number is said to be constant if it is θ independent after transforming it into a θ-weighted value function.*

For if $\tilde{b} = \langle [5,8,3,43], [5,8,2,2], [5,8,1,2] \rangle$ is a constant SVTN-number as $V_\theta(\tilde{b}) = 13.17$ (approx) whatever the value of θ is.

Theorem 4.1. *If a constant SVN-number be added to any row and/or any column of the cost matrix of an assignment problem in neutrosophic environment, then the resulting assignment problem has the same optimal solution as the original problem.*

Proof. Let $[\tilde{a}_{ij}]$ be the cost matrix and suppose two constant SVN-numbers $\tilde{\alpha}_i$, $\tilde{\beta}_j$ be added to the i-th row and j-th column respectively for $1 \leqslant i, j \leqslant m$. Let $[\tilde{a}'_{ij}]$ be the new cost matrix where $\tilde{a}'_{ij} = \tilde{a}_{ij} + \tilde{\alpha}_i + \tilde{\beta}_j$ and the two objective functions be $\tilde{z} = \mathfrak{R}_\theta \sum_{i=1}^{m} \sum_{j=1}^{m} a_{ij} x_{ij}, \tilde{z}' = \mathfrak{R}_\theta \sum_{i=1}^{m} \sum_{j=1}^{m} \tilde{a}'_{ij} x_{ij}$. Now,

$$\tilde{z}' = \mathfrak{R}_\theta \sum_{i=1}^{m} \sum_{j=1}^{m} \tilde{a}'_{ij} x_{ij} \Rightarrow V_\theta(\tilde{z}') = V_\theta \left(\sum_{i=1}^{m} \sum_{j=1}^{m} \tilde{a}'_{ij} x_{ij} \right)$$

$$\Rightarrow V_\theta(\tilde{z}') = \sum_{i=1}^{m} \sum_{j=1}^{m} V_\theta(\tilde{a}'_{ij} x_{ij})$$

$$\Rightarrow V_\theta(\tilde{z}') = \sum_{i=1}^{m} \sum_{j=1}^{m} V_\theta[(\tilde{a}_{ij} + \tilde{\alpha}_i + \tilde{\beta}_j) x_{ij}]$$

$$\Rightarrow V_\theta(\tilde{z}') = \sum_{i=1}^{m} \sum_{j=1}^{m} V_\theta(\tilde{a}_{ij} x_{ij}) + \sum_{i=1}^{m} \sum_{j=1}^{m} V_\theta(\tilde{\alpha}_i\, x_{ij}) + \sum_{i=1}^{m} \sum_{j=1}^{m} V_\theta(\tilde{\beta}_j x_{ij})$$

$$\Rightarrow V_\theta(\tilde{z}') = V_\theta \left(\sum_{i=1}^{m} \sum_{j=1}^{m} \tilde{a}_{ij} x_{ij} \right) + V_\theta \left(\sum_{i=1}^{m} \sum_{j=1}^{m} \tilde{\alpha}_i x_{ij} \right) + V_\theta \left(\sum_{i=1}^{m} \sum_{j=1}^{m} \tilde{\beta}_j x_{ij} \right)$$

$$\Rightarrow V_\theta(\tilde{z}') = V_\theta(\tilde{z}) + V_\theta\left(\sum_{i=1}^{m}\tilde{\alpha}_i\sum_{j=1}^{m}x_{ij}\right) + V_\theta\left(\sum_{i=1}^{m}\tilde{\beta}_j\sum_{j=1}^{m}x_{ij}\right)$$

$$\Rightarrow V_\theta(\tilde{z}') = V_\theta(\tilde{z}) + V_\theta\left(\sum_{i=1}^{m}\tilde{\alpha}_i\right) + V_\theta\left(\sum_{j=1}^{m}\tilde{\beta}_j\right) \text{ as } \left[\sum_{i=1}^{m}x_{ij} = \sum_{j=1}^{m}x_{ij} = 1\right]$$

$$\Rightarrow V_\theta(\tilde{z}' - \tilde{z}) = \sum_{i=1}^{m}V_\theta(\tilde{\alpha}_i) + \sum_{j=1}^{m}V_\theta(\tilde{\beta}_j)$$

$$\Rightarrow V_\theta(\tilde{z}' - \tilde{z}) = \kappa + \zeta \quad \left[\text{for } \sum_{i=1}^{m}V_\theta(\tilde{\alpha}_i) = \kappa,\ \sum_{j=1}^{m}V_\theta(\tilde{\beta}_j) = \zeta\right].$$

Thus the two objective functions $\tilde{z}'$ and $\tilde{z}$ differ by a constant not involving any decision variable x_{ij} and so the original problem as well as the new problem both attain same optimal solution. □

5. Numerical Example

Here, assignment problems with the costs as both SVTN-numbers and SVTrN-numbers have been solved by use of proposed method. For simplicity, we define the $\theta-$weighted value function for $n = 1$ in rest of this chapter.

Example 5.1. *A car owner wishes to run his three buses* $(B_1,\ B_2,\ B_3)$ *in three different routes* $(R_1,\ R_2,\ R_3)$. *The maintenance cost of each bus per kilometer in three different routes is given by the following cost matrix* $[\tilde{a}_{ij}]$ *whose elements are SVTN-numbers. Allot the route for each bus so that the maintenance cost in aggregate becomes minimum.*

$$[\tilde{a}_{ij}] = \begin{array}{c} \\ B_1 \\ B_2 \\ B_3 \end{array}\begin{array}{c} \begin{array}{ccc} R_1 & R_2 & R_3 \end{array} \\ \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33} \end{pmatrix} \end{array}$$

where

$$\tilde{a}_{11} = \langle[10,13,3,6],\ [10,13,8,5],\ [10,13,2,9]\rangle,$$
$$\tilde{a}_{12} = \langle[8,11,4,6],\ [8,11,6,9],\ [8,11,5,10]\rangle,$$
$$\tilde{a}_{13} = \langle[5,9,3,5],[5,9,2,9],[5,9,1,12]\rangle,$$
$$\tilde{a}_{21} = \langle[4,7,3,9],[4,7,1,6],[4,7,2,8]\rangle,$$
$$\tilde{a}_{22} = \langle[7,12,5,12],[7,12,4,8],[7,12,3,10]\rangle,$$

$$\tilde{a}_{23} = \langle[9,10,2,7],[9,10,6,12],[9,10,4,5]\rangle,$$
$$\tilde{a}_{31} = \langle[3,8,1,4],[3,8,2,7],[3,8,2,2]\rangle,$$
$$\tilde{a}_{32} = \langle[6,7,4,9],[6,7,5,5],[6,7,2,10]\rangle,$$
$$\tilde{a}_{33} = \langle[12,18,10,2],\ [12,18,8,7],\ [12,18,5,9]\rangle.$$

The problem can be put in the following form:

$$\text{Min}\,\tilde{z} = \mathfrak{R}_\theta\,\tilde{a}_{11}x_{11}+\tilde{a}_{12}x_{12}+\tilde{a}_{13}x_{13}+\tilde{a}_{21}x_{21}+\tilde{a}_{22}x_{22}+\tilde{a}_{23}x_{23}+\tilde{a}_{31}x_{31}+\tilde{a}_{32}x_{32}+\tilde{a}_{33}x_{33}$$

such that

$$\begin{aligned}
x_{11}+x_{12}+x_{13} &= 1,\\
x_{11}+x_{21}+x_{31} &= 1,\\
x_{21}+x_{22}+x_{23} &= 1,\\
x_{12}+x_{22}+x_{32} &= 1,\\
x_{31}+x_{32}+x_{33} &= 1,\\
x_{13}+x_{23}+x_{33} &= 1
\end{aligned}$$

with $x_{ij} \in \{0,1\}$.

The $\theta-$weighted value for the SVTN-numbers are calculated as:

$$V_\theta(\tilde{a}_{11}) = \frac{1}{6}(142-70\theta), \quad V_\theta(\tilde{a}_{12}) = \frac{1}{6}(122-63\theta), \quad V_\theta(\tilde{a}_{13}) = \frac{1}{6}(102-58\theta),$$
$$v_\theta(\tilde{a}_{21}) = \frac{1}{6}(77-38\theta), \quad v_\theta(\tilde{a}_{22}) = \frac{1}{6}(125-61\theta), \quad V_\theta(\tilde{a}_{23}) = \frac{1}{6}(121-59\theta),$$
$$V_\theta(\tilde{a}_{31}) = \frac{1}{6}(71-35\theta), \quad V_\theta(\tilde{a}_{32}) = \frac{1}{6}(86-42\theta), \quad V_\theta(\tilde{a}_{33}) = \frac{1}{6}(183-101\theta).$$

Assuming $\theta = 0.8$, we get the initial assignment Table 2 as follows (each entry taking two decimal places):

Table 2: Initial Assignment Table

14.33	11.93	9.27
7.77	12.70	12.30
7.17	8.73	17.03

Now applying the computational procedure as in crisp environment, we find the allocation of busses in different route with the optimal costs attained as follows :

$$B_1 \to R_3,\ B_2 \to R_1,\ B_3 \to R_2.$$

The optimal solutions are:

$$x_{13} = x_{21} = x_{32} = 1,\ x_{11} = x_{12} = x_{22} = x_{23} = x_{31} = x_{33} = 0.$$

and Min $\tilde{z} = \mathfrak{R}_\theta \tilde{a}_{13} + \tilde{a}_{21} + \tilde{a}_{32}$ which becomes Rs. 25.77 using θ-weighted value function for $\theta = 0.8$.

Remark 5.1. *Depending on θ chosen, the number of iteration in computational procedure to reach at optimality stage may vary only but the optimal solution will remain unchange. However, θ plays an important role to produce the aggregate optimal value of an assignment problem in neutrosophic environment. Since the total maintenance cost of a bus depends on so many factors, we assume θ as the degree of smoothness of road condition in the present problem. Following Table 3 shows the variation of optimal value with respect to different θ in the Example 4.1. Here $V_\theta(\tilde{z}) = \frac{1}{6}(265 - 138\theta)$.*

Table 3: Table for Variation of Optimal Value

θ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$V_\theta(\tilde{z})$	44.17	41.87	39.57	37.27	34.97	32.67	30.37	28.07	25.77	23.47	21.17

Example 5.2. *Four machines $(M_1,\ M_2,\ M_3,\ M_4)$ are available to perform four jobs $(J_1,\ J_2,\ J_3,\ J_4)$ in a company. The machines M_1 and M_3 can not perform the jobs J_3 and J_1 respectively. The adjacent matrix provides the approx required time (in minutes) to perform the jobs by different machines (given in SVTN-numbers and SVTrN-numbers). How would the jobs be allotted to minimize the total time. Provided that one machine will perform one job only.*

$$[\tilde{a}_{ij}] = \begin{array}{c} \\ M_1 \\ M_2 \\ M_3 \\ M_4 \end{array} \begin{array}{c} \begin{array}{cccc} J_1 & J_2 & J_3 & J_4 \end{array} \\ \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & -- & \tilde{a}_{14} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} & \tilde{a}_{24} \\ -- & \tilde{a}_{32} & \tilde{a}_{33} & \tilde{a}_{34} \\ \tilde{a}_{41} & \tilde{a}_{42} & \tilde{a}_{43} & \tilde{a}_{44} \end{pmatrix} \end{array}$$

where

$$\tilde{a}_{11} = \langle [5,8,4,3], [5,8,1,9], [5,8,2,6] \rangle,$$
$$\tilde{a}_{12} = \langle [10,8,2], [10,2,8], [10,5,13] \rangle,$$

$$\tilde{a}_{14} = \langle[7,11,2,5],\ [7,11,5,9],\ [7,11,1,7]\rangle,$$
$$\tilde{a}_{21} = \langle[15,8,9],\ [15,4,11],\ [15,6,10]\rangle,$$
$$\tilde{a}_{22} = \langle[3,7,2,4], [3,7,2,6], [3,7,1,10]\rangle,$$
$$\tilde{a}_{23} = \langle[5,8,4,10], [5,8,3,12], [5,8,2,9]\rangle,$$
$$\tilde{a}_{24} = \langle[6,5,8], [6,2,9], [6,3,5]\rangle,$$
$$\tilde{a}_{32} = \langle[10,14,8,7], [10,14,6,10], [10,14,3,4]\rangle,$$
$$\tilde{a}_{33} = \langle[9,2,3], [9,1,5], [9,3,8]\rangle,$$
$$\tilde{a}_{34} = \langle[8,12,6,5], [8,12,4,10], [8,12,5,11]\rangle,$$
$$\tilde{a}_{41} = \langle[8,4,8], [8,2,3], [8,5,10]\rangle,$$
$$\tilde{a}_{42} = \langle[9,13,2,5], [9,13,7,8], [9,13,5,9]\rangle,$$
$$\tilde{a}_{43} = \langle[7,11,6,5],\ [7,11,4,3],\ [7,11,2,8]\rangle,$$
$$\tilde{a}_{44} = \langle[12,10,4],\ [12,7,6],\ [12,3,3]\rangle.$$

Since the machines M_1 and M_3 are unable to perform the jobs J_3 and J_1 respectively, so we can assume two SVN-numbers ($\tilde{a}_{13}$ and $\tilde{a}_{31}$) whose θ-weighted value functions provide very large time in the cells $(1,3)$ and $(3,1)$ respectively.

$$\begin{aligned}\tilde{a}_{13} &= \langle[16,30,2,9],\ [16,30,8,15],\ [16,30,7,11]\rangle,\\ \tilde{a}_{31} &= \langle[18,22,6,13],\ [18,22,3,17],\ [18,22,9,28]\rangle.\end{aligned}$$

Then the problem can be put in the following form:

$$\begin{aligned}\text{Min } \tilde{z} &= \mathfrak{R}_\theta\tilde{a}_{11}x_{11} + \tilde{a}_{12}x_{12} + \tilde{a}_{13}x_{13} + \tilde{a}_{14}x_{14} + \tilde{a}_{21}x_{21} + \tilde{a}_{22}x_{22} + \tilde{a}_{23}x_{23} + \tilde{a}_{24}x_{24}\\ &\quad \tilde{a}_{31}x_{31} + \tilde{a}_{32}x_{32} + \tilde{a}_{33}x_{33} + \tilde{a}_{34}x_{34} + \tilde{a}_{41}x_{41} + \tilde{a}_{42}x_{42} + \tilde{a}_{43}x_{43} + \tilde{a}_{44}x_{44}\\ \text{such that } & x_{11} + x_{12} + x_{13} + x_{14} = 1,\\ & x_{11} + x_{21} + x_{31} + x_{41} = 1,\\ & x_{21} + x_{22} + x_{23} + x_{24} = 1,\\ & x_{12} + x_{22} + x_{32} + x_{42} = 1,\\ & x_{31} + x_{32} + x_{33} + x_{34} = 1,\\ & x_{13} + x_{23} + x_{33} + x_{43} = 1,\\ & x_{41} + x_{42} + x_{43} + x_{44} = 1,\\ & x_{14} + x_{24} + x_{34} + x_{44} = 1\\ & \text{with } x_{ij} \in \{0,1\}.\end{aligned}$$

The θ-weighted value for the SVTN-numbers are calculated as:

$$V_\theta(\tilde{a}_{11}) = \frac{1}{6}(90 - 52\theta), \qquad V_\theta(\tilde{a}_{12}) = \frac{1}{6}(134 - 80\theta),$$

$$V_\theta(\tilde{a}_{13}) = \frac{1}{6}(287 - 142\theta), \qquad V_\theta(\tilde{a}_{14}) = \frac{1}{6}(118 - 61\theta),$$
$$v_\theta(\tilde{a}_{21}) = \frac{1}{6}(191 - 100\theta), \qquad v_\theta(\tilde{a}_{22}) = \frac{1}{6}(73 - 41\theta),$$
$$V_\theta(\tilde{a}_{23}) = \frac{1}{6}(94 - 49\theta), \qquad v_\theta(\tilde{a}_{24}) = \frac{1}{6}(\text{S1} - 42\theta),$$
$$V_\theta(\tilde{a}_{31}) = \frac{1}{6}(273 - 146\theta), \qquad V_\theta(\tilde{a}_{32}) = \frac{1}{6}(149 - 78\theta),$$
$$V_\theta(\tilde{a}_{33}) = \frac{1}{6}(117 - 62\theta), \qquad V_\theta(\tilde{a}_{34}) = \frac{1}{6}(132 - 73\theta),$$
$$v_\theta(\tilde{a}_{41}) = \frac{1}{6}(102 - 50\theta), \qquad v_\theta(\tilde{a}_{42}) = \frac{1}{6}(137 - 68\theta),$$
$$V_\theta(\tilde{a}_{43}) = \frac{1}{6}(113 - 60\theta), \qquad v_\theta(\tilde{a}_{44}) = \frac{1}{6}(143 - 77\theta).$$

Taking $\theta = 0.4$, we get the initial assignment Table 4 as follows (each entry taking two decimal places):

Table 4: Initial Assignment Table

11.53	17	38.37	15.6
25.17	9.43	12.40	10.70
35.77	19.63	15.37	17.13
13.67	18.30	14.83	18.70

The computational procedure in crisp environment of Table 4 gives the final iteration as in Table 5.

Table 5: Optimal Assignment Table

0	3.82	25.68	1.15
17.39	0	3.46	0
21.56	3.77	0	0
0	2.98	0	2.11

Thus we get the allotment of jobs to the machines with a total minimum time taken as follows :

$$M_1 \to J_1,\ M_2 \to J_2,\ M_3 \to J_4,\ M_4 \to J_3.$$

The optimal solutions are:
$x_{11} = x_{22} = x_{34} = x_{43} = 1$, $x_{12} = x_{13} = x_{14} = x_{21} = x_{23} = x_{24} = x_{31} = x_{32} = x_{33} =$

$x_{41} = x_{42} = x_{44} = 0$. and Min $\tilde{z} =_{\Re_\theta} a_{11} + a_{22} + a_{34} + a_{43}$ which becomes 52.93 minutes using θ-weighted value function for $\theta = 0.4$.

(Sensitivity analysis in post optimization)

The variation of optimal time (taking two decimal places) with respect to different θ given in Table 6 for the Example 4.2 Here $V_\theta(\tilde{z}) = \frac{1}{6}(408 - 226\theta)$ where θ denotes the associate infrastructure facility provided by company, we claim.

Table 6: Table for Variation of Optimal Value

θ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$V_\theta(\tilde{z})$	68	64.23	60.47	56.7	52.93	49.17	45.4	41.63	37.87	34.1	30.33

6. Conclusion

In this paper, an SVN-number is presented in a new direction with the study of its characteristics. A linear ranking function is then defined to compare two or more such numbers. In continuation, a model of assignment problem is developed in neutrosophic environment where each entry of cost matrix is an SVN-number. The proposed concept is illustrated by two practical problems. The problems are stated and solved numerically. In post optimization period, the sensitivity analysis of these problems also have been performed with respect to different values of parameter. The fact has a nice significance in any kind of system solvable by the proposed approach. The notion of ranking function adopted here will bring a new arena in research and development of linear programming problem and its associated field, we expect.

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A LINEAR FRACTIONAL PROGRAMMING PROBLEM IN NEUTROSOPHIC ENVIRONMENT

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Abstract

In this paper a fractional linear programming problem is solved in Neutrosophic Environment. The proposed method for solving is based on modified simplex procedure for fractional function. This method is explained details with the help of Numerical illustration.

Keywords: Fractional Linear Programming Problem, Neutrosophic Environment, Modified Simplex Procedure, Fractional Function.

1. Introduction

Fractional programming problem is that in which the objective function is the ratio of numerator and denominator. These types of problems have attracted considerable research and interest. Since these are useful in production planning, financial and corporate planning, health care and hospital planning etc.

Algorithms for solving linear fractional programming problems are well known by many. Charnes and Copper [1] replaces a linear fractional program by one equivalent linear fractional program, in which one extra constraint and one extra variable has been added. The usual simplex algorithm computes the optimum solution. Chadha-Caldile [2] solves a system of linear of inequalities in which the objective function is expressed as one of the constraint along with the given set of linear constraints of the problem. Resently Tantawy [7] has suggested a feasible direction approach and a duality approach to solve a linear fractional programming problem. Here our aim is to find the solution of fractional programming problems in Neutrosophic environment (i.e. objective function is the ratio of numerator and denominator of linear functions). For it, we use modified simplex method. These methods are very easy to understand and apply.

Preliminaries are given in the next section. The steps of the proposed Modified Simplex Algorithm in Neutrosophic Environment are presented. Numerical illustration have been worked out in the paper and finally we present the references.

2. Preliminaries

Definition 2.1 (**Single Valued Neutrosophic Number**). *Let $w_a, u_a, y_a \in [0,1]$ be any real numbers. A single valued neutrosophic number $\tilde{a} = \{< a_1, b_1, c_1 >; w_a, u_a, y_a\}$ is defined as a special neutrosophic set on the real number set R, Whose truth-membership, indeterminacy-membership, and a flasty-membership are given as follows:-*

$$\mu_a(x) = \begin{cases} \frac{(x-a)w_a}{b_1-a_1} & (a_1 \leqslant x < b_1) \\ w_a & (x = b_1) \\ \frac{(c_1-x)w_a}{c_1-d_1} & (b_1 \leqslant x < c_1) \\ 0 & \textit{Otherwise} \end{cases}$$

$$\mu_a(x) = \begin{cases} \frac{(b_1-x+u_a(x-a))}{b_1-a_1} & (a_1 \leqslant x < b_1) \\ u_a & (x = b_1) \\ \frac{(x-b_1+u_a(c_1-x))}{c_1-d_1} & (b_1 \leqslant x < c_1) \\ 1 & \textit{Otherwise} \end{cases}$$

$$\mu_a(x) = \begin{cases} \frac{(b_1-x+y_a(x-a))}{b_1-a_1} & (a_1 \leqslant x < b_1) \\ y_a & (x = b_1) \\ \frac{(x-b_1y_a(c_1-x))}{c_1-d_1} & (b_1 \leqslant x < c_1) \\ 1 & \textit{Otherwise} \end{cases}$$

respectively.
If $a_1 \geqslant 0$ and at least $c_1 > 0$, then $\tilde{a} =< (a_1, b_1, c_1); w_a, u_a, y_a >$ is called a positive single valued triangular neutrosophic number, denoted by $\tilde{a} > 0$.

Similarly, If $c_1 \leqslant 0$ and at least $a_1 < 0$, then $\tilde{a} =< (a_1, b_1, c_1); w_a, u_a, y_a >$ is called a negative single valued triangular neutrosophic number, denoted by $\tilde{a} < 0$.

An ill-known quantity of the range, which is approximately equal to b_1 may be represented by a single valued triangular neutrosophic number $\tilde{a} = \{\langle a_1, b_1, c_1\rangle; w_a, u_a, y_a\}$.

Definition 2.2. *Let $\tilde{a}^n = \{\langle a_1, b_1, c_1\rangle; w_a, u_a, y_a\}$ and $\tilde{b}^n = \{\langle a_2, b_2, c_2\rangle; w_b, u_b, y_b\}$ be two single valued triangular neutrosophic numbers and $\gamma \neq 0$. Then*

1. $\tilde{a}^n + \tilde{b}^n = \{\langle a_1 + a_2, b_1 + b_2, c_1 + c_2\rangle; w_a \wedge w_b, u_a \vee u_b, y_a \vee y_b\}$

2. $\tilde{a}^n - \tilde{b}^n = \{\langle a_1 - c_2, b_1 - b_2, c_1 - a_2\rangle ; w_a \wedge w_b, u_a \vee u_b, y_a \vee y_b\}$

3. $$\tilde{a}^n.\tilde{b}^n = \begin{cases} \{\langle a_1a_2, b_1b_2, c_1c_2\rangle ; w_a \wedge w_b, u_a \vee u_b, y_a \vee y_b\}, (c_1 > 0, c_2 > 0) \\ \{\langle a_1c_2, b_1b_2, c_1a_2\rangle ; w_a \wedge w_b, u_a \vee u_b, y_a \vee y_b\}, (c_1 < 0, c_2 > 0) \\ \{\langle c_1c_2, b_1b_2, a_1a_2\rangle ; w_a \wedge w_b, u_a \vee u_b, y_a \vee y_b\}, (c_1 < 0, c_2 < 0) \end{cases}$$

4. $$\frac{\tilde{a}^n}{\tilde{b}^n} = \begin{cases} \left\{\left\langle \frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}\right\rangle ; w_a \wedge w_b, u_a \vee u_b, y_a \vee y_b\right\}, (c_1 > 0, c_2 > 0) \\ \left\{\left\langle \frac{a_1}{c_2}, \frac{b_1}{b_2}, \frac{c_1}{a_2}\right\rangle ; w_a \wedge w_b, u_a \vee u_b, y_a \vee y_b\right\}, (c_1 < 0, c_2 > 0) \\ \left\{\left\langle \frac{c_1}{c_2}, \frac{b_1}{b_2}, \frac{a_1}{a_2}\right\rangle ; w_a \wedge w_b, u_a \vee u_b, y_a \vee y_b\right\}, (c_1 < 0, c_2 < 0) \end{cases}$$

5. $$\gamma\tilde{a}^n = \begin{cases} \{(\gamma a_1^l, \gamma b_1^m, \gamma c_1^u); \tau_a \wedge \tau_b, i_a \vee i_b, \omega_a \vee \omega_b\}, (\gamma > 0) \\ \{(\gamma c_1^u, \gamma b_1^m, \gamma a_1^l); \tau_a \wedge \tau_b, i_a \vee i_b, \omega_a \vee \omega_b\}, (\gamma < 0). \end{cases}$$

2.1. Neutrosophic Linear Fractional Programming Problem

A maximization linear fractional programming problem may be stated as:

$$Max.\tilde{z}^n = \frac{c\tilde{x}^n + \alpha}{d\tilde{x}^n + \beta} \tag{2.1}$$

$$\begin{aligned} s.t \qquad & A\tilde{x}^n \leqslant \tilde{B}^n \\ & \tilde{x}^n \geqslant \{\langle 0, 0, 0\rangle ; 1, 0, 0\} \end{aligned}$$

Where $\tilde{x}^n, \tilde{z}^n, \tilde{B}^n$ are Neutrosophic Single Valued Triangular Number.

3. Modified Simplex Algorithm in Neutrosophic Environment

Step: 1. *Converts the inequality constraints to equations by introducing the non-negative slack or surplus variables. The coefficients of slack or surplus variables are always taken zero in the objective function.*

Step: 2. *Constructs the simplex table by using the following notations. Let* $\tilde{x}_A^n$ *be the initial basic feasible solution of the given problem such that*

$$A\tilde{x}_B^n = \tilde{B}^n$$

$$\begin{aligned} \tilde{x}_B^n &= \tilde{B}^n A^{-1} \\ \text{Where } \tilde{B}^n &=< (a,b,c); w_a, u_a, y_a >, \\ Z_1 &= c_B \tilde{x}_B + \alpha \\ Z_2 &= d_B \tilde{x}_B + \beta \end{aligned}$$

where c_B and d_B are the vectors having their components as the coefficients associated with the basic variables in the numerator and denominator of the objective function respectively.

Step: 3. *First we compute the values for $\tilde{z}_1, \tilde{z}_2$ and z. Where*

$$\begin{aligned} \tilde{z}_1 &= \sum \{(co - efficientofc_j) \times \tilde{B}^n\} + \alpha \\ \tilde{z}_2 &= \sum \{(co - efficientofd_j) \times \tilde{B}^n\} + \beta \\ z &= \frac{\tilde{z}_1}{z_2} \end{aligned}$$

Step: 4. *To calculate Δ_j^1 row is $c_j - \sum c_B \tilde{x}_B^n$ for each variable x_j. In this way we calculate the values of Δ_j^2 row is $d_j - \sum d_B \tilde{x}_B^n$ for each variable x_j.*

Step: 5. *Now, compute the evaluation Δ_j for each variable x_j (column vector x_j) by the formula*

$$\Delta_j = \tilde{Z}_2 \left(c_j - \sum c_B \tilde{x}_B^n \right) - \tilde{Z}_1 \left(d_j - \sum d_B \tilde{x}_B^n \right)$$

Step: 6. *To find the Entering variable, in the most positive value of Δ_j*

Step: 7. *Next we find the Minimum ratio column $\frac{\tilde{x}_{ij}}{\tilde{y}_{ij}}$.*

Step: 8. *Then we find the Leaving variable, to select the least positive value of the Minimum ratio $\frac{\tilde{x}_{ij}}{\tilde{y}_{ij}}$ column.*

Step: 9. *If all Δ_j less than (or) equal to $\{\langle 0,0,0\rangle ; 1,0,0\}$, the optimal solution is obtained. Then we get the required solution. Finally stop the iteration.*

Step: 10. *Otherwise, continued in Step-(3) to Step-(9).*

4. Numerical Example

Problem 4.1. *Find the solution of following linear fractional programming problem*

$$\begin{aligned} max.\tilde{Z} &= \frac{\tilde{x}_1 + 2\tilde{x}_2}{2\tilde{x}_1 - \tilde{x}_2 + 2} \\ s.t \quad -\tilde{x}_1 + 2\tilde{x}_2 &\leqslant \{\langle 1,2,3\rangle ; 0.6, 0.5, 0.3\} \\ \tilde{x}_1 + \tilde{x}_2 &\leqslant \{\langle 2,4,6\rangle ; 0.2, 0.6, 0.5\} \\ \tilde{x}_1^n, \tilde{x}_2^n &\geqslant \{\langle 0,0,0\rangle ; 1,0,0\} \end{aligned}$$

Solution: The Objective function is,

$$max.Z = \frac{\tilde{x}_1 + 2\tilde{x}_2 + 0\tilde{x}_3 + 0\tilde{x}_4 + 0}{2\tilde{x}_1 - \tilde{x}_2 + 0\tilde{x}_3 + 0\tilde{x}_4 + 2}$$

After adding slack variables, $\tilde{x}_3$ and $\tilde{x}_4$, the constraints become

$$\begin{aligned} -\tilde{x}_1 + 2\tilde{x}_2 + \tilde{x}_3 &= \{\langle 1,2,3\rangle ; 0.6, 0.5, 0.3\} \\ \tilde{x}_1 + \tilde{x}_2 + \tilde{x}_4 &= \{\langle 2,4,6\rangle ; 0.2, 0.6, 0.5\} \\ \tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 &\geqslant \{\langle 0,0,0\rangle ; 1,0,0\} \end{aligned}$$

Table:1.1

	$\alpha = 0$		c_j	1	2	0	0	Min.Ratio
B.V	$\beta = 2$		d_j	2	-1	0	0	$\frac{\tilde{x}_{ij}}{\tilde{y}_{ij}}$
$\tilde{x}_B$	d_B	c_B	$\tilde{B}$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	
$\tilde{x}_3$	0	0	$\{< 1,2,3 >; 0.6, 0.5, 0.3\}$	-1	2	1	0	
$\tilde{x}_4$	0	0	{¡2,4,6¿; 0.2,0.6,0.5}	1	1	0	1	
$\tilde{z}_1$				$\Delta_{11}^{(1)}$	$\Delta_{12}^{(1)}$	$\Delta_{13}^{(1)}$	$\Delta_{14}^{(1)}$	$\Delta_j^{(1)}$
$\tilde{z}_2$				$\Delta_{21}^{(1)}$	$\Delta_{22}^{(1)}$	$\Delta_{23}^{(1)}$	$\Delta_{24}^{(1)}$	$\Delta_j^{(2)}$
$\tilde{z}$				Δ_1	Δ_2	Δ_3	Δ_4	Δ_j

In table 1.2 calculations are given below.

$$\begin{aligned} \tilde{z}_1 &= \sum c_B \tilde{x}_B^n + \alpha = (0)(\{< 1,2,3 >; 0.6, 0.5, 0.3 >\}) \\ &\quad +(0)(\{< 2,4,6 >; 0.2, 0.6, 0.5\}) + 0 \\ &= \{< 0,0,0 >; 0.2, 0.6, 0.5\} \\ \tilde{z}_2 &= \sum d_B \tilde{x}_B^n + \beta = (0)(\{< 1,2,3 >; 0.6, 0.5, 0.3\}) \\ &\quad +(0)(\{< 2,4,6 >; 0.2, 0.6, 0.5\}) + 2 \\ &= \{< 2,2,2 >; 0.2, 0.6, 0.5\} \\ \tilde{z} &= \frac{\tilde{z}_1}{\tilde{z}_2} = \frac{\{< 0,0,0 >; 0.2, 0.6, 0.5\}}{\{< 2,2,2 >; 0.2, 0.6, 0.5\}} = \{< 0,0,0 >; 0.2, 0.6, 0.5\} \end{aligned}$$

$$\begin{aligned}
\Delta_j^{(1)} &= c_j - \sum c_B \tilde{x}_B \\
\Delta_{11}^{(1)} &= 1 - [(0)(-1) + (0)(1)] = 1 \\
\Delta_{12}^{(1)} &= 2 - [(0)(2) + (0)(1)] = 2 \\
\Delta_{13}^{(1)} &= 0 - [(0)(1) + (0)(0)] = 0 \\
\Delta_{14}^{(1)} &= 0 - [(0)(0) + (0)(1)] = 0 \\
\Delta_j^{(2)} &= d_j - \sum d_B \tilde{x}_B \\
\Delta_{21}^{(2)} &= 2 - [(0)(-1) + (0)(1)] = 2 \\
\Delta_{22}^{(2)} &= -1 - [(0)(2) + (0)(1)] = -1 \\
\Delta_{23}^{(2)} &= 0 - [(0)(1) + (0)(0)] = 0 \\
\Delta_{24}^{(2)} &= 0 - [(0)(0) + (0)(1)] = 0 \\
\Delta_j &= \tilde{z}_2 \times \Delta_j^{(1)} - \tilde{z}_1 \times \Delta_j^{(2)} \\
\Delta_1 &= (\{< 2,2,2 >; 0.2, 0.6, 0.5\})(1) - (\{< 0,0,0 >; 0.2, 0.6, 0.5\})(2) \\
&= \{< 2,2,2 >; 0.2, 0.6, 0.5\} \\
\Delta_2 &= (\{< 2,2,2 >; 0.2, 0.6, 0.5\})(2) - (\{< 0,0,0 >; 0.2, 0.6, 0.5\})(-1) \\
&= \{< 4,4,4 >; 0.2, 0.6, 0.5\} \\
\Delta_3 &= (\{< 2,2,2 >; 0.2, 0.6, 0.5\})(0) - (\{< 0,0,0 >; 0.2, 0.6, 0.5\})(0) \\
&= \{< 0,0,0 >; 0.2, 0.6, 0.5\} \\
\Delta_4 &= (\{< 2,2,2 >; 0.2, 0.6, 0.5\})(0) - (\{< 0,0,0 >; 0.2, 0.6, 0.5\})(0) \\
&= \{< 0,0,0 >; 0.2, 0.6, 0.5\}
\end{aligned}$$

Table:1.2

	$\alpha = 0$		c_j	1	2	0	0	Min.Ratio
B.V	$\beta = 2$		d_j	2	-1	0	0	$\frac{\tilde{x}_{ij}}{\tilde{y}_{ij}}$
$\tilde{x}_B$	d_B	c_B	$\tilde{B}$	$\tilde{x}_1$	$\tilde{x}_2 \downarrow$	$\tilde{x}_3$	$\tilde{x}_4$	
$\leftarrow \tilde{x}_3$	0	0	$\{\langle 1,2,3\rangle; 0.6, 0.5, 0.3\}$	-1	2	1	0	$\{< \frac{1}{2}, 1, \frac{3}{2} >; 0.6, 0.5, 0.3\}$
$\tilde{x}_4$	0	0	$\{< 2,4,6 >; 0.2, 0.6, 0.5\}$	1	1	0	1	$\{\langle 2,4,6\rangle; 0.2, 0.6, 0.5\}$
$\tilde{z}_1 =$	$\{\langle 0,0,0\rangle; 0.2, 0.6, 0.5\}$			1	2	0	0	$\Delta_j^{(1)}$
$\tilde{z}_2 =$	$\{\langle 2,2,2\rangle; 0.2, 0.6, 0.5\}$			2	-1	0	0	$\Delta_j^{(2)}$
$\tilde{z} = \{\langle 0,0,0\rangle; 0.2, 0.6, 0.5\}$				$\{\langle 2,2,2\rangle; 0.2, 0.6, 0.5\}$	$\{\langle 4,4,4\rangle; 0.2, 0.6, 0.5\}$	$\{\langle 0,0,0\rangle; 0.2, 0.6, 0.5\}$	$\{\langle 0,0,0\rangle; 0.2, 0.6, 0.5\}$	Δ_j

$$\begin{aligned}
\text{Since } \tilde{z}_1 &= \{\langle 0,0,0\rangle; 0.2, 0.6, 0.5\}, \\
\tilde{z}_2 &= \{\langle 2,2,2\rangle; 0.2, 0.6, 0.5\} \\
\text{and } \tilde{z} &= \{\langle 0,0,0\rangle; 0.2, 0.6, 0.5\}.
\end{aligned}$$

Therefore $\Delta_j = (\{\langle 2,2,2\rangle; 0.2, 0.6, 0.5\}) \times \Delta_j^{(1)}$ as shown in table 1.2. Entering variable is $\tilde{x}_2$ and Leaving variable is $\tilde{x}_3$. in table 1.2 Go to next table.

In table 1.3 calculations are given below.

$$\begin{aligned}
\tilde{z}_1 &= \sum c_B \tilde{x}_B + \alpha \\
&= (2)\left(\left\{< \frac{1}{2}, 1, \frac{3}{2} >; 0.6, 0.5, 0.3\right\}\right) + (0)\left(\left\{< \frac{-3}{2}, 3, \frac{9}{2} 9/2 >; 0.2, 0.6, 0.5\right\}\right) + 0 \\
&= \{< 1, 2, 3 >; 0.2, 0.6, 0.5\} \\
\tilde{z}_2 &= \sum d_B \tilde{x}_B + \beta(-1) \\
&= \left(\left\{< \frac{1}{2}, 1, \frac{3}{2} >; 0.6, 0.5, 0.3\right\}\right) + (0)\left(\left\{< \frac{-3}{2}, 3, \frac{9}{2} 9/2 >; 0.2, 0.6, 0.5\right\}\right) + 2 \\
&= \left\{< \frac{3}{2}, 1, \frac{1}{2} >; 0.2, 0.6, 0.5\right\} \\
\tilde{z} &= \frac{\tilde{z}_1}{\tilde{z}_2} = \frac{\{< 1, 2, 3 >; 0.2, 0.6, 0.5\}}{\left\{< \frac{3}{2}, 1, \frac{1}{2} >; 0.2, 0.6, 0.5\right\}} = \left\{< \frac{2}{3}, 2, 6 >; 0.2, 0.6, 0.5\right\}
\end{aligned}$$

$$\Delta_j^{(1)} = c_j - \sum c_B \tilde{x}_B$$

$$\Delta_{11}^{(1)} = 1 - \left[(2)\left(\frac{-1}{2}\right) + (0)\left(\frac{3}{2}\right)\right] = 2$$

$$\Delta_{12}^{(1)} = 2 - [(2)(1) + (0)(0)] = 0$$

$$\Delta_{13}^{(1)} = 0 - \left[(2)(\frac{1}{2}) + (0)\left(\frac{-1}{2}\right)\right] = -1$$

$$\Delta_{14}^{(1)} = 0 - [(2)(0) + (0)(1)] = 0$$

$$\begin{aligned}
\Delta_j^{(2)} &= d_j - \sum d_B \tilde{x}_B \\
\Delta_{21}^{(2)} &= 2 - \left[(-1)\left(\frac{-1}{2}\right) + (0)\left(\frac{3}{2}\right)\right] = \frac{3}{2} \\
\Delta_{22}^{(2)} &= -1 - [(-1)(1) + (0)(0)] = -0 \\
\Delta_{23}^{(2)} &= 0 - \left[(-1)\left(\frac{1}{2}\right) + (0)\left(\frac{-1}{2}\right)\right] = \frac{1}{2} \\
\Delta_{24}^{(2)} &= 0 - [(-1)(0) + (0)(1)] = 0 \\
\Delta_j &= \tilde{z}_2 \times \Delta_j^{(1)} - \tilde{z}_1 \times \Delta_j^{(2)} \\
\Delta_1 &= \left(\left\{< \frac{3}{2}, 1, \frac{1}{2} >; 0.2, 0.6, 0.5\right\}\right) \times (2) - (\{< 1, 2, 3 >; 0.2, 0.6, 0.5\}) \times \left(\frac{3}{2}\right) \\
&= \left\{< \frac{3}{2}, -1, \frac{-7}{2} >; 0.2, 0.6, 0.5\right\} \\
\Delta_2 &= \left(\left\{< \frac{3}{2}, 1, \frac{1}{2} >; 0.2, 0.6, 0.5\right\}\right) \times (0) - (\{< 1, 2, 3 >; 0.2, 0.6, 0.5\}) \times (0) \\
&= \{< 0, 0, 0 >; 0.2, 0.6, 0.5\}
\end{aligned}$$

$$\Delta_3 = \left(\left\{< \frac{3}{2}, 1, \frac{1}{2} >; 0.2, 0.6, 0.5\right\}\right) \times (-1) - (\{< 1, 2, 3 >; 0.2, 0.6, 0.5\}) \times \left(\frac{1}{2}\right)$$
$$= \{< -2, -2, -2 >; 0.2, 0.6, 0.5\}$$
$$\Delta_4 = \left(\left\{< \frac{3}{2}, 1, \frac{1}{2} >; 0.2, 0.6, 0.5\right\}\right) \times (0) - (\{< 1, 2, 3 >; 0.2, 0.6, 0.5\}) \times (0)$$
$$= \{< 0, 0, 0 >; 0.2, 0.6, 0.5\}$$

Table:1.3

	$\alpha = 0$		c_j	1	2	0	0	Min.Ratio
B.V	$\beta = 2$		d_j	2	-1	0	0	$\frac{\tilde{x}_{ij}}{\tilde{y}_{ij}}$
$\tilde{x}_B$	d_B	c_B	$\tilde{B}^n$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	
$\tilde{x}_2$	-1	2	$\{< \frac{1}{2}, 1, \frac{3}{2} >; 0.6, 0.5, 0.3\}$	$\frac{-1}{2}$	1	$\frac{1}{2}$	0	-
$\tilde{x}_4$	0	0	$\{\left\langle \frac{-3}{2}, 3, \frac{9}{2}\right\rangle; 0.2, 0.6, 0.5\}$	$\frac{3}{2}$	0	$\frac{-1}{2}$	1	-
$\tilde{z}_1 = \{\langle 1, 2, 3\rangle; 0.2, 0.6, 0.5\}$				2	0	-1	0	$\Delta_j^{(1)}$
$\tilde{z}_2 = \{\left\langle \frac{3}{2}, 1, \frac{1}{2}\right\rangle; 0.2, 0.6, 0.5\}$				$\frac{3}{2}$	0	$\frac{1}{2}$	0	$\Delta_j^{(2)}$
$\tilde{z} = \{\left\langle \frac{2}{3}, 2, 6\right\rangle; 0.2, 0.6, 0.5\}$				$\{\left\langle \frac{3}{2}, -1, \frac{-7}{2}\right\rangle; 0.2, 0.6, 0.5\}$	$\{\langle 0, 0, 0\rangle; 0.2, 0.6, 0.5\}$	$\{\langle -2, -2, -2\rangle; 0.2, 0.6, 0.5\}$	$\{\langle 0, 0, 0\rangle; 0.2, 0.6, 0.5\}$	Δ_j

Here $\tilde{x}_1 = \{\langle 0, 0, 0\rangle; 1, 0, 0\}$,

$$\tilde{x}_2 = \left\{\left\langle \frac{1}{2}, 1, \frac{3}{2}\right\rangle; 0.6, 0.5, 0.3\right\}$$

and $\tilde{z} = \frac{\tilde{z}_1}{\tilde{z}_2} = \left\{\left\langle \frac{2}{3}, 2, 6\right\rangle; 0.2, 0.6, 0.5\right\}$.

Since all $\Delta_j \leqslant \{\langle 0, 0, 0\rangle; 1, 0, 0\}$.

$\therefore$ The solution is the optimal basic feasible solution.

Problem 4.2. *Find the solution of following linear fractional programming problem:*

$$max.\tilde{Z} = \frac{3\tilde{x}_1 + \tilde{x}_2}{3\tilde{x}_1 + \tilde{x}_2 + 6}$$
$$s.t \quad 5\tilde{x}_1 + 3\tilde{x}_2 \leqslant \{\langle 5, 6, 7\rangle; 0.2, 0.7, 0.5\}$$
$$7\tilde{x}_1 + \tilde{x}_2 \leqslant \{\langle 3, 6, 9\rangle; 0.4, 0.6, 0.3\}$$
$$\tilde{x}_1, \tilde{x}_2 \geqslant \{\langle 0, 0, 0\rangle; 1, 0, 0\}$$

Solution: After adding slack variables, $\tilde{x}_3$ and $\tilde{x}_4$, the constraints become

$$5\tilde{x}_1 + 3\tilde{x}_2 + \tilde{x}_3 = \{\langle 5, 6, 7\rangle; 0.2, 0.7, 0.5\}$$
$$7\tilde{x}_1 + \tilde{x}_2 + \tilde{x}_4 = \{\langle 3, 6, 9\rangle; 0.4, 0.6, 0.3\}$$
$$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \geqslant \{\langle 0, 0, 0\rangle; 1, 0, 0\}$$

Table:2.1

	$\alpha=0$		c_j	2	1	0	0	Min.Ratio
B.V	$\beta=2$		d_j	3	1	0	0	$\frac{\tilde{x}_{ij}}{\tilde{y}_{ij}}$
$\tilde{x}_B$	d_B	c_B	$\tilde{B}$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	
$\tilde{x}_3$	0	0	$\{\langle 5,6,7\rangle;$ $0.2,0.7,0.5\}$	5	3	1	0	
$\tilde{x}_4$	0	0	$\{\langle 3,6,9\rangle;$ $0.4,0.6,0.3\}$	7	1	0	1	
$\tilde{z}_1$				$\Delta_{11}^{(1)}$	$\Delta_{12}^{(1)}$	$\Delta_{13}^{(1)}$	$\Delta_{14}^{(1)}$	$\Delta_j^{(1)}$
$\tilde{z}_2$				$\Delta_{21}^{(1)}$	$\Delta_{22}^{(1)}$	$\Delta_{23}^{(1)}$	$\Delta_{24}^{(1)}$	$\Delta_j^{(2)}$
$\tilde{z}$				Δ_1	Δ_2	Δ_3	Δ_4	Δ_j

Similarly we find the value of the Table:2.2 (In the Problem 4.1 calculation as well as).

Table:2.2

	$\alpha=0$		c_j	2	1	0	0	Min.Ratio
B.V	$\beta=2$		d_j	3	1	0	0	$\frac{\tilde{x}_{ij}}{\tilde{y}_{ij}}$
$\tilde{x}_B$	d_B	c_B	$\tilde{B}$	$\tilde{x}_1\downarrow$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	
$\tilde{x}_3$	0	0	$\{\langle 5,6,7\rangle;$ $0.2,0.7,0.5\}$	5	3	1	0	$\{\left\langle \frac{5}{5},\frac{6}{5},\frac{7}{5}\right\rangle;$ $0.2,0.7,0.5\}$
$\leftarrow$ $\tilde{x}_4$	0	0	$\{\langle 3,6,9\rangle;$ $0.4,0.6,0.3\}$	7	1	0	1	$\left\{\left\langle \frac{3}{7},\frac{6}{7},\frac{9}{7}\right\rangle;\right.$ $0.4,0.6,0.3\}$
$\tilde{z}_1=\{\langle 0,0,0\rangle;$ $0.2,0.7,0.5\}$				2	1	0	0	$\Delta_j^{(1)}$
$\tilde{z}_2=\{\langle 6,6,6\rangle;$ $0.2,0.7,0.5\}$				3	1	0	0	$\Delta_j^{(2)}$
$\tilde{z}=\{\langle 0,0,0\rangle;$ $0.2,0.7,0.5\}$				$\{\langle 12,12,12\rangle;$ $0.2,0.7,0.5\}$	$\{\langle 6,6,6\rangle;$ $0.2,0.7,0.5\}$	$\{\langle 0,0,0\rangle;$ $0.2,0.7,0.5\}$	$\{\langle 0,0,0\rangle;$ $0.2,0.7,0.5\}$	Δ_j

$$\text{Since } \tilde{z}_1 = \{\langle 0,0,0\rangle; 0.2,0.7,0.5\},$$
$$\tilde{z}_2 = \{\langle 6,6,6\rangle; 0.2,0.7,0.5\},$$
$$\text{and } \tilde{z} = \{\langle 0,0,0\rangle; 0.2,0.7,0.5\}.$$

$\therefore \Delta_j = \left[\{\langle 6,6,6\rangle; 0.2,0.7,0.5\} \times \Delta_j^{(1)}\right]$ as shown in table 2.2.
Entering variable is $\tilde{x}_1$ and Leaving variable is $\tilde{x}_4$. in table 2.2 Go to next table.
Similarly we find the value of the Table:2.3 (In the Problem 4.1 calculation as well as).

Table:2.3

	$\alpha=0$		c_j	2	1	0	0	Min.Ratio
B.V	$\beta=6$		d_j	3	1	0	0	$\frac{\tilde{x}_{ij}}{\tilde{y}_{ij}}$
$\tilde{x}_B$	d_B	c_B	$\tilde{B}$	$\tilde{x}_1$	$\tilde{x}_2\downarrow$	$\tilde{x}_3$	$\tilde{x}_4$	
$\leftarrow \tilde{x}_3$	0	0	$\left\{\left\langle\frac{10}{7},\frac{12}{7},\frac{34}{7}\right\rangle;\ 0.2,0.7,0.5\right\}$	0	$\frac{16}{7}$	1	$\frac{-5}{7}$	$\left\{\left\langle\frac{-5}{8},\frac{3}{4},\frac{17}{8}\right\rangle;\ 0.2,0.7,0.5\right\}$
$\tilde{x}_1$	3	2	$\left\{\left\langle\frac{3}{7},\frac{6}{7},\frac{9}{7}\right\rangle;\ 0.4,0.6,0.3\right\}$	1	$\frac{1}{7}$	0	$\frac{1}{7}$	$\{\langle 3,6,9\rangle;\ 0.4,0.6,0.3\}$
$\tilde{z}_1=\left\{\left\langle\frac{6}{7},\frac{12}{7},\frac{18}{7}\right\rangle;\ 0.2,0.7,0.5\right\}$				0	$\frac{5}{7}$	0	$\frac{-2}{7}$	$\Delta_j^{(1)}$
$\tilde{z}_2=\left\{\left\langle\frac{51}{7},\frac{60}{7},\frac{69}{7}\right\rangle;\ 0.2,0.7,0.5\right\}$				0	$\frac{4}{7}$	0	$\frac{-3}{7}$	$\Delta_j^{(2)}$
$\tilde{z}=\left\{\left\langle\frac{6}{51},\frac{1}{5},\frac{18}{69}\right\rangle;\ 0.2,0.7,0.5\right\}$				$\{\langle 0,0,0\rangle;\ 0.2,0.7,0.5\}$	$\left\{\left\langle\frac{183}{49},\frac{252}{49},\frac{321}{49}\right\rangle;\ 0.2,0.7,0.5\right\}$	$\{\langle 0,0,0\rangle;\ 0.2,0.7,0.5\}$	$\left\{\left\langle\frac{-12}{7},\frac{-12}{7},\frac{-12}{7}\right\rangle;\ 0.2,0.7,0.5\right\}$	Δ_j

$$\text{Since } \tilde{z}_1 = \left\{\left\langle\frac{6}{7},\frac{12}{7},\frac{18}{7}\right\rangle;0.2,0.7,0.5\right\},$$
$$\tilde{z}_2 = \left\{\left\langle\frac{51}{7},\frac{60}{7},\frac{69}{7}\right\rangle;0.2,0.7,0.5\right\}$$
$$\text{and } \tilde{z} = \left\{\left\langle\frac{6}{51},\frac{1}{5},\frac{18}{69}\right\rangle;0.2,0.7,0.5\right\}.$$

$$\therefore \Delta_j = \begin{bmatrix} \left(\left\{\left\langle\frac{51}{7},\frac{60}{7},\frac{69}{7}\right\rangle;0.2,0.7,0.5\right\}\right)\times\Delta_j^{(1)} \\ -\left(\left\{\left\langle\frac{6}{7},\frac{12}{7},\frac{18}{7}\right\rangle;0.2,0.7,0.5\right\}\right)\times\Delta_j^{(2)} \end{bmatrix} \text{ as shown in table:2.3.}$$

Entering variable is $\tilde{x}_2$ and Leaving variable is $\tilde{x}_3$. in table:2.3 Go to next table.
Similarly we find the value of the Table:2.4 (In the Problem 4.1 calculation as well as).

Table:2.4

	$\alpha=0$		c_j	2	1	0	0	Min.Ratio
B.V	$\beta=6$		d_j	3	1	0	0	$\frac{\tilde{x}_{ij}}{\tilde{y}_{ij}}$
$\tilde{x}_B$	d_B	c_B	$\tilde{B}$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	
$\tilde{x}_2$	1	1	$\left\{\left\langle\frac{-5}{8},\frac{3}{4},\frac{17}{8}\right\rangle;\ 0.2,0.7,0.5\right\}$	0	1	$\frac{7}{16}$	$\frac{-5}{16}$	
$\tilde{x}_1$	3	2	$\left\{\left\langle\frac{1}{8},\frac{3}{4},\frac{11}{8}\right\rangle;\ 0.2,0.7,0.5\right\}$	1	0	$\frac{-1}{6}$	$\frac{3}{16}$	
$\tilde{z}_1=\left\{\left\langle\frac{-3}{8},\frac{9}{4},\frac{39}{8}\right\rangle;\ 0.2,0.7,0.5\right\}$				0	0	$\frac{-5}{16}$	$\frac{-1}{16}$	$\Delta_j^{(1)}$
$\tilde{z}_2=\left\{\left\langle\frac{46}{8},9,\frac{98}{8}\right\rangle;\ 0.2,0.7,0.5\right\}$				0	0	$\frac{-1}{4}$	$\frac{-1}{4}$	$\Delta_j^{(2)}$
$\tilde{z}=\left\{\left\langle\frac{-3}{46},\frac{1}{4},\frac{39}{98}\right\rangle;\ 0.2,0.7,0.5\right\}$				$\{\langle 0,0,0\rangle;\ 0.2,0.7,0.5\}$	$\{\langle 0,0,0\rangle;\ 0.2,0.7,0.5\}$	$\left\{\left\langle\frac{-53}{32},\frac{-9}{4},\frac{-167}{64}\right\rangle;\ 0.2,0.7,0.5\right\}$	$\left\{\left\langle\frac{-29}{4},0,\frac{29}{4}\right\rangle;\ 0.2,0.7,0.5\right\}$	Δ_j

$$\text{Here } \tilde{x}_1 = \left\{\left\langle\frac{1}{8},\frac{3}{4},\frac{11}{8}\right\rangle;0.2,0.7,0.5\right\},$$

$$\tilde{x}_2 = \left\{\left\langle \frac{-5}{8}, \frac{3}{4}, \frac{17}{8}\right\rangle; 0.2, 0.7, 0.5\right\}$$
$$\text{and } \tilde{z} = \frac{\tilde{z}_1}{\tilde{z}_2} = \left\{\left\langle \frac{-3}{46}, \frac{1}{4}, \frac{39}{98}\right\rangle; 0.2, 0.7, 0.5\right\}.$$

Since all $\Delta_j \leqslant \{\langle 0,0,0\rangle; 1,0,0\}$.

$\therefore$ The current solution is the optimal basic feasible solution.

5. Conclusion

In this work, we have presented the simplex algorithm method for solving Neutrosophic linear fractional programming problem.

The solution of the given Problem (1) is

$$\tilde{x}_1 = \{\langle 0,0,0\rangle; 1,0,0\},$$
$$\tilde{x}_2 = \left\{\left\langle \frac{1}{2}, 1, \frac{3}{2}\right\rangle; 0.6, 0.5, 0.3\right\}$$
$$\text{and } \tilde{z} = \frac{\tilde{z}_1}{\tilde{z}_2} = \left\{\left\langle \frac{2}{3}, 2, 6\right\rangle; 0.2, 0.6, 0.5\right\}$$

The solution of the given Problem (2) is

$$\tilde{x}_1 = \left\{\left\langle \frac{1}{8}, \frac{3}{4}, \frac{11}{8}\right\rangle; 0.2, 0.7, 0.5\right\},$$
$$\tilde{x}_2 = \left\{\left\langle \frac{-5}{8}, \frac{3}{4}, \frac{17}{8}\right\rangle; 0.2, 0.7, 0.5\right\}$$
$$\text{and } \tilde{z} = \frac{\tilde{z}_1}{\tilde{z}_2} = \left\{\left\langle \frac{-3}{46}, \frac{1}{4}, \frac{39}{98}\right\rangle; 0.2, 0.7, 0.5\right\}.$$

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A STUDY ON FUZZY LINDELOF CLOSED SPACES

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Abstract

In this paper, we investigate two new spaces on Lindelof closed space and fuzzy Lindelof closed space, the main purpose of this work is to study the relation between fuzzy Lindelof closed and other spaces such as fuzzy Lindelof, fuzzy lightly compact, fuzzy semi regular.

Keywords: Fuzzy Topological Spaces, Fuzzy Compactness, Fuzzy Lindelof Spaces, Fuzzy Lindelof Closed Spaces.

1. Introduction

The Theory of Fuzzy sets is introduced by Zadeh, and the fuzzy topology is defined by Chang. Many mathematicians have tried to extend to fuzzy set theory the main notations of general topology. In this paper we define an Lindelof closed space and a fuzzy Lindelof closed space, the main purpose of this work is to study the relation between fuzzy lindelof closed and other spaces such as fuzzy Lindelof, fuzzy lightly compact, fuzzy semi regular, C_{II}, fuzzy paracompact, fuzzy separable, and others. If μ is a fuzzy set in a fuzzy topological space (X, T) then the closure and the interior of μ will be as usual defined by $\bar{\mu} = \Lambda\{\lambda : \lambda \geqslant \mu, 1 - \lambda \in T\}$ and $\mu^o = v\{\lambda : \lambda \leqslant \mu, \lambda \in T\}$ respectively. A fuzzy set μ is called fuzzy regular open, if $\mu = (\bar{\mu})^0$. Let (X, T) be a fuzzy topological space, the fuzzy regular open sets in T form a base for a unique fuzzy topology T_0 called the fuzzy semi-regular topology on X associated with T. A fuzzy topology T is fuzzy semi-regular, if and only if $T = T_0$. In this paper we show that (X, T_0) is fuzzy Lindelof closed if (X, T) is fuzzy Lindelof. In another direction. Let D be non-empty set. Let $\geqslant$ be a semi-order on D. The pair $(D, \geqslant)$ is called a directed set, directed by $\geqslant$, if and only if for every pair $m, n \in D$, there exists a $p \in D$ such that $p \geqslant m$ and $p \geqslant n$. Let X be an ordinary set. Let χ be the collection of all the fuzzy points in X. The function $S : D \to \chi$. In other words, a fuzzy net is a pair $(S, \geqslant)$ such that S is a function : D $\to \chi$ and $\geqslant$ directs the domain of S. For $n \in D, S(n)$ is

often denoted by S_n and hence a net S often denoted by $\{S_n, n \in D\}$. In this paper we show that a fuzzy topological space X is fuzzy Lindelof closed space if for every net $\{K_n, n \in D\}$ of fuzzy closed sets in X such that $F - \limsup_D (K_n) = 0$, there exists $n_0 \in D$ for which $K_n = 0$, for every $n \in D, n \geqslant n_0$. Also we show that X is fuzzy Lindelof closed space if for every $\alpha \in I$, for every net $\{K_n, n \in D\}$ of fuzzy closed sets in X such that $F - \limsup_D (K_n) = 1 - \alpha$ and for every $\varepsilon \in (0, \alpha]$, there exists an element $n_0 \in D$ for which $K_n \leqslant 1 - \alpha + \varepsilon$, for every $n \in D, n \geqslant n_0$.

2. Preliminaries

Definition 2.1. A fuzzy topological space (X, δ) is said to be fuzzy lightly compact if for all $\{\mu_i\}_{i\in\Omega} \subseteq \delta$ with $\sup\{\mu_i\} = 1$, there exists an $n_0 \in N$ such that $\sup\{\overline{\mu_i}\}_{i=1}^{n_0} = 1$.

Definition 2.2. Let λ be a fuzzy set in a fuzzy topological space (X, δ).λ is said to be fuzzy compact in the Lowen's sense if for all family of fuzzy open sets cover $\{\lambda_l \mid l \in L\}$ such that $\lambda \leqslant \vee\{\lambda_l \mid l \in L\}$ and for all $\epsilon > 0$ there exist a finite subfamily $\{\lambda_l \mid l \in L^*\}$ such that $\lambda - \epsilon \leqslant \vee\{\lambda_1 \mid l \in L^*\}$.

Definition 2.3. A fuzzy topological space (X, δ) is called fuzzy compact if and only if for every family Ψ of fuzzy open sets of X and for every $\alpha \in I$ such that $\vee\{U : U \in \Psi\} \geqslant \alpha$ and for every $\epsilon \in (0, \alpha]$ there exists a finite subfamily Ψ^* of Ψ such that $v\{U : U \in \Psi^*\} \geqslant \alpha - \epsilon$.

Definition 2.4. Let λ be a fuzzy set in a fuzzy topological space (X, δ).λ is said to be fuzzy paracompact if for every open cover in the sense of Lowen H of λ and for every $\epsilon \in (0, \alpha]$, there exists an open refinement D of H which is both locally finite in λ and cover of $\lambda - \epsilon$ in the sense of Lowen.

Definition 2.5. Let (X, τ) be a topological space and $w(\tau)$ be the set of all semi continuous function from (X, τ) to the unit interval $I = [0, 1]$ equipped with the usual topology, then $(X, w(\tau))$ is called induced fuzzy topological space by (X, τ).

Definition 2.6. A fuzzy topological space (X, δ) is said to be fuzzy Lindelof, if for each family $H \subset \delta$ and for each $\alpha \in I$ such that $\bigvee_{h\in H} h \geqslant \alpha$, there exists for each $\epsilon \in (0, \alpha]$ a countable subset H^* of H such that $\bigvee_{h\in H^*} V \geqslant \alpha - \epsilon$.

Definition 2.7. A fuzzy topological space (X, δ) is said to be separable iff there exist a countable sequence of fuzzy points $\{p_i, i = 1, 2, \ldots\}$ such that for every member $\lambda \neq 0$ of δ there exist a p_i such that $p_i \in \lambda$.

Definition 2.8. A fuzzy topological space (X, T) is said to be C_{II}, if there exists a countable base for T.

Theorem 2.1. *Let $\{A_n : n \in N\}$ be a net of fuzzy closed sets in Y such that $A_{n_1} \leqslant A_{n_2}$ if and only if $n_2 \leqslant n_1$. Then $F - \lim_{N} \sup (A_n) = \wedge \{A_n : n \in N\}$.*

3. Fuzzy Lindelof Closed Spaces

In this section we define the concept of fuzzy Lindelof closed space and we discuss the relation between fuzzy Lindelof closed and the other fuzzy spaces.

Definition 3.1. A topological space (X, T) is said to be Lindelof closed space if and only if for each open cover $\{A_l : l \in L\}$ of X, i.e $X = Y_{l \in L} A_l$, there exists a countable subfamily $\{A_l : l \in L^*\}$ of $\{A_l : l \in L\}$ whose closures cover X.

Definition 3.2. A fuzzy topological space (X, T) is said to be fuzzy Lindelof closed space if and only if for every family Ψ of fuzzy open sets of X and for every $\alpha \in I$ such that $\vee\{U : U \in \Psi\} \geqslant \alpha$ and for every $\varepsilon \in (0, \alpha]$ there exists a countable subfamily Ψ^* of Ψ whose closures such that $\vee \left\{\overline{U} : U \in \Psi^*\right\} \geqslant \alpha - \varepsilon$.

Proposition 3.1. *A space (X, T_0) is fuzzy Lindelof closed, if (X, T) is fuzzy Lindelof closed.*

Proposition 3.2. *A fuzzy semi-regular space (X, T) is fuzzy Lindelof closed $\Leftrightarrow$ (X, T_0) is fuzzy Lindelof closed.*

Proposition 3.3. *A space (X, T) is fuzzy Lindelof closed, if (X, T) is fuzzy compact.*

Proposition 3.4. *A space (X, T) is fuzzy Lindelof closed, if (X, T) is fuzzy Lindelof.*

The converse of Proposition (3.4) is not necessarily true as the following example shows.

Example 3.1. Let $X = R$ be the set of all real numbers, and $C \subseteq R - \{0\}$. Define

$\lambda_1 : R \to [0, 1]$ as $\lambda_1(x) = 0$ for all $x \in X$,

$$\lambda_2 : R \to [0, 1] \text{ as } \lambda_2(x) = \begin{cases} 1 \text{ if } x = 0 \\ 0 \text{ if } x \neq 0 \end{cases}$$

$$\lambda_C : R \to [0, 1] \text{ as } \lambda_C(x) = \begin{cases} 1 \text{ if } x \in C\ Y\{0\} \\ 0 \text{ if } x \notin C\ Y\{0\} \end{cases}$$

Let $\delta = \{\lambda_1, \lambda_2, \lambda_c \mid C \subseteq R - \{0\}\}$. Then (X, δ) is fuzzy topological space. For each $0 \neq \lambda \in \delta$, we have $\bar{\lambda} = 1$, then for any family Ψ of fuzzy open sets of X such that $\vee\{U : U \in \Psi\} \geqq 1$, there exists $0 \neq U^* \in \Psi \Rightarrow \overline{U^*} = 1$. Then (X, δ) is fuzzy Lindelof closed. However, (X, δ) is not fuzzy Lindelof space. Let $\Psi = \{\lambda_{\{r\}} \mid r \in R - \{0\}\}$ be a family of fuzzy open sets of X. Given $\varepsilon > 0$, and let $r = \varepsilon$. Then at $x = \varepsilon \Rightarrow \lambda_{\{\varepsilon\}}(x) = 1 > 1 - \varepsilon$. So $\bigvee_{r \in \{-\{0\}} \lambda_{\{r\}} = 1 > 1 - \varepsilon$. Then Ψ is fuzzy open cover of X. But if L is a countable subset of $R - \{0\}$, then we can find $x' \in R - \{0\} \subset X$ and $x' \notin L$. It follows that at $x = x'$ we have $\lambda_{\{r\}}(x) = 0$ for all $r \in L$, and certainly $\bigvee_{r \in L} \lambda_{\{r\}} < 1$. Then Ψ is cover of X by members of δ has not countable sub-cover of X.

Theorem 3.1. *Every fuzzy lightly compact space (X, δ) is fuzzy Lindelof closed.*

Proof. Let $\{\lambda_l\}_{l \in L} \subseteq \delta$ be a fuzzy open cover of X, $\vee\{\lambda_l\}_{l \in L} = \sup\{\lambda_l : l \in L\} = 1$ then there exists $J \subseteq L$ such that $\lambda_{lj} \in \{\lambda_l\}_{l \in L}$ with $\sup\{\lambda_{lj}\} = 1$ for $j \in J$. But (X, δ) is fuzzy lightly compact and $\{\lambda_{lj}\}_{j \in J} \subseteq \delta$ with $\sup\{\lambda_{lj}\} = 1$. Therefore $\{\lambda_{lj}\}_{j \in J}$ has a finite subfamily $\{\lambda_{l_j}{}^k : k = 1, 2, \ldots, n\}$ whose closures such that $\sup\left\{\overline{\lambda_{l_j}{}^k}\right\}_{k=1}^{n} = 1$, but each finite family is a countable, then for each family $\{\lambda_l\}_{l \in L} \subseteq \delta$ with $\vee\{\lambda_l\}_{l \in L} = 1$, there exists a countable subfamily of $\{\lambda_l\}_{l \in L}$ whose closures cover of X. Then (X, δ) is fuzzy Lindelof closed. □

Theorem 3.2. *If $(X, w(\delta))$ is fuzzy Lindelof closed, then (X, δ) is Linelof closed.*

Proof. Let $\{A_j : j \in J\}$ be an open cover of (X, δ), i.e $\underset{j \in J}{Y} A_j = X$.

Then $\bigvee_{j \in J} \chi_{A_j} = \sup\{\chi_{A_j}; j \in J\} = 1$ and $\{\chi_{A_j}; j \in J\}$ is an open Lowen's cover of $(X, w(\delta))$. By the assumption of the fuzzy lindelof closed of $(X, w(\delta))$, choose $\varepsilon > 0$, then there exists a countable open Lowen's sub-cover $\{\chi_{A_{jk}}; k \in N\}$ such that $\bigvee_{k \in N} \bar{\chi}_{A_{jk}} \geqslant 1 - \varepsilon$. Since ε is arbitrary then $X = \underset{k \in N}{Y} \bar{A}_{jk}$. □

Theorem 3.3. *Let f be a F-continuous surjection map between fuzzy topological spaces (X, δ) and (Y, π). If (X, δ) is fuzzy Lindelof closed then (Y, π) is verifies the same property.*

Proof. Let α be a constant fuzzy set in Y. Let $K = \{\lambda_j\}_{j \in J} \subset \pi$ such that $\bigvee_{j \in J} \{\lambda_j\} \geqslant \alpha$, and let $\varepsilon \in (0, \alpha]$, since f is F-continuous, then $M = \{f^{-1}(\lambda_j); j \in J\}$ is an L-cover of the constant fuzzy set β in X such that $f(\beta) = \alpha$. Since X is fuzzy Lindelof closed,

then there exists a fuzzy open countable subset M^* of M such that $\bigvee_{\gamma\in M^*}\{\bar{\gamma}\} \geqslant \beta-\varepsilon$. Then $K^* = \{f(\gamma);\gamma\in M^*\}$ is a fuzzy open countable subfamily of K such that $\bigvee_{\lambda_j\in K^*}\left\{\overline{\lambda_j}\right\} \geqslant \alpha-\varepsilon$. Then (Y,π) is fuzzy Lindelof closed. □

Theorem 3.4. *If a fuzzy topological space (X,δ) is C_{II}, then it is also fuzzy Lindelof closed space.*

Proof. Let $\{\lambda_j\}_{j\in J}\subset\delta$ such that $\bigvee_{j\in J}\{\lambda_j\}\geqslant\alpha$ for each $\alpha\in I$. Since (X,δ) is C_{II} then there exists a countable subfamily $\psi=\{\psi_1,1=1,2,\ldots\}$ of δ such that $\lambda_j=\underset{k=1}{v}\{\psi_{jk}\}$, where t my be infinitely. Let $\psi^*=\{\psi_{jk}\}, j\in J, k=1,2,\ldots,t,\psi^*$ is countable because it is subfamily of ψ. Let $x\in X$ and since $\bigvee_{j\in J}\{\lambda_j\}\geqslant\alpha$, there exist $m\in J$ such that $\lambda_m(x)\geqslant\alpha$ and we have $\lambda_m=\bigvee_{k=1}\{\psi_{jk}\}$ which implies that $\bigvee_{k=1}\{\psi_{jk}\}\geqslant\alpha$ and then ψ^* is open cover. Let $\varepsilon\in(0,1]$, then $\bigvee_{k=1}\{\bar{\psi}_{jk}\}\geqslant\bigvee_{k=1}\{\psi_{jk}\}\geqslant\alpha-\varepsilon$. Finally (X,δ) is fuzzy Lindelof closed. □

Theorem 3.5. *Let (X,δ) be a fuzzy paracompact separable topological space, then (X,δ) is fuzzy Lindelof closed.*

Proof. Let $H=\{\lambda_j\}_{j\in J}\subset\delta$ be a family such that $\bigvee_{j\in J}\{\lambda_j\}\geqslant\alpha$ for each $\alpha\in I$. Let H^* be a δ-open refinement of H which is locally finite and $\bigvee_{h\in H^*}h\geqslant\alpha-\varepsilon,\varepsilon\in(0,\alpha]$. X has countable sequence of fuzzy points $\{p_i,i=1,2,\ldots\}$ such that for every $h\neq 0$ there exists a $p_i\in h$. Then the family $\{h;h\in H^*\}$ is at most countable, otherwise since each h contains at least one p_i. This implies that there would be some p_n contained in uncountable many $h\in H$ which would contradiction locally finite. Choose for each $h_i\in H^*$ an element $\lambda_i\in H$ such that $h_i<\lambda_i\leqslant\bar{\lambda}_i$. Then there exists an open countable H' subset of H such that $\bigvee_{\lambda\in H'}\{\bar{\lambda}\}\geqslant\alpha-\varepsilon$. Therefore (X,δ) is fuzzy Lindelof closed. □

Theorem 3.6. *Let (X,δ) be a C_{II} fuzzy topological space is, then the continuous image of fuzzy paracompact is fuzzy Lindelof closed.*

Proof. Let be $f:(X,\delta)\to(Y,\pi)$ F-continuous and let β be a fuzzy paracompact subset of X. For every subspace (W,δ_W) of C_{II} fuzzy topological space (X,δ) is C_{II}, since by assumption δ has a countable base $L=\{L_i\},i=1,2,\ldots$, then $\{W\wedge L_i\}\subset\delta_W,i=$

$1, 2, ..,$ be a countable base for δ_W such that (W, δ_W) subspace of (X, δ), then (W, δ_W) is C_{II} and we have (W, δ_W) is separable. So β is separable. Then by Theorem (3.5) β is Lindelof closed fuzzy, and finally $f(\beta)$ is Lindelof closed fuzzy by Theorem (3.3). □

4. Conclusion

In this paper we have presented an Lindelof closed and fuzzy Lindelof closed spaces. Using that we have proved some theorems.

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A STUDY ON FUZZY ARITHMETIC OPERATIONS (PROPERTIES)

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Abstract

We analyze the fuzzy numbers (or intervals) which seems to be of interest in the study of some properties of fuzzy arithmetic operations and, in particular, in the analysis of fuzziness, of shape-preservation (symmetry) and distributivity of multiplication and division. By the use of the same decomposition, we suggest an approximation of multiplication and division to reduce the overestimation effect and/or to obtain total-distributivity of multiplication and left-distributivity of division. Finally, we compare the proposed approximation with the results of standard (α-cuts based) fuzzy mathematics and with other new definitions of fuzzy arithmetic operations that recently appeared in the literature.

Keywords: Arithmetic Operations, Fuzzy numbers, Distributive Operations.

1. Introduction

The main purpose of this paper is discuss about Arithmetic Operations on Fuzzy Numbers. The scientific literature on fuzzy arithmetic operations is rich of several approaches to define fuzzy operations having many desired properties that are not always present in the classical extension principle approach or its approximations: shape preservation reduction of the overestimation effect, requisite constraints distributivity of multiplication and division. These problems are essentially approached by joining representations of fuzzy quantities and fuzzy operations.

In this paper, the decomposition is used to analyze some properties of fuzzy operations and, particularly, distributivity of multiplication (and division) and symmetry of the results of fuzzy operations. The decomposition is also used to suggest some approximations of fuzzy operations that reduce the range of fuzziness (with respect to the classical exact operations) and assure distributivity.

2. Standard Fuzzy Arithmetic Operations

If $u = (u^-, u^+)$ and $v = (v^-, v^+)$ are two given fuzzy intervals, the standard arithmetic operations are defined as follows:

Definition 2.1. *(Standard addition, scalar multiplication, subtraction)*

For $\alpha \in [0,1]$:

$$[u+v]_\alpha = [u_\alpha^- + v_\alpha^-, u_\alpha^+ + v_\alpha^+] \tag{2.1}$$

$$[ku]_\alpha = [\min\{ku_\alpha^-, ku_\alpha^+\}, \max\{ku_\alpha^-, ku_\alpha^+\}], \quad k \in \mathbb{R}. \tag{2.2}$$

In particular, if $k = -1, [-u]_\alpha = [-u_\alpha^+, -u_\alpha^-]$,

$$[u-v]_\alpha = [u_\alpha^- - v_\alpha^+, u_\alpha^+ - v_\alpha^-], \alpha \in [0,1]. \tag{2.3}$$

Definition 2.2. *(Standard multiplication and division)*

For $\alpha \in [0,1], [uv]_\alpha = [(uv)_\alpha^-, (uv)_\alpha^+]$ *with*

$$\begin{aligned} (uv)_\alpha^- &= \min\{u_\alpha^- v_\alpha^-, u_\alpha^- v_\alpha^+, u_\alpha^+ v_\alpha^-, u_\alpha^+ v_\alpha^+\} \\ (uv)_\alpha^+ &= \max\{u_\alpha^- v_\alpha^-, u_\alpha^- v_\alpha^+, u_\alpha^+ v_\alpha^-, u_\alpha^+ v_\alpha^+\}. \end{aligned} \tag{2.4}$$

If $0 \notin [v_0^-, v_0^+]$, $\left[\frac{u}{v}\right]_\alpha = \left[\left(\frac{u}{v}\right)_\alpha^-, \left(\frac{u}{v}\right)_\alpha^+\right]$ *with*

$$\begin{aligned} \left(\frac{u}{v}\right)_\alpha^- &= \min\left\{\frac{u_\alpha^-}{v_\alpha^-}, \frac{u_\alpha^-}{v_\alpha^+}, \frac{u_\alpha^+}{v_\alpha^-}, \frac{u_\alpha^+}{v_\alpha^+}\right\} \\ \left(\frac{u}{v}\right)_\alpha^+ &= \max\left\{\frac{u_\alpha^-}{v_\alpha^-}, \frac{u_\alpha^-}{v_\alpha^+}, \frac{u_\alpha^+}{v_\alpha^-}, \frac{u_\alpha^+}{v_\alpha^+}\right\}. \end{aligned} \tag{2.5}$$

It is also well known that, in general, distributivity of multiplication and left-distributivity of division are not valid, except for special cases.

3. Properties of Arithmetic Operations

In this section we analyze standard arithmetic operations between fuzzy numbers in terms of the decomposition

$$u = (\widehat{u}, \widetilde{u}, \overline{u}) \in \widehat{\mathbb{F}} \times \mathbb{P} \times \mathbb{S}_0 \tag{3.1}$$

and we study, in particular, distributivity and symmetry associated to multiplication and division.

Remember that (3.1) is equivalent to

$$\begin{cases} u_\alpha^+ = \widehat{u} + \widetilde{u}_\alpha + \overline{u}_\alpha \\ u_\alpha^- = \widehat{u} + \widetilde{u}_\alpha - \overline{u}_\alpha \end{cases} \quad \forall \alpha \in [0,1]. \tag{3.2}$$

3.1. Addition, Scalar Multiplication and Subtraction

Addition, scalar multiplication and subtraction are immediate. Let $u = (\widehat{u}, \widetilde{u}, \overline{u})$ and $v = (\widehat{v}, \widetilde{v}, \overline{v})$ be two fuzzy numbers, then we can write:

$$\begin{aligned} u + v &= (\widehat{u} + \widehat{v}, \widetilde{u} + \widetilde{v}, \overline{u} + \overline{v}) && (3.3)\\ ku &= (k\widehat{u}, k\widetilde{u}, |k|\overline{u}), \text{ for } k \in \mathbb{R} && (3.4)\\ u - v &= u + (-v) = (\widehat{u} - \widehat{v}, \widetilde{u} - \widetilde{v}, \overline{u} + \overline{v}). && (3.5) \end{aligned}$$

It may be of interest to note that addition and difference have the same 0-symmetric fuzzy components $\overline{u} + \overline{v}$ and they differ (only) in the crisp and the profile parts. On the other hand, it is well known that:

$$u = (0, 0, \overline{u}) \in \mathbb{S}_0 \Longleftrightarrow u = -u$$

and that

$$u, v \in \mathbb{S}_0 \Longleftrightarrow u + v = u - v = v - u = -u - v.$$

These facts can be a little formalized; consider the following relation on $\mathbb{F}$($u \sim v$ if they differ only for the $\mathbb{S}_0-$ component):

$$u \sim v \Longleftrightarrow \begin{cases} \widehat{u} = \widehat{v} \\ \widetilde{u}_\alpha = \widetilde{v}_\alpha \forall \alpha \in [0, 1] \end{cases}$$

which is an equivalence relation on $\mathbb{F}$. The equivalence class corresponding to $0 = (0, 0, 0) \in \mathbb{S}_0$ (the null element of addition in $\mathbb{F}$) is the whole $\mathbb{S}_0$, i.e. $[0]_\sim = \mathbb{S}_0$; in fact, with respect to $\sim$ all the elements of $\mathbb{S}_0$ are equivalent to 0 and in particular $(u - u) \sim 0$ $\forall u \in \mathbb{F}$, as $(u - u) = (\widehat{u}, \widetilde{u}, \overline{u}) - (\widehat{u}, \widetilde{u}, \overline{u}) = (0, 0, 2\overline{u}) \in \mathbb{S}_0$.

3.2. Multiplication

Consider now the multiplication of $u, v \in \mathbb{F}$ where $u = (\widehat{u}, \widetilde{u}, \overline{u})$ and $v = (\widehat{v}, \widetilde{v}, \overline{v})$ are fuzzy numbers with

$$\begin{cases} u_\alpha^+ = \widehat{u} + \widetilde{u}_\alpha + \overline{u}_\alpha \\ u_\alpha^- = \widehat{u} + \widetilde{u}_\alpha - \overline{u}_\alpha \end{cases} \text{ and } \begin{cases} V_\alpha^+ = \widehat{v} + \widetilde{v}_\alpha + \overline{v}_\alpha \\ V_\alpha^- = \widehat{v} + \widetilde{v}_\alpha - \overline{v}_\alpha. \end{cases}$$

The $\alpha-$ cuts of the fuzzy product uv are given (see the Appendix) by:

$$\begin{aligned} [uv]_\alpha &= [u_\alpha^-, u_\alpha^+]\,[v_\alpha^-, v_\alpha^+] \\ &= [\min\{(\widehat{u} + \widetilde{u}_\alpha)\, v_\alpha^- - \overline{u}_\alpha\, |v_\alpha^-|\,, (\widehat{u} + \widetilde{u}_\alpha)\, v_\alpha^+ - \overline{u}_\alpha\, |v_\alpha^+|\} \\ &\quad \max\{(\widehat{u} + \widetilde{u}_\alpha)\, v_\alpha^- + \overline{u}_\alpha\, |v_\alpha^-|\,, (\widehat{u} + \widetilde{u}_\alpha)\, v_\alpha^+ + \overline{u}_\alpha\, |v_\alpha^+|\}]\,. \end{aligned} \tag{3.6}$$

Distributivity of Multiplication

We use (3.6) to analyze the distributivity property of the multiplication.

Given three fuzzy numbers in terms of their components

$$u = (\widehat{u}, \widetilde{u}, \overline{u}); v = (\widehat{v}, \widetilde{v}, \overline{v}); z = (\widehat{z}, \widetilde{z}, \overline{z}),$$

we obtain from (3.6) the $\alpha-$ cuts

$$[(u+v)z]_\alpha = \left[\begin{array}{l} \min\left\{\begin{array}{l}(\widehat{u}+\widehat{v}+\widetilde{u}_\alpha+\widetilde{v}_\alpha)\, z_\alpha^- - (\overline{u}_\alpha+\overline{v}_\alpha)\,|z_\alpha^-| \\ (\widehat{u}+\widehat{v}+\widetilde{u}_\alpha+\widetilde{v}_\alpha)\, z_\alpha^+ - (\overline{u}_\alpha+\overline{v}_\alpha)\,|z_\alpha^+|\end{array}\right\}, \\ \max\left\{\begin{array}{l}(\widehat{u}+\widehat{v}+\widetilde{u}_\alpha+\widetilde{v}_\alpha)\, z_\alpha^- - (\overline{u}_\alpha+\overline{v}_\alpha)\,|z_\alpha^-|, \\ (\widehat{u}+\widehat{v}+\widetilde{u}_\alpha+\widetilde{v}_\alpha)\, z_\alpha^+ + (\overline{u}_\alpha+\overline{v}_\alpha)\,|z_\alpha^+|\end{array}\right\}\end{array}\right]$$

and

$$[uz+vz]_\alpha = \left[\begin{array}{c} \min\left\{(\widehat{u}+\widetilde{u}_\alpha)\, z_\alpha^- - \overline{u}_\alpha\,|z_\alpha^-|, (\widehat{u}+\widetilde{u}_\alpha)\, z_\alpha^+ - \overline{u}_\alpha\,|z_\alpha^+|\right\} \\ +\min\left\{(\widehat{v}+\widetilde{v}_\alpha)\, z_\alpha^- - \overline{v}_\alpha\,|z_\alpha^-|, (\widehat{v}+\widetilde{v}_\alpha)\, z_\alpha^+ - \overline{v}_\alpha\,|z_\alpha^+|\right\}, \\ \max\left\{(\widehat{u}+\widetilde{u}_\alpha)\, z_\alpha^- + \overline{u}_\alpha\,|z_\alpha^-|, (\widehat{u}+\widetilde{u}_\alpha)\, z_\alpha^+ + \overline{u}_\alpha\,|z_\alpha^+|\right\} \\ +\max\left\{(\widehat{v}+\widetilde{v}_\alpha)\, z_\alpha^- + \overline{v}_\alpha\,|z_\alpha^-|, (\widehat{v}+\widetilde{v}_\alpha)\, z_\alpha^+ + \overline{v}_\alpha\,|z_\alpha^+|\right\}\end{array}\right].$$

Theorem 3.1. *The distributivity $(u+v)z = uz + vz$ is valid if and only if, for each $\alpha \in [0,1]$, at least one of the following four systems of inequalities is satisfied:*

$$\left\{\begin{array}{l}(\widehat{u}+\widetilde{u}_\alpha)\, z_\alpha^- - \overline{u}_\alpha\,|z_\alpha^-| \leqslant (\widehat{u}+\widetilde{u}_\alpha)\, z_\alpha^+ - \overline{u}_\alpha\,|z_\alpha^+| \\ (\widehat{v}+\widetilde{v}_\alpha)\, z_\alpha^- - \overline{v}_\alpha\,|z_\alpha^-| \leqslant (\widehat{v}+\widetilde{v}_\alpha)\, z_\alpha^+ - \overline{v}_\alpha\,|z_\alpha^+| \\ (\widehat{u}+\widetilde{u}_\alpha)\, z_\alpha^+ + \overline{u}_\alpha\,|z_\alpha^+| \leqslant (\widehat{u}+\widetilde{u}_\alpha)\, z_\alpha^- + \overline{u}_\alpha\,|z_\alpha^-| \\ (\widehat{v}+\widetilde{v}_\alpha)\, z_\alpha^+ + \overline{v}_\alpha\,|z_\alpha^+| \leqslant (\widehat{v}+\widetilde{v}_\alpha)\, z_\alpha^- + \overline{v}_\alpha\,|z_\alpha^-|\end{array}\right. \tag{3.7}$$

$$\left\{\begin{array}{l}(\widehat{u}+\widetilde{u}_\alpha)\, z_\alpha^- - \overline{u}_\alpha\,|z_\alpha^-| \leqslant (\widehat{u}+\widetilde{u}_\alpha)\, z_\alpha^+ - \overline{u}_\alpha\,|z_\alpha^+| \\ (\widehat{v}+\widetilde{v}_\alpha)\, z_\alpha^- - \overline{v}_\alpha\,|z_\alpha^-| \leqslant (\widehat{v}+\widetilde{v}_\alpha)\, z_\alpha^+ - \overline{v}_\alpha\,|z_\alpha^+| \\ (\widehat{u}+\widetilde{u}_\alpha)\, z_\alpha^- + \overline{u}_\alpha\,|z_\alpha^-| \leqslant (\widehat{u}+\widetilde{u}_\alpha)\, z_\alpha^+ + \overline{u}_\alpha\,|z_\alpha^+| \\ (\widehat{v}+\widetilde{v}_\alpha)\, z_\alpha^- + \overline{v}_\alpha\,|z_\alpha^-| \leqslant (\widehat{v}+\widetilde{v}_\alpha)\, z_\alpha^+ + \overline{v}_\alpha\,|z_\alpha^+|\end{array}\right. \tag{3.8}$$

$$\left\{\begin{array}{l}(\widehat{u}+\widetilde{u}_\alpha)\, z_\alpha^+ - \overline{u}_\alpha\,|z_\alpha^+| \leqslant (\widehat{u}+\widetilde{u}_\alpha)\, z_\alpha^- - \overline{u}_\alpha\,|z_\alpha^-| \\ (\widehat{v}+\widetilde{v}_\alpha)\, z_\alpha^+ - \overline{v}_\alpha\,|z_\alpha^+| \leqslant (\widehat{v}+\widetilde{v}_\alpha)\, z_\alpha - \overline{v}_\alpha\,|z_\alpha^-| \\ (\widehat{u}+\widetilde{u}_\alpha)\, z_\alpha^+ + \overline{u}_\alpha\,|z_\alpha^+| \leqslant (\widehat{u}+\widetilde{u}_\alpha)\, z_\alpha^- + \overline{u}_\alpha\,|z_\alpha^-| \\ (\widehat{v}+\widetilde{v}_\alpha)\, z_\alpha^+ + \overline{v}_\alpha\,|z_\alpha^+| \leqslant (\widehat{v}+\widetilde{v}_\alpha)\, z_\alpha^- + \overline{v}_\alpha\,|z_\alpha^-|\end{array}\right. \tag{3.9}$$

$$\left\{\begin{array}{l}(\widehat{u}+\widetilde{u}_\alpha)\, z_\alpha^+ - \overline{u}_\alpha\,|z_\alpha^+| \leqslant (\widehat{u}+\widetilde{u}_\alpha)\, z_\alpha^- - \overline{u}_\alpha\,|z_\alpha^-| \\ (\widehat{v}+\widetilde{v}_\alpha)\, z_\alpha^+ - \overline{v}_\alpha\,|z_\alpha^+| \leqslant (\widehat{v}+\widetilde{v}_\alpha)\, z_\alpha^- - \overline{v}_\alpha\,|z_\alpha^-| \\ (\widehat{u}+\widetilde{u}_\alpha)\, z_\alpha^- + \overline{u}_\alpha\,|z_\alpha^-| \leqslant (\widehat{u}+\widetilde{u}_\alpha)\, z_\alpha^+ + \overline{u}_\alpha\,|z_\alpha^+| \\ (\widehat{v}+\widetilde{v}_\alpha)\, z_\alpha^- + \overline{v}_\alpha\,|z_\alpha^-| \leqslant (\widehat{v}+\widetilde{v}_\alpha)\, z_\alpha^+ + \overline{v}_\alpha\,|z_\alpha^+|\,.\end{array}\right. \tag{3.10}$$

Proof. We have the following dependencies:

$$\begin{array}{lll} (3.7) & \Longrightarrow & |z_\alpha^-| \geqslant |z_\alpha^+| \\ (3.8) & \Longrightarrow & \widehat{u} + \widetilde{u}_\alpha \geqslant 0, \widehat{v} + \widetilde{v}_\alpha \geqslant 0 \\ (3.9) & \Longrightarrow & \widehat{u} + \widetilde{u}_\alpha \leqslant 0, \widehat{v} + \widetilde{v}_\alpha \leqslant 0 \\ (3.10) & \Longrightarrow & |z_\alpha^-| \leqslant |z_\alpha^+| . \end{array}$$

The following propositions give relevant cases of validity of the distributivity and are immediately verified (at our knowledge, some of them are new in the fuzzy literature).

The first two properties say that the distributivity is valid if $z \in \mathbb{S}_0$ and, independently of their sign, the fuzzy numbers u and v have "homogeneous" asymmetries, i.e. $(\widehat{u} + \widetilde{u}_\alpha)(\widehat{v} + \widetilde{v}_\alpha) \geqslant 0$. □

Proposition 3.1. *Given $z \in \mathbb{S}_0$ and $\widehat{u} + \widetilde{u}_\alpha \geqslant 0, \widehat{v} + \widetilde{v}_\alpha \geqslant 0$ then system (3.8) is satisfied.*

Proof. In fact, $|z_\alpha^-| = -z_\alpha^- = z_\alpha^+ > 0$ and (3.8) reduces simply to $(\widehat{u} + \widetilde{u}_\alpha) z_\alpha^+ \geqslant 0$ and $(\widehat{v} + \widetilde{v}_\alpha) z_\alpha^+ \geqslant 0$ □

Proposition 3.2. *Given $z \in \mathbb{S}_0$ and $\widehat{u} + \widetilde{u}_\alpha \leqslant 0, \widehat{v} + \widetilde{v}_\alpha \leqslant 0$ then system (3.9) is satisfied.*

Proof. In fact, $|z_\alpha^-| = -z_\alpha^- = z_\alpha^+ > 0$ and (3.9) reduces simply to $(\widehat{u} + \widetilde{u}_\alpha) z_\alpha^+ \leqslant 0$ and $(\widehat{v} + \widetilde{v}_\alpha) z_\alpha^+ \leqslant 0$ □

Proposition 3.3. *Given $z \in \mathbb{F}_+$ and $u, v \in \mathbb{F}_0$ then system (3.10) is satisfied.*

Proof. In this case, z has $z_\alpha^- = |z_\alpha^-|, z_\alpha^+ = |z_\alpha^+|$, and $z_\alpha^- \leqslant z_\alpha^+$; u and v have $-\overline{u}_\alpha \leqslant \widehat{u} + \widetilde{u}_\alpha \leqslant \overline{u}_\alpha$ and $-\overline{v}_\alpha \leqslant \widehat{v} + \widetilde{v}_\alpha \leqslant \overline{v}_\alpha$; then system (3.10) is immediate. □

Proposition 3.4. *Given $z \in \mathbb{F}_-$ and $u, v \in \mathbb{F}_0$ then system (3.7) is satisfied.*

Proof. In this case, z has $z_\alpha^- = -|z_\alpha^-|, z_\alpha^+ = -|z_\alpha^+|$, and $z_\alpha^- \leqslant z_\alpha^+$; u and v have $-\overline{u}_\alpha \leqslant \widehat{u} + \widetilde{u}_\alpha \leqslant \overline{u}_\alpha$ and $-\overline{v}_\alpha \leqslant \widehat{v} + \widetilde{v}_\alpha \leqslant \overline{v}_\alpha$; then system (3.7) is immediate. □

3.3. Division

We complete this section by analyzing the division between fuzzy numbers $u \in \mathbb{F}$ and $z \in \mathbb{F}_- \cup \mathbb{F}_+$ The reciprocal of z is the fuzzy number given by z^{-1} with $\alpha - cuts$

$$\left[z^{-1}\right]_\alpha = \left[\frac{1}{z_\alpha^+}, \frac{1}{z_\alpha^-}\right]$$

and the division is defined as

$$\frac{u}{z} = uz^{-1}.$$

Note that if $u = (0, 0, \overline{u}_\alpha) \in \mathbb{S}_0$ then $\dfrac{u}{z} \in \mathbb{S}_0$ as the $\alpha-$ cuts are

$$\left[\frac{u}{z}\right]_\alpha = \left[-\overline{u}_\alpha \max\left\{\frac{1}{z_\alpha^+}, -\frac{1}{z_\alpha^-}\right\}, \overline{u}_\alpha \max\left\{\frac{1}{z_\alpha^+}, -\frac{1}{z_\alpha^-}\right\}\right].$$

If $z = (\widehat{z}, 0, \overline{z}) \in \mathbb{S}_- \cup \mathbb{S}_+$ is symmetric, then it is easy to see that $z^{-1} = \left(\widehat{z}^{-1}, \widetilde{z^{-1}}, \overline{z^{-1}}\right)$ is Not symmetric as

$$\widetilde{z_\alpha^{-1}} = \frac{\overline{z}_\alpha^2}{\widehat{z}\left(\widehat{z}^2 - \overline{z}_\alpha^2\right)}$$

$$\overline{z_\alpha^{-1}} = \frac{\overline{z}_\alpha}{\widehat{z}^2 - \overline{z}_\alpha^2}.$$

The distributivity of the division, i.e. the equality

$$\frac{u+v}{z} = \frac{u}{z} + \frac{v}{z}$$

can be written in terms of the distributivity of the multiplication:

$$(u+v)z^{-1} = uz^{-1} + vz^{-1}.$$

4. Conclusion

In this paper, we discussed about the Standard Fuzzy Arithmetic Operations and Properties of Arithmetic Operations Addition, Subfraction, Multiplication and Division.

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A STUDY ON FUZZY MULTI-IDEALS OF NEAR RINGS

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Abstract

Fuzzy multisets have come to the forefront of scientists' interest and have been used for algebraic structures such as groups, rings, and near rings. In this paper, we first summarize the knowledge about algebraic structure of fuzzy multisets such as fuzzy multi-subnear rings and fuzzy multi-ideals of near rings. Then we recall the results from our related previous work, where we defined different operations on fuzzy multi-ideals of near rings and we generalized some known results for fuzzy ideals of near rings to fuzzy multi-ideals of near rings.

Keywords: Near Ring, Fuzzy Multi-Ideal, Fuzzy Multi-Subnear Ring.

1. Introduction

Zadeh L.A introduced the concept of fuzzy sets in 1965 [51], after the introduction of fuzz sets several researchers explored on the generalization of the notion of fuzzy set. Kumbhojkar H.V and Bapat M.S [27] defined not-so fuzzy ideals, Palaniappan N and Arjunan K [39] defined the homomorphism, anti homomorphism of a fuzzy and an anti fuzzy ideals, Chandrasekhara Rao K and V Swaminathan [8] defined the anti homomorphisms in near rings.

S. Abou-Zaid [1] introduced the notion of a fuzzy subnear-ring, and studied fuzzy left (resp. right) ideals of a near-ring, and gave some properties of fuzzy prime ideals of a near-ring.

W. Liu [30] has studied fuzzy ideals of a ring, and many researchers [12, 24, 25, 40] are engaged in extending the concepts. S. Abou-Zaid [1] introduced the notion of a fuzzy subnear-ring, and studied fuzzy ideals of a near-ring, and many followers [17, 20, 24] discussed further properties of fuzzy ideals in near-rings. In [2], R. Biswas introduced the concept of anti fuzzy subgroups of groups, and K.H. Kim and Y.B. Jun studied the notion of anti fuzzy R-subgroups of near-ring [22].

To highlight the connection between fuzzy multisets and algebraic structures from an anti-fuzzification point of view. Moreover, this research proposes the generalization of the results known for anti-fuzzy ideals of near rings. It is known that the notion of fuzzy multiset is well entrenched in solving many real life problems.

The concept of fuzzy set was introduced by Zadeh. Since then these ideal have been applied to other algebraic structure like groups, rings, modules, topologies and so on. The notations of fuzzy sub near ring and ideals where introduced by S.Abou-Zaid in 1991. However, the fantastic growth of literature, the vitality of the field, and the applications of this theory provided a denial for this impression. This is well evident from contributions in many international journals such as fuzzy sets and systems, information sciences, information and control, journal of Mathematical Analysis and Applications, Rocky Mountain Journal of Mathematics, etc.,

In this paper, we study different operations on fuzzy multi-ideals of near rings and we generalized some known results for fuzzy ideals of near rings to fuzzy multi-ideals of near rings.

2. Fuzzy Multi-Ideals of Near Rings

Definition 2.1. *Let $(R, +, \cdot)$ be a near ring. A fuzzy multiset A (with fuzzy count function CM_A) over R is a fuzzy multi-subnear ring of R if for all $x, y \in R$, the following conditions hold.*

1. $CM_A(x) \wedge CM_A(y) \leqslant CM_A(x - y)$;
2. $CM_A(x) \wedge CM_A(y) \leqslant CM_A(xy)$.

Definition 2.2. *Let $(R, +, \cdot)$ be a near ring. A fuzzy multiset A (with fuzzy count function CM_A) over R is a fuzzy multi-ideal of R if for all $x, y \in R$, the following conditions hold.*

1. $CM_A(x) \wedge CM_A(y) \leqslant CM_A(x - y)$;
2. $CM_A(x) \wedge CM_A(y) \leqslant CM_A(xy)$;
3. $CM_A(y) \leqslant CM_A(x + y - x)$;
4. $CM_A(y) \leqslant CM_A(xy)$;
5. $CM_A(a) \leqslant CM_A((x + a)y - xy)$ *for all* $a \in R$.

Proposition 2.1. *Let $(R, +, \cdot)$ be a near ring with zero element $0 \in R$ and A be a fuzzy multi-ideal of R. Then*

1. $CM_A(x) \leqslant CM_A(0)$;
2. $CM_A(-x) = CM_A(x)$;
3. $CM_A(x+y-x) = CM_A(y)$;
4. $CM_A(a) \leqslant CM_A(x)$ *for all* $x \in < a >= \{ra : R \in R\}$;
5. $\bigwedge\limits_{1 \leqslant i \leqslant n} CM_A(x_i) \leqslant CM_A(x_1 + \cdots + x_n)$ *for all* $x_i \in R$.

Remark 2.1. *Let* $(R, +, \cdot)$ *be a near ring with zero element* $0 \in R$ *and* A *be a fuzzy multiset of* R *with* $CM_A(x) = CM_A(0)$ *for all* $x \in R$. *Then* A *is a fuzzy multi-ideal of* R *and it is called the constant fuzzy multi-ideal.*

Next, we deal with some operations on fuzzy multi-subnear rings (multi-ideals) of near rings such as intersection, union, and product.

Proposition 2.2. *Let* $(R, +, \cdot)$ *be a near ring and* A, B *be fuzzy multi-subnear rings of* R. *Then* $A \cap B$ *is a fuzzy multi-subnear ring of* R.

Corollary 2.1. *Let* $(R, +, \cdot)$ *be a near ring and* A_i *be a fuzzy multi-subnear ring of* R *for* $i = 1, \ldots, n$. *Then* $\bigcap\limits_{i=1}^{n} A_i$ *is a fuzzy multi-subnear ring of* R.

Proposition 2.3. *Let* $(R, +, \cdot)$ *be a near ring and* A, B *be fuzzy multi-ideals of* R. *Then* $A \cap B$ *is a fuzzy multi-ideal of* R.

Corollary 2.2. *Let* $(R, +, \cdot)$ *be a near ring and* A_i *be a fuzzy multi-ideal of* R *for* $i = 1, \ldots, n$. *Then* $\bigcap\limits_{i=1}^{n} A_i$ *is a fuzzy multi-ideal of* R.

Proposition 2.4. *Let* R, S *be near rings with fuzzy multisets* A, B, *respectively. If* A, B *are fuzzy multi-subnear rings (multi-ideals) of* R, S, *then* $A \times B$ *is fuzzy multi-subnear rings (multi-ideal) of* $R \times S$, *where* $CM_{A \times B}(r, s) = CM_A(r) \wedge CM_B(s)$ *for all* $(r, s) \in R \times S$.

Proposition 2.5. *Let* R_i *be near rings with fuzzy multiset* A_i *for* $i = 1, \ldots, n$. *If* A_i *is a fuzzy multi-subnear ring (multi-ideal) of* R_i, *then* $\prod\limits_{i=1}^{n} A_i$ *is a fuzzy multi-ideal of* $\prod\limits_{i=1}^{n} R_i$. R_i *where* $CM_{\prod_{i=1}^{n} A_i}((r_1, \ldots, r_n)) = \bigwedge\limits_{i=1}^{n} CM_{A_i}(r_i)$ *for all* $(r_1, \ldots, r_n) \in \prod\limits_{i=1}^{n} R_i$.

Notation 1 Let $(R, +, \cdot)$ be a near ring, A be a fuzzy multiset of R, and $CM_A(x) = (\mu_A^1(x), \mu_A^2(x), \ldots, \mu_A^p(x))$. Then

- $CM_A(x) = 0$ if $\mu_A^1(x) = 0$,
- $CM_A(x) > 0$ if $\mu^1 A(x) > 0$,
- $CM_A(x) = 1$ if $CM_A(x) = (1, 1, 1, 1, \ldots, S \text{ times})$

where $s = \max\{k \in \mathbb{N} : CM_A(y) = (\mu_A^1(y), \mu_A^2(y), \ldots, \mu_A^k(y)), \mu_A^k(y) \neq 0, y \in R\}$.

Definition 2.3. *Let $(R, +, \cdot)$ be a near ring and A be a fuzzy multiset of H. Then $A_* = \{x \in R : CM_A(x) > 0\}$ and $A^* = \{x \in R : CM_A(x) = CM_A(0)\}$.*

Proposition 2.6. *Let $(R, +, \cdot)$ be a near ring and A be a fuzzy multi-subnear ring (multi-ideal) of R. Then A_* is either the empty set or a subnear ring (ideal) of R.*

Proposition 2.7. *Let $(R, +, \cdot)$ be a near ring with a fuzzy multi-ideal A. Then the fuzzy multiset of $P_n(R)$ defined as $CM(a_0 + a_1x + \cdots + a_nx) = CM_A(a_0)$ is a fuzzy multi-ideal of $P_n(R)$.*

Notation 2 Let $(R, +, \cdot)$ be a near ring, A be a fuzzy multiset of R and

$$CM_A(x) = (\mu_A^1(x), \mu_A^2(x), \ldots, \mu_A^p(x)).$$

We say that $CM_A(x) \geqslant (t_1, \ldots, t_k)$ if $p \geqslant k$ and $\mu_A^i(x) \geqslant t_i$ for all $i = 1, \ldots, k$. If $CM_A(x) \not\geqslant (t_1, \ldots, t_k)$ and $(t_1, \ldots, t_k) \not\geqslant CM_A(x)$, then we say that $CM_A(x)$ and $(t_1, \ldots, t_k)$ are not comparable.

Notation 3 Let $(R, +, \cdot)$ be a near ring, A a fuzzy multiset of R with fuzzy count function CM, and $t = (t_1, \ldots, t_k)$, where $t_i \in \cup[0, 1]$ for $i = 1, \ldots, k$ and $t_1 \geqslant t_2 \geqslant \cdots \geqslant t_k$. Then $CM^t = \{x \in R : CM(x) \geqslant t\}$.

Theorem 2.1. *Let $(R, +, \cdot)$ be a near ring, A a fuzzy multiset of R with fuzzy count function CM and $t = (t_1, \ldots, t_k)$, where $t_i \in [0, 1]$ for $i = 1, \ldots, k$ and $t_1 \geqslant t_2 \geqslant \ldots \geqslant t_k$. Then A is a fuzzy multi-subnear ring of R if and only if CM_t is either the empty set or a subnear ring of R.*

Theorem 2.2. *Let $(R, +, \cdot)$ be a near ring, A a fuzzy multiset of R with fuzzy count function CM and $t = (t_1, \ldots, t_k)$, where $t_i \in [0, 1]$ for $i = 1, \ldots, k$ and $t_1 \geqslant t_2 \geqslant \ldots \geqslant t_k$. Then A is a fuzzy multi-ideal of R if and only if CM_t is either the empty set or an ideal of R.*

Corollary 2.3. *Let $(R, +, \cdot)$ be a near ring. If every subnear ring of R is an ideal of R then every fuzzy multi-subnear ring of R is a fuzzy multi-ideal of R.*

Proposition 2.8. *Let $(R, +, \cdot)$ be a near ring and I be an ideal of R. Then $I = CM_t$ for some fuzzy multi-ideal CM of $R, t = (t_1, \ldots, t_k)$, where $t_i \in [0, 1]$ for $i = 1, \ldots, k$ and $t_1 \geqslant t_2 \geqslant \cdots \geqslant t_k$.*

Corollary 2.4. *Let $(R, +, \cdot)$ be a near ring. Then R has at least one fuzzy multi-ideal beside the constant fuzzy multi-ideal.*

Definition 2.4. *Let $(R, +, \cdot)$ be a near ring, A and B be fuzzy multi-subnear rings (ideals) of R with fuzzy count functions CM_A and CM_B, respectively. If for all $x, y \in R$:*

1. *$CM_A(x) = CM_A(y)$ if and only if $CM_B(x) = CM_B(y)$,*
2. *$CM_A(x) \leqslant CM_A(y)$ if and only if $CM_B(x) \leqslant CM_B(y)$, and*
3. *$CM_A(x), CM_A(y)$ are not comparable if and only if $CM_B(x), CM_B(y)$ are not comparable,*

then A and B are equivalent fuzzy multi-subnear rings (ideals) of R.

3. Conclusion

In this paper, we found a new link between algebraic structures and fuzzy multisets by introducing fuzzy multi-ideals of near rings and studying their properties. The various basic operations, definitions and theorems related to fuzzy multi-ideals of near rings have been discussed. The results in this paper can be considered as a generalization of the results known for fuzzy ideals of near rings. Moreover, our results are considered as a generalization for fuzzy ideals of rings. This is because every ring is a near ring. The aim of this paper was to highlight the connection between fuzzy multisets and algebraic structures. It is well known that the concept of fuzzy multiset is well established in dealing with many real life problems. So, the algebraic structure defined concerning them in this paper would help to approach these problems with a different perspective. The benefit of the paper is the link found between algebraic structures and fuzzy multisets and studying their properties.

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A STUDY ON MULTIPLICATION OPERATION ON FUZZY NUMBERS

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Abstract

A fuzzy number is simply an ordinary number whose precise value is somewhat uncertain. Fuzzy numbers are used in statistics, computer programming, engineering, and experimental science. The arithmetic operators on fuzzy numbers are basic content in fuzzy mathematics. Operation of fuzzy number can be generalized from that of crisp interval. The operations of interval are discussed. Multiplication operation on fuzzy numbers is defined by the extension principle. Based on extension principle, nonlinear programming method, analytical method, computer drawing method and computer simulation method are used for solving multiplication operation of two fuzzy numbers. The nonlinear programming method is a precise method also, but it only gets a membership value as given number and it is a difficult problem for solving nonlinear programming. The analytical method is most precise, but it is hard to a -cuts interval when the membership function is complicated. The computer drawing method is simple, but it need calculate the a -cuts interval. The computer simulation method is the most simple, and it has wide applicability, but the membership function is rough. Each method is illuminated by examples.

Keywords: Index Terms, Fuzzy Number, Membership Function, Extension Principle, α-cuts; Nonlinear Programming.

1. Introduction

A. Fuzzy Number

If a fuzzy set is convex and normalized, and its membership function is defined in R and piecewise continuous, it is called as fuzzy number. So fuzzy number (fuzzy set) represents a real number interval whose boundary is fuzzy.

Fuzzy number is expressed as a fuzzy set defining a fuzzy interval in the real number R. Since the boundary of this interval is ambiguous, the interval is also a fuzzy set.

Generally a fuzzy interval is represented by two end points a_1 and a_3 and a peak point a_2 as $[a_1, a_2, a_3]$. The α-cut operation can be also applied to the fuzzy number. If we denote α-cut interval for fuzzy number A as A_α, the obtained interval A_α is defined as $A_\alpha = [a_1^\alpha, a_3^\alpha]$.

We can also know that it is an ordinary crisp interval.

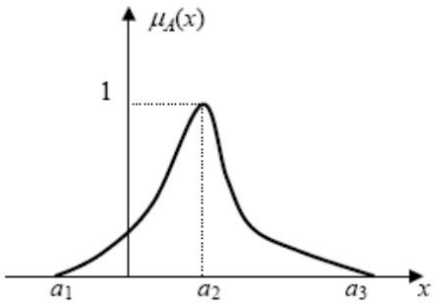

Figure 1: Fuzzy Number $A = [a_1, a_2, a_3]$

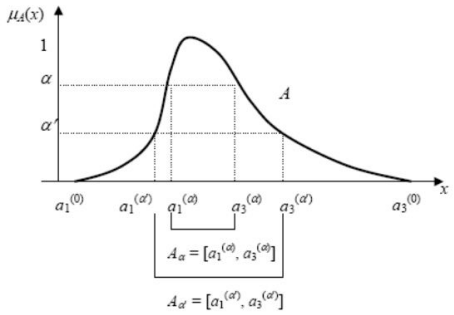

Figure 2: α**-cut of fuzzy number** $(\alpha' < \alpha) \Rightarrow (A_\alpha \subset A_{\alpha'})$

Fuzzy number should be normalized and convex. Here the condition of normalization implies that maximum membership value is 1. $\exists x_0 \in R$, $\mu_{\tilde{A}}(x_0) = 1$.

B. Operation of α-cut Interval

The convex condition is that the line by α-cut is continuous and α-cut interval satisfies the following relation. $A_\alpha = [a_1^\alpha, a_3^\alpha]$

$$(\alpha' < \alpha) \Rightarrow (a_1^{(\alpha')} \leqslant a_1^{(\alpha)},\ a_3^{(\alpha')} \geqslant a_3^{(\alpha)}.$$

The convex condition may also be written as,

$$(\alpha' < \alpha) \Rightarrow (A_\alpha \subset A_{\alpha'}).$$

Operation of fuzzy number can be generalized from that of crisp interval. Lets have a look at the operations of interval.

$$A = [a_1, a_3],\ \ B = [b_1, b_3] \qquad \forall a_1, a_3, b_1, b_3 \in \mathbb{R}.$$

Assuming A and B as numbers expressed as interval, main operations of interval are

(i) Addition

$$[a_1, a_3]\,(+)\,[b_1, b_3] = [a_1 + b_1, a_3 + b_3].$$

(ii) Subtraction

$$[a_1, a_3]\,(-)\,[b_1, b_3] = [a_1 - b_3, a_3 - b_1].$$

(iii) Multiplication

$$\begin{aligned}[a_1, a_3]\,(\cdot)\,[b_1, b_3] &= [a_1 \cdot b_1 \wedge a_1 \cdot b_3 \wedge a_3 \cdot b_1 \wedge a_3 \cdot b_3\,, \\ &\qquad a_1 \cdot b_1 \vee a_1 \cdot b_3 \vee a_3 \cdot b_1 \vee a_3 \cdot b_3]\,.\end{aligned}$$

(iv) Division

$$\begin{aligned}[a_1, a_3]\,(/)\,[b_1, b_3] &= [a_1/b_1 \wedge a_1/b_3 \wedge a_3/b_1 \wedge a_3/b_3\,, \\ &\qquad a_1/b_1 \vee a_1/b_3 \vee a_3/b_1 \vee a_3/b_3]\end{aligned}$$

excluding the case $b_1 = 0$ or $b_3 = 0$.

(v) Inverse Interval

$$[a_1, a_3]^{-1} = [1/a_1 \wedge 1/a_3, 1/a_1 \vee 1/a_3]$$

excluding the case $a_1 = 0$ or $a_3 = 0$.

When previous sets A and B is defined in the positive real number $\mathbb{R}^+$, the operations of multiplication, division, and inverse interval are written as

(iii') Multiplication

$$[a_1, a_3]\,(\cdot)\,[b_1, b_3] = [a_1 \cdot b_1, a_3 \cdot b_3]\,.$$

(iv') Division

$$[a_1, a_3]\,(/)\,[b_1, b_3] = [a_1/b_3, a_3/b_1]\,.$$

(v') Inverse Interval

$$[a_1, a_3]^{-1} = [1/a_3, 1/a_1]\,.$$

(vi) Minimum

$$[a_1, a_3]\,(\wedge)\,[b_1, b_3] = [a_1 \wedge b_1, a_3 \wedge b_3]\,.$$

(vii) Maximum

$$[a_1, a_3]\,(\vee)\,[b_1, b_3] = [a_1 \vee b_1, a_3 \vee b_3]\,.$$

Example 1.1. *There are two intervals A and B,* $A = [3, 5], B = [-2, 7]$. *Then following operation might be set.*

$$\begin{aligned}A(+)B &= [3 - 2, 5 + 7] = [1, 12] \\ A(-)B &= [3 - 7, 5 - (-2)] = [-4, 7] \\ A(\cdot)B &= [3 \cdot (-2) \wedge 3 \cdot 7 \wedge 5 \cdot (-2) \wedge 5 \cdot 7, 3 \cdot (-2) \vee 3 \cdot 7 \vee 5 \cdot (-2) \vee 5 \cdot 7] \\ &= [-10, 35] \\ A\,(/)\,B &= [3/(-2) \wedge 3/7 \wedge 5/(-2) \wedge 5/73/(-2) \vee 3/7 \vee 5/(-2) \vee 5/7] = [-2.5, 5/7]\end{aligned}$$

$$\begin{aligned} A^{-1} &= [3,5]^{-1} = [1/5, 1/3] \\ B^{-1} &= [-2,7]^{-1} = [1/(-2) \wedge 1/7, 1/(-2) \vee 1/7] = [-1/2, 1/7] \\ A(\wedge)B &= [3 \wedge (-2), 5 \wedge 7] = [-2, 5] \\ A(\vee)B &= [3 \vee (-2), 5 \vee 7] = [3, 7]. \end{aligned}$$

C. Operation of Fuzzy Numbers

Based on the extension principle, arithmetic operations on fuzzy numbers are defined by following:

If $\widetilde{M}$ and $\widetilde{N}$ are fuzzy numbers, membership of $\widetilde{M}(*)\widetilde{N}$ is defined as follow:

$$\mu_{\widetilde{M}(*)\widetilde{N}}(z) = \sup_{z=x*y} \min\left\{\mu_{\widetilde{M}}(x), \mu_{\widetilde{N}}(y)\right\}, \tag{1.1}$$

where $*$ stands for any of the four arithmetic operations.

$$\begin{aligned} \mu_{\widetilde{M}(+)\widetilde{N}}(z) &= \sup_{z=x+y} \min\left\{\mu_{\widetilde{M}}(x), \mu_{\widetilde{N}}(y)\right\} \\ \mu_{\widetilde{M}(-)\widetilde{N}}(z) &= \sup_{z=x-y} \min\left\{\mu_{\widetilde{M}}(x), \mu_{\widetilde{N}}(y)\right\} \\ \mu_{\widetilde{M}(/)\widetilde{N}}(z) &= \sup_{z=x/y} \min\left\{\mu_{\widetilde{M}}(x), \mu_{\widetilde{N}}(y)\right\}. \end{aligned}$$

Therefore multiplication operation on fuzzy numbers is expressed as

$$\mu_{\widetilde{M}(x)\widetilde{N}}(z) = \sup_{z=x\times y} \min\left\{\mu_{\widetilde{M}}(x), \mu_{\widetilde{N}}(y)\right\}. \tag{1.2}$$

The procedure of addition or subtraction is simple, but the procedure of multiplication or division is complex.

2. Nonlinear Programming Method

Based on multiplication operation on fuzzy numbers, multiplication operation problem is formulated as a nonlinear programming.

$$\max v \text{ s.t.} \quad \mu_{\widetilde{M}}(x) \geqslant v; \qquad \mu_{\widetilde{N}}(y) \geqslant v; \qquad xy = z_0. \tag{2.1}$$

Given z_0, we can get the maximum $v_{\max}$ of nonlinear Programming (1.1). $v_{\max}$ is membership value of z_0.

Example 2.1. Suppose that the membership of $\widetilde{M}, \widetilde{N}$ are $\mu_{\widetilde{M}}(x) = e^{-(x-2)^2}$ (Figure 3) and $\mu_{\widetilde{N}}(y) = e^{-(y+2)^2}$ $\widetilde{Q} = \widetilde{M}(\times)\widetilde{N}$ is formulated as following:

$$\max v \text{ s.t.} \quad e^{-(x-2)^2} \geqslant v; \qquad e^{-(y+2)^2} \geqslant v; \qquad xy = z_0. \tag{2.2}$$

When $z_0 = -12, -11, -10, -9, -8, -7, -6, -5, -4, -3, -2, -1$ respectively, we can get the $v_{\max}$ correspondingly. The membership value of z_0 is shown in Table 1.

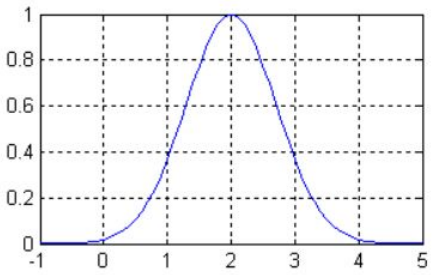

Figure 3: Membership function of $\widetilde{M}$

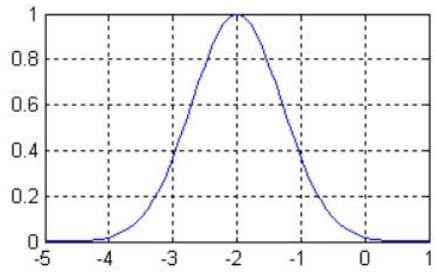

Figure 4: Membership function of $\widetilde{N}$

Table 1: z_0 and its Membership Value

z_0	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9
$v_{\max}$	0.0065	0.0110	0.0183	0.0300	0.0482	0.0759	0.1172	0.1767	0.2590	0.3679
z_0	-8	-7	-6	-5	-4	-3	-2	-1	0	1
$v_{\max}$	0.5034	0.6590	0.8171	0.9458	1.0000	0.9307	0.7095	0.3679	0.0183	0.0067

3. Conclusion

In this paper, we discussed about the concept of fuzzy number, Operation of α-cut interval, Operation of Fuzzy numbers and Nonlinear Programming Method with example.

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A STUDY ON ANALYTICAL SOLUTIONS FOR STOCHASTIC DIFFERENTIAL EQUATIONS VIA MARTINGALE PROCESSES

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Abstract

In this paper, we propose some analytical solutions of stochastic differential equations related to Martingale processes. In the first resolution, the answers of some stochastic differential equations are connected to other stochastic equations just with diffusion part (or drift free). The second suitable method is to convert stochastic differential equations into ordinary ones that it is tried to omit diffusion part of stochastic equation by applying Martingale processes. Finally, solution focuses on change of variable method that can be utilized about stochastic differential equations which are as function of Martingale processes like Wiener process, exponential Martingale process and differentiable processes.

Keywords: Martingale Process, Itô Formula, Change of Variable, Differentiable Process, Analytical Solution.

1. Introduction

The purpose of this article is to put forward some analytical and numerical solutions to solve the Itô stochastic differential equation (SDE):

$$\begin{cases} \mathrm{d}X(t) &= \mathcal{A}(X(t),t)\mathrm{d}t + \mathcal{B}(X(t),t)\mathrm{d}W_t, \\ X(0) &= X_0, \end{cases} \tag{1.1}$$

where $W(t)$ is a Wiener process and triple $(\Omega, \mathcal{F}, \mathbb{P})$ is a probability space under some conditions and special relations between drift and volatility.

Both the drift vector $\mathcal{A} : \mathbb{R} \times [0,T] \longrightarrow \mathbb{R}$ and the diffusion matrix $a := \mathcal{B}\mathcal{B}^T : \mathbb{R} \times [0,T] \longrightarrow \mathbb{R}$ are considered Borel measurable and locally bounded functions. It is assumed that X_0 is a non-random vector. As usual, $\mathcal{A}$ and $\mathcal{B}$ are globally Lipschitz in $\mathbb{R}$ that is:

$$|\mathcal{A}(X,t) - \mathcal{A}(Y,t)| + |\mathcal{B}(X,t) - \mathcal{B}(Y,t)| \leqslant D|X-Y|, \quad X, Y \in \mathbb{R} \quad \text{and} \quad t \in [0,T],$$

and result in the linear growth condition:

$$|\mathcal{A}(X,t)| + |\mathcal{B}(X,t)| \leqslant C(1 + |X|).$$

These conditions guarantee the Eq. (1.1) has a unique t-continuous solution adapted to the filtration $\mathcal{F}_t t \geqslant 0$ generated by $W(t)$ and

$$E\left[\int_0^T |X(s)|^2 \, \mathrm{d}s\right] < \infty. \tag{1.2}$$

It is generally accepted that, analytical solutions of partial and ordinary differential equations are so important particularly in physics and engineering, whereas most of them do not have an exact solution and even a limited number of these equations, (e.g., in classical form), have implicit solutions. Analytical methods and solutions, especially in stochastic differential equations, could be excessive fundamental in some cases therefore we draw to take a comparison and analyze computation error between them and different numerical methods. Numerous numerical methods can be applied to solve stochastic differential equations like Monte Carlo simulation method, finite elements and finite differences.

2. Change of Measure and Martingale Process

In this section under some conditions, we intend to make a Martingale process from a random one in $\mathbb{L}^2(\mathbb{R} \times [0,T])$, where T is called maturity time. The exponential Martingale process associated with $\lambda(t)$ is defined as follows:

$$\mathcal{Z}_t^\lambda = \exp\left(\int_0^t \lambda(s)\, \mathrm{d}W_s - \frac{1}{2}\int_0^t \lambda^2(s)\, \mathrm{d}s\right). \tag{2.1}$$

It can be indicated by Itô formula that $\mathcal{Z}_t^\lambda$ is a Martingale due to the drift-free property:

$$\mathrm{d}\mathcal{Z}_t^\lambda = \lambda \mathcal{Z}_t^\lambda \mathrm{d}W_t, \quad \mathcal{Z}_t^\lambda(0) = 1. \tag{2.2}$$

Theorem 2.1. *Suppose that stochastic processes X_t verify in differential equation:*

$$\mathrm{d}X_t = \mu(X_t,t)\mathrm{d}t + \sigma(X_t,t)\mathrm{d}W_t, \tag{2.3}$$

and let $\lambda(t) := -\mu(X_t,t)/\sigma(X_t,t)$. Therefore, $X\mathcal{Z}_t^\lambda$ is a Martingale process.

Proof. With attention to real function $\lambda(t)$, we have:

$$\begin{cases} \mathrm{d}X = \mu(X,t)\mathrm{d}t + \sigma(X,t)\mathrm{d}W_t = -\lambda(t)\sigma(X,t)\mathrm{d}t + \sigma(X,t)\mathrm{d}W_t, \\ \mathrm{d}\mathcal{Z}_t^\lambda = \mathcal{Z}_t^\lambda \lambda \mathrm{d}W_t. \end{cases}$$

By utilizing Itô product formula, we get:

$$\begin{aligned}\mathrm{d}(X\mathcal{Z}_t^\lambda) &= X\mathrm{d}(\mathcal{Z}_t^\lambda) + \mathcal{Z}_t^\lambda \mathrm{d}X + \mathrm{d}X\mathrm{d}(\mathcal{Z}_t^\lambda) \\ &= \lambda X\mathcal{Z}_t^\lambda \mathrm{d}W_t + \mu(X,t)\mathcal{Z}_t^\lambda \mathrm{d}t + \sigma(X,t)\mathcal{Z}_t^\lambda \mathrm{d}W_t + \lambda\sigma(X,t)\mathcal{Z}_t^\lambda \mathrm{d}t.\end{aligned}$$

According to theorem assumption, we obtain:

$$\mathrm{d}(X\mathcal{Z}_t^\lambda) = \mathcal{Z}_t^\lambda(X\lambda + \sigma(X,t))\mathrm{d}W_t. \tag{2.4}$$

It emphasizes that $X\mathcal{Z}_t^\lambda$ is a P-Martingale. □

Therefore, $\lambda(t) = \dfrac{-\mu(X,t)}{\sigma(X,t)}$ is the sufficient condition for following SDEs equivalence:

$$\mathrm{d}X = \mu(X,t)\mathrm{d}t + \sigma(X,t)\mathrm{d}W_t \Leftrightarrow \mathrm{d}(X\mathcal{Z}_t^\lambda) = \mathcal{Z}_t^\lambda(X\lambda(t) + \sigma(X,t))\mathrm{d}W_t. \tag{2.5}$$

Consequently, by solving the obtained equation in Eq. (2.4), we obtain the following result when $\mathcal{Z}_0^\lambda = 1$:

$$X\mathcal{Z}_t^\lambda = \int_0^t \mathcal{Z}_t^\lambda(X\lambda(s) + \sigma(X,t))\,\mathrm{d}W_t + X_0. \tag{2.6}$$

By taking mathematical expectation from both sides of Eq. (2.6):

$$E^P[X\mathcal{Z}_t^\lambda] = X_0 \Rightarrow E^P[X] = X_0(\mathcal{Z}_t^\lambda)^{-1}. \tag{2.7}$$

In addition, to compute the variance of this stochastic process:

$$\begin{aligned}E^P[(X\mathcal{Z}_t^\lambda)^2] &= X_0^2 + E\left[\int_0^t (\mathcal{Z}_s^\lambda)^2(X\lambda(s) + \sigma(X,t))^2\,\mathrm{d}s\right] \quad (\text{ by Itŏ isometry }) \\ &= X_0^2 + \int_0^t (\mathcal{Z}_s^\lambda)^2 E\left(\left[(X\lambda(s) + \sigma(X,t))^2\right]\right)\,\mathrm{d}s.\ \mathrm{var}\,(X\mathcal{Z}_t^\lambda) = (\mathcal{Z}_t^\lambda)^2\,\mathrm{var}\,(X) \\ &= \int_0^t (\mathcal{Z}_s^\lambda)^2 E\left(\left[(X\lambda(s) + \sigma(X,t))^2\right]\right)\,\mathrm{d}s.\end{aligned} \tag{2.8}$$

Applying (2.4) and using numerical approximation by EM method, we have:

$$\begin{aligned}\Delta X_i\mathcal{Z}_{t_i}^\lambda &= \mathcal{Z}_{t_i}^\lambda(X_i\lambda(t_i) + \sigma_i)\Delta W_i. \\ X_{t_{i+1}}\mathcal{Z}_{t_{i+1}}^\lambda &= X_{t_i}\mathcal{Z}_{t_i}^\lambda + \mathcal{Z}_{t_i}^\lambda(X_{t_i}\lambda(t_i) + \sigma_i)\Delta W_i. \\ X_{t_{i+1}} &= (\mathcal{Z}_{t_{i+1}}^\lambda)^{-1}\mathcal{Z}_{t_i}^\lambda(X_{t_i} + (X_{t_i}\lambda(t_i) + \sigma_i)\Delta W_i).\end{aligned}$$

Direct calculations would lead to the conclusion that:

$$R_{t_i} = (\mathcal{Z}_{t_{i+1}}^\lambda)^{-1}\mathcal{Z}_{t_i}^\lambda = \exp\left(-\int_{t_i}^{t_{i+1}} \lambda(s)\,\mathrm{d}W_s + \frac{1}{2}\int_{t_i}^{t_{i+1}} |\lambda^2(s)|\,\mathrm{d}s\right).$$

So the following Milstein recursive method is inferred as a good numerical method to find $X(t_{i+1})$:

$$X_{t_{i+1}} = R_{t_i}(X_{t_i} + (X_{t_i}\lambda(t_i) + \sigma_i)\Delta W_i) + \frac{1}{2}R_{t_i}^2\lambda(t_i)\,(X_{t_i}\lambda(t_i) + \sigma_i)\,(\Delta^2 W_i - \Delta t_i). \quad (2.9)$$

In example, we compare this method with usual Milstein method in the case that a stochastic differential equation contains drift and volatility both parts and indicate that this method could be better in some cases.

3. Change of Variable Method

This section intends to analyze the change of variable method like, to get explicitly the solution of arbitrary SDE:

$$\mathrm{d}X = \mathcal{A}(X,t)\mathrm{d}t + \mathcal{B}(X,t)\mathrm{d}W_t, \quad X(0) = x.$$

By finding appropriate variables $u(Y) = X$ and their conditions so that Y is the answer of a well-known SDEs related to Martingale processes.

$$\mathrm{d}Y = f(X,t)\mathrm{d}t + g(X,t)\mathrm{d}W_t, \quad y(0) = y.$$

For more explanation and different conditions under which they are possible. Now we consider following various cases.
Case 1 Consider the following SDE:

$$\mathrm{d}Y = a(t)\mathrm{d}t + b(t)\mathrm{d}W_t. \quad (3.1)$$

Applying Itô formula for $u(Y) = X$, to (3.1), we get:

$$\begin{cases} u'(a(t)) + \dfrac{1}{2}u''b^2(t) = \mathcal{A}(u(Y),t), \\ u'b(t) = \mathcal{B}(u(Y),t). \end{cases} \quad (3.2)$$

Thus, it concludes that:

$$\frac{a(t)}{b(t)}\mathcal{B} + \frac{1}{2}\mathcal{B}\mathcal{B}' = \mathcal{A} \Rightarrow \frac{\mathcal{A}}{\mathcal{B}} - \frac{1}{2}\mathcal{B}' = \frac{a(t)}{b(t)}. \quad (3.3)$$

Finally, the equation $\dfrac{\partial}{\partial Y}\left(\dfrac{\mathcal{A}}{\mathcal{B}} - \dfrac{1}{2}\mathcal{B}'\right) = 0$ is necessary condition to solve an equation via change of variable in (3.1) $\left(\mathcal{B}' = \dfrac{\partial \mathcal{B}}{\partial X}\right)$.
Case 2 Consider the exponential Martingale process SDE (2.1):

$$\begin{cases} \mathrm{d}Y = \lambda(t)Y\mathrm{d}W_t, \\ Y(0) = Y_0. \end{cases} \quad (3.4)$$

Applying Itô formula for $u(Y) = X$, to (3.4), we acquire:

$$\begin{cases} u'\lambda Y = \mathcal{B}(u,t) = \lambda(t) Y \hat{\mathcal{B}}(u) \quad \text{or} \quad u' = \hat{\mathcal{B}}(u), \\ \dfrac{1}{2} u''\lambda^2 Y^2 = \mathcal{A}(u,t). \end{cases} \tag{3.5}$$

So from the last equality, we have $\dfrac{\mathcal{B}'}{\lambda(t)} - \dfrac{2\mathcal{A}}{\mathcal{B}} = \lambda(t)$. Therefore, $\dfrac{\partial}{\partial u}\left(\mathcal{B}'_u - \dfrac{2\lambda(t)\mathcal{A}}{\mathcal{B}}\right) = 0$ is necessary condition to solve SDE, with this change of variable.

Case 3 Consider the well-known equation:

$$\begin{cases} \mathrm{d}Y = a(t) Y \mathrm{d}t + b(t) Y \mathrm{d}W_t, \\ Y(0) = Y_0. \end{cases} \tag{3.6}$$

Which is Black-Scholes equation with exact solution

$$Y_0 = \exp\left(\int_0^t b(s)\mathrm{d}W_s + \int_0^t \left(a(s) - \frac{1}{2} b^2(s)\right) \mathrm{d}s\right).$$

Applying Itô formula for $u(Y) = X$, to (3.6), we get:

$$\begin{cases} u' a(t) Y + \dfrac{1}{2} u'' b^2(t) Y^2 = \mathcal{A}(u,t), \\ u' Y b(t) = \mathcal{B}(u,t) = b(t) Y \hat{\mathcal{B}}(u). \end{cases} \tag{3.7}$$

For this reason, $u' = \hat{\mathcal{B}}(u)$ and we have:

$$\frac{a(t)}{b(t)} = \frac{\mathcal{A}}{\mathcal{B}} - \frac{1}{2}(\mathcal{B}'_u - b(t)) = \gamma(u,t). \tag{3.8}$$

It means that $\dfrac{\partial}{\partial u}\gamma(u,t) = 0$, is a necessary condition to solve the initial stochastic differential equation by this change of variable.

Case 4 Another appropriate and prominent case is as follows:

$$\begin{cases} \mathrm{d}Y_t = f(Y_t, t)\mathrm{d}t + c(t) Y_t \mathrm{d}W_t, \\ Y(0) = Y_0. \end{cases} \tag{3.9}$$

This kind of equations, applying Itô formula on $X_t = Y_t \mathcal{Z}_t^c(t)^{-1}$, is converted to a ordinary differential equations.

Theorem 3.1. *The stochastic differential equations in (3.9) given by continuous functions $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ and $C : \mathbb{R} \to \mathbb{R}$ can be written as:*

$$\mathrm{d}(Y_t(\mathcal{Z}_t^c(t))^{-1}) = (\mathcal{Z}_t^c(t))^{-1} f(Y_t, t)\mathrm{d}t, \tag{3.10}$$

where $\mathcal{Z}_t^c(t)$ is an exponential Martingale process.

To be more precise, using change of variable $V = X(Z_t^{c(t)})^{-1}$, it is enough to solve

$$\begin{cases} X_t' = (Z_t^{c(t)})^{-1} f(X_t Z_t^{c(t)}), \\ X(0) = X_0. \end{cases} \tag{3.11}$$

Applying Itô formula for $u(Y) = M_t$, in (3.9) we get:

$$\begin{aligned} &\mathrm{d}M_t = M_t' \mathrm{d}Y + \frac{1}{2} M_t'' (\mathrm{d}Y)^2. \\ &\begin{cases} f(Y,t)M_t' + \frac{1}{2} M_t'' c^2(t) Y^2 = \mathcal{A}(M_t, t), (1) \\ c(t) Y M_t' = \mathcal{B}(M_t, t),\ u(Y_0) = M_0. (2) \end{cases} \end{aligned} \tag{3.12}$$

According to (3.12), we have $\mathcal{B}(M_t, t) = c(t)\hat{\mathcal{B}}(M_t)$. Besides, if the new stochastic differential equation is related to a Martingale process, we have $\mathcal{A}(M_t, t) = 0$ and:

$$f(Y,t) = -\frac{c^2(t)Y}{2}(\hat{\mathcal{B}}(M_t)' - 1). \tag{3.13}$$

Again, applying Itô formula for $\phi(M_t) = V_t$ to Martingale equation contributes to

$$\mathrm{d}M_t = \mathcal{B}(M_t, t)\mathrm{d}W_t = c(t)\hat{\mathcal{B}}(M_t)\mathrm{d}W_t,$$

we can achieve to a novel group of stochastic differential equation that its solution is as a function of a Martingale process.

4. Conclusion

In this paper, a couple of analytical solutions of some determined set of stochastic differential equations was indicated via making the Martingale process from a stochastic process. Converting stochastic differential equations to ordinary ones as another suitable method was posed. Indeed, it is tried to omit diffusion part of stochastic equation by applying Martingale processes. In addition, change of variable method on SDEs related to Martingale processes was discussed.

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A STUDY ON STOCHASTIC INTEGRATION ON SPACES OF MODELLED DISTRIBUTIONS

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Abstract

The Besov spaces of modelled distributions are shown to be UMD Banach spaces and of martingale type 2. As a consequence, this gives access to a rich stochastic integration theory and to existence and uniqueness results for mild solutions of semilinear stochastic partial differential equations in these spaces of modelled distributions and for distribution-valued SDEs.

Keywords: UMD and M-type 2 Banach Spaces, Regularity Structures, Stochastic Integration in Banach Spaces.

1. Introduction

This paper is devoted to prove that the Besov spaces $\mathcal{D}^{\gamma}_{p,q}$ of modelled distributions are UMD Banach spaces and of martingale type 2. These Banach space properties open the door to apply highly developed stochastic integration theory on the spaces $\mathcal{D}^{\gamma}_{p,q}$. For example one can integrate predictable $\mathcal{D}^{\gamma}_{p,q}$-valued processes with respect to Brownian motion. One successful application of stochastic integration on Banach spaces lies in the area of stochastic partial differential equations, which we will discuss in more details below. For a more comprehensive introduction and treatment of stochastic integration on Banach spaces.

Let $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ be a complete filtered probability space, $I \subset \mathbb{R}, \mathbb{F} := (\mathcal{F}_t)_{t\in I}$ be an increasing family of sub-σ-algebra of $\mathcal{F}$ and X be a Banach space with norm $\|\cdot\|_X$. The expectation operator with respect to $\mathbb{P}$ is denoted $\mathbb{E}$ and the corresponding conditional expectation by $\mathbb{E}[\,\cdot\,|\mathcal{F}_t]$ for $t \in I$. A process $(M_t)_{t\in I}$ is a X-valued martingale if and only if $M_t \in L^1(\Omega, \mathcal{F}_t, \mathbb{P}; X)$ for all $t \in I$ and

$$\mathbb{E}[M_t|\mathcal{F}_s] = M_s \quad \mathbb{P}\text{-}a.s., \quad \text{for all } s, t \in I \text{ with } s \leqslant t.$$

A sequence $(\xi_i)_{i\in\mathbb{N}}$ is called martingale difference if $(\sum_{i=0}^{n} \xi_i)_{n\in\mathbb{N}}$ is a X-valued martingale. To rely on stochastic integration theory on Banach spaces, one needs to require some additional properties on the Banach space X.

2. Stochastic Integration on Spaces of Modelled Distributions

Definition 2.1. *Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space.*

- *A Banach space $(X, \|\cdot\|_X)$ is of martingale type p for $p \in [1, \infty)$ if any X-valued martingale $(M_n)_{n\in\mathbb{N}}$ satisfies*

$$\sup_n \mathbb{E}\left[\|M_n\|_X^p\right] \leqslant C_p(X) \sum_{n\in\mathbb{N}} \mathbb{E}\left[\|M_n - M_{n-1}\|_X^p\right]$$

for some constant $C_p(X) > 0$ independent of the martingale $(M_n)_{n\in\mathbb{N}}$ and $M_{-1} := 0$.

- *A Banach space $(X, \|\cdot\|_X)$ if of type p for $p \in [1, 2]$ if any finite sequence $\epsilon_1, \ldots, \epsilon_n : \Omega \to \{-1, 1\}$ of symmetric and i.i.d. random variables and for any finite sequence $x_1, \ldots, x_n$ of elements of X the inequality*

$$\mathbb{E}\left[\left\|\sum_{i=1}^n \epsilon_i x_i\right\|_X^p\right] \leqslant K_p(X) \sum_{i=1}^n \|x_i\|_X^p$$

holds for some constant $K_p(X) > 0$.

- *A Banach space $(X, \|\cdot\|_X)$ is called an UMD space or is said to have the unconditional martingale difference property if for any $p \in (1, \infty)$, for any martingale difference $(\xi_j)_{j\in\mathbb{N}}$ and for any sequence $(\epsilon_i)_{i\in\mathbb{N}} \subset \{-1, 1\}$ the inequality*

$$\mathbb{E}\left[\left\|\sum_{i=1}^n \epsilon_i \xi_j\right\|_X^p\right] \leqslant \tilde{K}_p(X)\mathbb{E}\left[\left\|\sum_{i=1}^n \xi_i\right\|_X^p\right]$$

holds for all $n \in \mathbb{N}$, where $\tilde{K}_p(X) > 0$ is some constant.

Let us remark that Hilbert spaces and finite dimensional Banach spaces are always UMD spaces.

Coming back to a regularity structure $\mathcal{T} = (A, T, G)$ with an associated model (Π, Γ) and let us assume now additionally that each T_α is an UMD space for $\alpha \in A$. Under this assumption the space $T_\gamma^- = \bigoplus_{\alpha<\gamma} T_\alpha$ is again an UMD space, since A is locally finite and T is a finite product of UMD spaces.

Proposition 2.1. Let $\mathcal{T} = (A, T, G)$ be a regularity structure with a model (Π, Γ) as in the Definition **??**. Suppose that $\gamma \in \mathbb{R}$ and that the Banach space T_α is an UMD space for every $\alpha \in A$. Then, the space $\mathcal{D}_{p,q}^\gamma$ is an UMD spaces, too, for $1 < p < \infty$ and $1 < q < \infty$. If the Banach space T_γ^- is additionally of type 2, then $\mathcal{D}_{p,q}^\gamma$ is of martingale type 2 for every $p \geqslant 2$ and $q \geqslant 2$.

Proof. Since every T_α with $\alpha \in A_\gamma$ is an UMD space by assumption, every $L^p(\mathbb{R}^d; T_\alpha)$ is also an UMD space. Furthermore, let μ be the Borel measure on $\mathbb{R}$ defined by

$$\mu(h) := \frac{1}{\|h\|_{\mathfrak{s}}^{|\mathfrak{s}|}} \, \mathrm{d}h,$$

the corresponding L^q-space $L^q_\mu(B(0,1); L^p(\mathbb{R}^d; T_\alpha))$ is again an UMD space for every $\alpha \in A_\gamma$. Consequently the finite product space

$$\prod_{\alpha \in A_\gamma} \Big(L^p(\mathbb{R}^d; T_\alpha) \times L^q_\mu(B(0,1); L^p(\mathbb{R}^d; T_\alpha)) \Big)$$

is an UMD space. We will show that $\mathcal{D}^\gamma_{p,q}$ is a closed linear subspace in the above product space and we can conclude that $\mathcal{D}^\gamma_{p,q}$ is an UMD space.

For this purpose we define for every $\alpha \in A_\gamma$ the following mappings

$$\Phi^\alpha_1 : \mathcal{D}^\gamma_{p,q} \to L^p(\mathbb{R}^d; T_\alpha) \quad \text{via} \quad f \mapsto f^\alpha$$

and

$$\Phi^\alpha_2 : \mathcal{D}^\gamma_{p,q} \to L^q_\mu(B(0,1); L^p(\mathbb{R}^d; T_\alpha)) \quad \text{via} \quad f \mapsto \Big[h \mapsto \frac{f^\alpha(\cdot + h) - (\Gamma_{\cdot+h,\cdot} f(\cdot))^\alpha}{\|h\|_{\mathfrak{s}}^{\gamma-\alpha}} \Big],$$

where f^α is the projection of f onto T_α and $\dfrac{f^\alpha(\cdot + h) - (\Gamma_{\cdot+h,\cdot} f(\cdot))^\alpha}{\|h\|_{\mathfrak{s}}^{\gamma-\alpha}}$ is an element in $L^p(\mathbb{R}^d; T_\alpha)$ such that

$$\frac{f^\alpha(\cdot + h) - (\Gamma_{\cdot+h,\cdot} f(\cdot))^\alpha}{\|h\|_{\mathfrak{s}}^{\gamma-\alpha}}(x) = \frac{f^\alpha(x + h) - (\Gamma_{x+h,x} f(x))^\alpha}{\|h\|_{\mathfrak{s}}^{\gamma-\alpha}}$$

for all $x \in \mathbb{R}^d$.

Clearly, the mapping $\Big(\Phi^\alpha_1 \times \Phi^\alpha_2 \Big)_{\alpha \in A_\gamma}$ is an isometry from $\mathcal{D}^\gamma_{p,q}$ onto its image in the product space

$$\prod_{\alpha \in A_\gamma} \Big(L^p(\mathbb{R}^d; T_\alpha) \times L^q_\mu(B(0,1); L^p(\mathbb{R}^d; T_\alpha)) \Big),$$

so that we can embed $\mathcal{D}^\gamma_{p,q}$ into the above product space as a closed linear subspace. The space $\mathcal{D}^\gamma_{p,q}$ is therefore UMD, too. The previous construction is similar. Since every UMD space of type 2 is a Banach space of martingale type 2 as shown, one concludes that $L^p(\mathbb{R}^d; T_\alpha)$ and $L^q_\mu(B(0,1); L^p(\mathbb{R}^d; T_\alpha))$ are of martingale type 2 for every $p \in [2, \infty), q \in [2, \infty)$ and $\alpha \in A_\gamma$, and the same argument as before applies. □

we can now formulate and prove our main theorem. Like in the Fubini theorem the order of reconstruction and stochastic integration can be interchanged:

Theorem 2.1. *Let $\gamma > \alpha_0 := \inf A, \alpha_0 \notin \mathbb{Z}$ and $\mathcal{T} = (A, T, G)$ be a regularity structure together with a model (Π, Γ) and T_α is an UMD space for every $\alpha \in A$. Let $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ be a complete filtered probability space and W be Brownian motion on $[0, T]$ for some $T \in (0, \infty)$. Let H be a $\mathcal{D}^\gamma_{p,q}$-valued process for some $1 < p < \infty$ and $1 < q < \infty$ which is locally L^2-stochastically integrable with respect to W, then the order of "integration" can be interchanged*

$$\left\langle \mathcal{R}\big((H \bullet W)\big), \psi \right\rangle = \left(\big\langle \mathcal{R}(H), \psi \big\rangle \bullet W \right) \tag{2.1}$$

for every test function $\psi \in \mathcal{B}^r$ with $r > |\alpha_0|$. Here $(H \bullet W)$ stands for the stochastic integral of H with respect to W and $\mathcal{R}$ denotes a reconstruction operator for $\mathcal{T} = (A, T, G)$ and (Π, Γ).

Proof. **Step 1:** First we assume that H is an elementary process which can be written as

$$H(\omega, t) = \sum_{n=1}^{N} \sum_{m=1}^{M} \mathbf{1}_{(t_{n-1}, t_n]}(t) \mathbf{1}_{A_{mn}}(\omega) f_{mn}$$

where $0 = t_0 < t_1 < \cdots < t_N = T, A_{mn} \in \mathcal{F}_{t_{n-1}}$ for all $m = 1, \ldots, M$ and are pairwise disjoint, $f_{mn} \in \mathcal{D}^\gamma_{p,q}$ for all m and n. Here $\mathbf{1}_{A_{mn}}$ denotes the indicator function of the set $A_{n,m}$.

Then it holds that for all $t \in [0, T]$,

$$(H \bullet W)_t = \sum_{n=1}^{N} \sum_{m=1}^{M} \mathbf{1}_{A_{mn}} (W_{t \wedge t_n} - W_{t \wedge t_{n-1}}) f_{mn},$$

and therefore $\mathcal{R}\big((H \bullet W)_t\big) = \sum_{n=1}^{N} \sum_{m=1}^{M} \mathbf{1}_{A_{mn}} (W_{t \wedge t_n} - W_{t \wedge t_{n-1}}) \mathcal{R} f_{mn}$ as well as

$$\left\langle \mathcal{R}\big((H \bullet W)_t\big), \psi \right\rangle = \sum_{n=1}^{N} \sum_{m=1}^{M} \mathbf{1}_{A_{mn}} (W_{t \wedge t_n} - W_{t \wedge t_{n-1}}) \langle \mathcal{R} f_{mn}, \psi \rangle.$$

On the other hand, we have

$$\big\langle \mathcal{R}(H)(\omega, t), \psi \big\rangle = \sum_{n=1}^{N} \sum_{m=1}^{M} \mathbf{1}_{(t_{n-1}, t_n]}(t) \mathbf{1}_{A_{mn}}(\omega) \langle \mathcal{R} f_{mn}, \psi \rangle$$

which is an real-valued elementary process. Hence, we indeed have

$$\left(\left\langle \mathcal{R}(H), \psi \right\rangle \bullet W\right)_t = \sum_{n=1}^{N} \sum_{m=1}^{M} \mathbf{1}_{A_{mn}} (W_{t\wedge t_n} - W_{t\wedge t_{n-1}}) \langle \mathcal{R} f_{mn}, \psi \rangle.$$

Obviously now we obtain (2.1) for all elementary processes H. □

Step 2: Now suppose that H is a L^2-stochastically integrable process. There exists a sequence $(H_n)_{n\geqslant 1}$ of elementary processes such that

$$H_n \to H \quad \text{in} \quad L^2(\Omega, \mathbb{P}; \gamma(L^2([0,T], \mathrm{d}t); \mathcal{D}^{\gamma}_{p,q})),$$

where $\gamma(L^2([0,T], \mathrm{d}t); \mathcal{D}^{\gamma}_{p,q})$ denotes the space of γ-radonifying operators from the space $L^2([0,T], \mathrm{d}t)$ into $\mathcal{D}^{\gamma}_{p,q}$

$$(H \bullet W) = \lim (H_n \bullet W) \quad \text{in} \quad L^2(\Omega; C([0,T]; \mathcal{D}^{\gamma}_{p,q})).$$

Now we choose an $\bar{\alpha} < \alpha_0$ with $\lfloor \bar{\alpha} \rfloor = \lfloor \alpha_0 \rfloor$. For $\gamma > 0$ and Theorem **??** (for $\gamma < 0$) we know that $\mathcal{R} : \mathcal{D}^{\gamma}_{p,q} \to \mathcal{B}^{\bar{\alpha}}_{p,q}$ is a continuous linear mapping, which implies that

$$\mathcal{R}\big((H \bullet W)\big) = \lim_{n\to\infty} \mathcal{R}\big((H_n \bullet W)\big)$$

uniformly in $t \in [0,T]$ with respect to the Besov topology on $\mathcal{B}^{\bar{\alpha}}_{p,q}$. Since $\mathcal{B}^{\bar{\alpha}}_{p,q}$ can be embedded in the dual of C^r_0 for $r \geqslant \lfloor \bar{\alpha} \rfloor = \lfloor \alpha_0 \rfloor$, we can derive that

$$\left\langle \mathcal{R}\big((H \bullet W)\big), \psi \right\rangle = \lim_{n\to\infty} \left\langle \mathcal{R}\big((H_n \bullet W)\big), \psi \right\rangle$$

in $L^2(\Omega; C([0,T]; \mathbb{R}))$ for any $\psi \in \mathcal{B}^r \subset C^r_0$.

On the other hand, since the operator $\mathcal{R}$ and the dual pairing $\langle \cdot, \psi \rangle$ are continuous, the ideal property of γ-radonifying operators implies that $\langle \mathcal{R}(H), \psi \rangle$ is L^2-stochastically integrable with respect to W and

$$\mathbb{E}\Big[\|\langle \mathcal{R}(H_n), \psi \rangle - \langle \mathcal{R}(H), \psi \rangle\|^2_{L^2([0,T], \mathrm{d}t)} \Big] \leqslant \|\mathcal{R}\|^2 \|\psi\|^2_{C^r_0} \, \mathbb{E}\Big[\|H_n - H\|^2_{\gamma(L^2([0,T], \mathrm{d}t); \mathcal{D}^{\gamma}_{p,q})} \Big],$$

which implies that $\langle \mathcal{R}(H_n), \psi \rangle$ converges to $\langle \mathcal{R}(H), \psi \rangle in L^2(\Omega \times [0,T], \mathbb{P} \times \mathrm{d}t)$ as n tends to infinity and therefore by It o isometry we obtain that

$$\left(\left\langle \mathcal{R}(H), \psi \right\rangle \bullet W\right) = \lim_{n\to\infty} \left(\left\langle \mathcal{R}(H_n), \psi \right\rangle \bullet W\right)$$

in $L^2(\Omega; C([0,T]; \mathbb{R}))$ for any $\psi \in \mathcal{B}^r \subset C^r_0$. Since we have

$$\left\langle \mathcal{R}\big((H_n \bullet W)\big), \psi \right\rangle = \left(\left\langle \mathcal{R}(H_n), \psi \right\rangle \bullet W\right)$$

for every n by the result from Step 1, we obtain (2.1) for such H.

Step 3: Now suppose that H is locally L^2-stochastically integrable with respect to W. A standard localization argument together with the result from Step 2 then provides that (2.1) holds for all such H.

3. Conclusion

In this paper, we discussed about the stochastic integration theory and proved theorem on stochastic integration on spaces of modelled distributions.

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A STUDY ON PARABOLIC SYSTEMS OF SPDE's OF 2m-th ORDER

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Abstract

We unify and extend the semigroup and the PDE approaches to stochastic maximal regularity of time-dependent semilinear parabolic problems with noise given by a cylindrical Brownian motion. We treat random coefficients that are only progressively measurable in the time variable. For 2m-th order systems with VMO regularity in space, we obtain $(L^p(L^q))$ estimates for all $(p > 2)$ and $(q \geqslant 2)$, leading to optimal space-time regularity results. For second order systems with continuous coefficients in space, we also include a first order linear term, under a stochastic parabolicity condition, and obtain $(L^p(L^p))$ estimates together with optimal space-time regularity. For linear second order equations in divergence form with random coefficients that are merely measurable in both space and time, we obtain estimates in the tent spaces $(T^{p,2}_{\sigma})$ of Coifman–Meyer–Stein. This is done in the deterministic case under no extra assumption, and in the stochastic case under the assumption that the coefficients are divergence free.

Keywords: Stochastic PDEs, Maximal Regularity, VMO Coefficients, Measurable Coefficients, Sobolev Spaces.

1. Introduction

In this paper we develop an $L^p(L^q)$-theory for systems of SPDEs of order $2m$. The case $m = 1$ (where more can be proven) will be considered. A similar setting was considered but under a regularity assumption on the coefficients of the operator A. At the same time it extends some of the results. Finally we mention that the temporal weights allow us to obtain an $L^p(L^q)$-theory for a wider class of initial values than usually considered.

The main novelty in the results below are that the coefficients $a_{\alpha\beta}$ are allowed to be matrix-valued, random, and in the time-variable we do not assume any smoothness of the coefficients. The precise assumptions are stated below.

In this paper we consider the following system of stochastic PDEs on $[0,T]\times\mathbb{R}^d$:

$$\begin{cases} dU(t)+A(t)U(t)dt &= f(t,U(t))dt+\sum\limits_{n\geqslant 1} g_n(t,U(t))dw_n(t), \\ U(0) &= u_0, \end{cases} \tag{1.1}$$

where w_n is a sequence of independent standard Brownian motions. The function $U:[0,T]\times\Omega\to L^p(\mathbb{R}^d;\mathbb{C}^N)$ is the unknown.

The operator A is given by

$$(A(t,\omega)\phi)(x)=(-1)^m\sum_{|\alpha|,|\beta|=m} a_{\alpha\beta}(t,\omega,x)D^\alpha D^\beta\phi(x),\ x\in\mathbb{R}^d, t\in[0,T],\omega\in\Omega.$$

and there will be no need to consider lower order terms as they can be absorbed in the function f.

Note that if $|\phi(y)-\phi(z)|\leqslant\omega(\varepsilon)$ for all $y,z\in\mathbb{R}^d$ satisfying $|x-y|<\varepsilon$, then $\mathrm{osc}_{r,x}\leqslant\omega(r)$.

Assumption 1.1. **(i)** *The functions $a_{\alpha\beta}:[0,T]\times\Omega\times\mathbb{R}^d\to\mathbb{C}^{N\times N}$ are strongly progressively measurable.*

(ii) *There exist $\mu\in(0,1)$ and $K>0$ such that*

$$Re\left(\sum_{|\alpha|=|\beta|=m}\xi^\alpha\xi^\beta(a_{\alpha\beta}(t,\omega,x)\theta,\theta)_{\mathbb{C}^N}\right)\geqslant\mu|\xi|^{2m}|\theta|^2,$$

and $|a_{\alpha\beta}(t,\omega,x)|_{\mathbb{C}^{N\times N}}\leqslant K$ for all $\xi\in\mathbb{R}^d,\theta\in\mathbb{C}^N,x\in\mathbb{R}^d,t\in[0,T]$ and $\omega\in\Omega$.

(iii) *Let $\gamma\in(0,1)$. Assume there exists an $R\in(0,\infty)$ such that for all $|\alpha|,|\beta|=m, r\in(0,R], x\in\mathbb{R}^d, t\in[0,T]and\omega\in\Omega$,*

$$osc_{r,x}(a_{\alpha\beta}(t,\omega,\cdot))\leqslant\gamma.$$

Note that in (t,ω) only measurability is assumed.

For the Ω-independent setting, a slightly less restrictive condition appears. We choose the above formulation assumption in order to make the assumptions easier to state. However, it is possible to extend the results of this section to their setting.

Concerning f and g_n we make the following assumptions:

Assumption 1.2. **(1)** *The function $f:[0,T]\times\Omega\times H^{2m,q}(\mathbb{R}^d;\mathbb{C}^N)\to L^q(\mathbb{R}^d;\mathbb{C}^N)$ is strongly progressively measurable, $f(\cdot,\cdot,0)\in L^p(\Omega;L^p(I,w_\alpha;H^{2m,q}(\mathbb{R}^d;\mathbb{C}^N)))$,*

and there exist L_f and $\widetilde{L}_f$ such that for all $t \in [0,T], \omega \in \Omega$, and $u, v \in H^{2m,q}(\mathbb{R}^d;\mathbb{C}^N)$,

$$\begin{aligned}\|f(t,\omega,u) - f(t,\omega,v)\|_{L^q(\mathbb{R}^d,\mathbb{C}^N)} &\leqslant L_f \|D^{2m}u - D^{2m}v\|_{L^q(\mathbb{R}^d;\mathbb{C}^N)} \\ &\quad + \widetilde{L}_f \|u - v\|_{H^{2m-1,q}(\mathbb{R}^d;\mathbb{C}^N)}.\end{aligned}$$

(2) *The functions $g_n : [0,T] \times \Omega \times H^{2m,q}(\mathbb{R}^d;\mathbb{C}^N) \to H^{m,q}(\mathbb{R}^d;\mathbb{C}^N)$ are strongly progressive measurable, $(g_n)_{n\geqslant 1}(\cdot,\cdot,0) \in L^p(\Omega; L^p(I, w_\alpha; H^{m,q}(\mathbb{R}^d;\ell^2(\mathbb{C}^N))))$ and there exist $L_g, \widetilde{L}_g$ such that for all $t \in [0,T], \omega \in \Omega$, and $u, v \in H^{2m,q}(\mathbb{R}^d;\mathbb{C}^N)$,*

$$\begin{aligned}\|(g_n(t,\omega,u) - g_n(t,\omega,v))_{n\geqslant 1}\|_{H^{m,q}(\mathbb{R}^d;\ell^2(\mathbb{C}^N))} &\leqslant L_g \|D^{2m}u - D^{2m}v\|_{L^q(\mathbb{R}^d;\mathbb{C}^N)} \\ &\quad + \widetilde{L}_g \|u - v\|_{H^{2m-1,q}(\mathbb{R}^d;\mathbb{C}^N)}.\end{aligned}$$

The nonlinearity f can depend on $u, D^1u, \ldots, D^{2m}u$ in a Lipschitz continuous way as long as the dependence on $D^{2m}u$ has a small Lipschitz constant. One could allow lower order terms in A, but one can just put them into the function f. Similarly, g can depend on $u, D^1u, \ldots, D^mu$ in a Lipschitz continuous way as long as the dependence on D^mu has a small Lipschitz constant.

Theorem 1.1. *Let $T \in (0,\infty), p \in (2,\infty), q \in [2,\infty)$ and $\alpha \in [0, \frac{p}{2} - 1)$ (or $q = r = 2$ and $\alpha = 0$). Set $\delta = 1 - \dfrac{1+\alpha}{p}$ and $I = (0,T)$. Assume there exist a constant $\varrho = \varrho(p,q,\alpha,T,m,N,d,R,K,\mu)$ such that Assumptions 1.1 and 1.2 hold with $\gamma,\ L_f,\ L_g \in (0,\ \varrho\ )$. Then for any $u_0 \in L^p(\Omega,\mathcal{F}_0; B^{2m\delta}_{q,p}(\mathbb{R}^d))$, the problem (1.1) has a unique strong solution $U \in L^p_{\mathcal{F}}(\Omega; L^p(I, w_\alpha; H^{2m,q}(\mathbb{R}^d;\mathbb{C}^N)))$. Moreover, there exist constants $C, C_\varepsilon, C_{\varepsilon,\theta}$ depending on $p,q,\alpha,T,m,N,d,R,K,\mu$ and the Lipschitz constants of F and G such that*

$$\begin{aligned}\|U\|_{L^p(\Omega;C(\bar{I};B^{2m\delta}_{q,p}(\mathbb{R}^d)))} &\leqslant CK_{u_0,f,g}, \\ \|U\|_{L^p(\Omega;C([\varepsilon,T];B^{2m(1-\frac{1}{p})}_{q,p}(\mathbb{R}^d)))} &\leqslant C_\varepsilon K_{u_0,f,g},\ \ \varepsilon \in (0,T]. \\ \|U\|_{L^p(\Omega;H^{\theta,p}(I,w_\alpha;H^{2m(1-\theta),q}(\mathbb{R}^d;\mathbb{C}^N)))} &\leqslant C_\theta K_{u_0,f,g},\ \ \theta \in \left[0,\tfrac{1}{2}\right) \\ \|U\|_{L^p(\Omega;C^{\theta-\frac{1+\alpha}{p}}(\bar{I};H^{2m(1-\theta),q}(\mathbb{R}^d;\mathbb{C}^N)))} &\leqslant C_\theta K_{u_0,f,g},\ \ \theta \in \left(\tfrac{1+\alpha}{p},\tfrac{1}{2}\right) \\ \|U\|_{L^p(\Omega;C^{\theta-\frac{1}{p}}([\varepsilon,T];H^{2m(1-\theta),q}(\mathbb{R}^d;\mathbb{C}^N)))} &\leqslant C_{\varepsilon,\theta} K_{u_0,f,g},\ \ \theta \in \left(\tfrac{1}{p},\tfrac{1}{2}\right), \varepsilon \in (0,T].\end{aligned}$$

where

$$\begin{aligned}K_{u_0,f,g} = \|u_0\|_{L^p(\Omega;B^{2m\delta}_{q,p}(\mathbb{R}^d))} &+ \|f(\cdot,\cdot,0)\|_{L^p(\Omega;L^p(I,w_\alpha,L^q(\mathbb{R}^d;\mathbb{C}^N)))} \\ &+ \|g(\cdot,\cdot,0)\|_{L^p(\Omega;L^p(I,w_\alpha,H^{m,q}(\mathbb{R}^d;\ell^2(\mathbb{C}^N))))}.\end{aligned}$$

Furthermore, if U^1, U^2 are the strong solution of (1.1) with initial value $u_0^1, u_0^2 \in L^p(\Omega, \mathcal{F}_0; B^{2m\delta}_{q,p}(\mathbb{R}^d))$ respectively, then each of the above estimates holds with U replaced by $U^1 - U^2$, and $K_{u_0,f,g}$ replaced by $K_{u_0^1-u_0^2,f,g}$ on the right-hand side.

Proof. Let $X_0 = L^q(\mathbb{R}^d;\mathbb{C}^N)$ and $X_1 = W^{2m,q}(\mathbb{R}^d;\mathbb{C}^N)$. Since the coefficients $a_{\alpha\beta}$ are uniformly bounded, Assumption holds (with $B = 0$). Let $X_\theta = H^{2m\theta,q}(\mathbb{R}^d;\mathbb{C}^N)$ for $\theta \in (0,1)$. Note that if $\theta \in [0,1]$ and $2m\theta \in \mathbb{N}$, then $X_\theta = W^{2m\theta,q}(\mathbb{R}^d;\mathbb{C}^N)$. Let $X_{\theta,p} := (X_0, X_1)_{\theta,p} = B^{2m\theta}_{q,p}(\mathbb{R}^d;\mathbb{C}^N)$.

On X_0 consider $A_0 = (1-\Delta)^m I_N with D(A_0) = X_1$, where I_N stands for the $N \times N$ diagonal operator. Then by theorem $0 \in \rho(A_0)$ and the operator A_0 has a bounded H^∞-calculus of angle 0. Furthermore we claim that for any $\varepsilon > 0$, the function $F = f$ satisfies Assumption with constants

$$L_F = L_f + \varepsilon \text{ and } \widetilde{L}_F = C_\varepsilon \widetilde{L}_f.$$

Indeed, it suffices to note that for any $k \in \{1, \dots, 2m-1\}$

$$\|u\|_{H^{k,q}(\mathbb{R}^d;\mathbb{C}^N)} \leqslant \varepsilon \|D^{2m}u\|_{L^q(\mathbb{R}^d;\mathbb{C}^N)} + C_{\varepsilon,q,m}\|u\|_{L^q(\mathbb{R}^d;\mathbb{C}^N)}. \tag{1.2}$$

for all $\varepsilon > 0$.

Since $\gamma(\ell^2, X_{\frac{1}{2}}) = H^{m,q}(\mathbb{R}^d;\ell^2(\mathbb{C}^N))$ isomorphically, in a similar way as above one sees that the function $G = g$ satisfies Assumption with $L_G = L_g + \varepsilon$ and $\widetilde{L}_G = C_\varepsilon \widetilde{L}_g$. □

2. Parabolic Systems of SPDEs of Second Order

In this section we discuss $L^p(L^q)$-theory for systems of second order SPDEs with rough initial values. However, this time we will consider $B \neq 0$. Related problems have been discussed in an $L^p(L^p)$-setting with smooth initial values and in an $L^p(\Omega; L^2((0,T)\times\mathbb{R}^d))$-setting and a Hölder setting, with vanishing initial values.

In this section we consider the following system of stochastic PDEs on $[0,T]\times\mathbb{R}^d$:

$$\begin{cases} dU(t) + A(t)U(t)dt &= f(t,U(t))dt + \sum_{n\geqslant 1}(b_n(t)U(t) + g_n(t,U(t))dw_n(t), \\ U(0) &= u_0, \end{cases} \tag{2.1}$$

where w_n is a sequence of independent standard Brownian motions. The function $U : [0,T]\times\Omega \to L^p(\mathbb{R}^d;\mathbb{C}^N)$ is the unknown.

The operators A and b_n are given by

$$(A(t,\omega)\phi)(x) = -\sum_{i,j=1}^{d} a_{ij}(t,\omega,x)\partial_j\partial_k\phi(x), \quad x \in \mathbb{R}^d, t \in [0,T], \omega \in \Omega.$$

$$(b_n(t,\omega)\phi)(x) = \Big(\sum_{j=1}^{d} \sigma_{jkn}(t,\omega,x)\partial_j\phi_k(x)\Big)_{k=1}^{N}, \quad x \in \mathbb{R}^d, t \in [0,T], \omega \in \Omega.$$

There is no need to consider lower order terms in A or b_n since they can be absorbed in the functions f and g_n, respectively.

We make the following assumptions on the coefficients.

Assumption 2.1. **(1)** *The functions* $a_{ij} : [0,T] \times \Omega \times \mathbb{R}^d \to \mathbb{C}^{N\times N}$ *and* $\sigma_{jkn} : [0,T] \times \Omega \times \mathbb{R}^d \to \mathbb{R}$ *are strongly progressively measurable.*

(2) *There exist* $\mu \in (0,1)$ *and* $K > 0$ *such that* $|a_{ij}(t,\omega,x)|_{\mathbb{C}^{N\times N}} \leqslant K$ *and* $\|(\sigma_{jkn}(t,\omega,\cdot))_{n\geqslant 1}\|_{W^{1,\infty}(\mathbb{R}^d;\ell^2)} \leqslant K$.

$$Re\Big(\sum_{i,j=1}^{d} \xi_i\xi_j((a_{ij}(t,\omega,x) - \Sigma_{ij}(t,\omega,x))\theta,\theta)_{\mathbb{C}^N}\Big) \geqslant \mu|\xi|^2|\theta|^2,$$

for all $\xi \in \mathbb{R}^d, \theta \in \mathbb{C}^N, x \in \mathbb{R}^d, t \in [0,T]$. *Here for each fixed numbers* $i,j \in \{1,\dots,d\}$, $\Sigma_{ij}(t,\omega,x)$ *is the* $N \times N$ *diagonal matrix with diagonal elements* $(\frac{1}{2}\sum_{n\geqslant 1}\sigma_{ikn}(t,\omega,x)\sigma_{jkn}(t,\omega,x))_{k=1}^{N}$.

(3) *Assume there exists an increasing continuous function* $\zeta : [0,\infty) \to [0,\infty)$ *with* $\zeta(0) = 0$ *such that for all* $i,j,x,y \in \mathbb{R}^d, t \in [0,T]$ *and* $\omega \in \Omega$,

$$|a_{i,j}(t,\omega,x) - a_{i,j}(t,\omega,y)| + \sum_{n\geqslant 1}|\sigma_{jkn}(t,\omega,x) - \sigma_{jkn}(t,\omega,y)|^2 \leqslant \zeta(|x-y|).$$

Theorem 2.1. *Let* $T \in (0,\infty), p \in (2,\infty), q \in [2,\infty)$ *and* $\alpha \in [0, \frac{p}{2} - 1)$ *(or* $p = q = 2$ *and* $\alpha = 0$*). Set* $\delta = 1 - \frac{1+\alpha}{p}$ *and* $I = (0,T)$. *Suppose Assumption 2.1 holds, Assumption 1.2 holds with* $m = 1$, *and suppose further that* a_{ij} *and* σ_{jkn} *are x-independent. Then for any* $u_0 \in L^p(\Omega,\mathcal{F}_0; B^{2\delta}_{q,p}(\mathbb{R}^d))$, *the problem (2.1) has a unique strong solution* $U \in L^p_{\mathcal{F}}(\Omega; L^p(I, w_\alpha; H^{2,q}(\mathbb{R}^d;\mathbb{C}^N)))$. *Moreover, there exist constants* $C, C_\varepsilon, C_{\varepsilon,\theta}$ *depending on* p,q,α,T,N,d,K,μ *and the Lipschitz constants of* f *and* g *such*

that

$$\begin{aligned}
\|U\|_{L^p(\Omega;C(\bar{I};B^{2\delta}_{q,p}(\mathbb{R}^d)))} &\leqslant CK_{u_0,f,g},\\
\|U\|_{L^p(\Omega;C([\varepsilon,T];B^{2(1-\frac{1}{p})}_{q,p}(\mathbb{R}^d)))} &\leqslant C_\varepsilon K_{u_0,f,g},\quad \varepsilon\in(0,T].\\
\|U\|_{L^p(\Omega;H^{\theta,p}(I,w_\alpha;H^{2(1-\theta),q}(\mathbb{R}^d;\mathbb{C}^N)))} &\leqslant C_\theta K_{u_0,f,g},\quad \theta\in[0,\tfrac{1}{2})\\
\|U\|_{L^p(\Omega;C^{\theta-\frac{1+\alpha}{p}}(\bar{I};H^{2(1-\theta),q}(\mathbb{R}^d;\mathbb{C}^N)))} &\leqslant C_\theta K_{u_0,f,g},\quad \theta\in(\tfrac{1+\alpha}{p},\tfrac{1}{2})\\
\|U\|_{L^p(\Omega;C^{\theta-\frac{1}{p}}([\varepsilon,T];H^{2(1-\theta),q}(\mathbb{R}^d;\mathbb{C}^N)))} &\leqslant C_{\varepsilon,\theta} K_{u_0,f,g},\quad \theta\in(\tfrac{1}{p},\tfrac{1}{2}),\varepsilon\in(0,T].
\end{aligned}$$

where

$$\begin{aligned}
K_{u_0,f,g} = \|u_0\|_{L^p(\Omega;B^{2\delta}_{q,p}(\mathbb{R}^d))} &+ \|f(\cdot,\cdot,0)\|_{L^p(\Omega;L^p(I,w_\alpha,L^q(\mathbb{R}^d;\mathbb{C}^N)))}\\
&+ \|(g_n(\cdot,\cdot,0))_{n\in\mathbb{N}}\|_{L^p(\Omega;L^p(I,w_\alpha,H^{1,q}(\mathbb{R}^d;\ell_2)))}.
\end{aligned}$$

Furthermore, if U^1, U^2 are the strong solution of (2.1) with initial value $u_0^1, u_0^2 \in L^p(\Omega,\mathcal{F}_0;B^{2\delta}_{q,p}(\mathbb{R}^d))$ respectively, then each of the above estimates holds with U replaced by U^1-U^2, and $K_{u_0,f,g}$ replaced by $K_{u_0^1-u_0^2,f,g}$ on the right-hand side.

Proof. Define the function spaces $X_\theta = H^{2\theta,q}$ and $X_{\theta,p} = (X_0,X_1)_{\theta,p}$ as in Theorem 1.1 with $m = 1$. In order to prove the result we will check the conditions of Theorem again. The proof is the same as for Theorem 1.1, but this time we need to check $(A,B)\in \mathrm{SMR}(p,\alpha,T)$ with a nonzero B. Indeed, let $B(t)u = \sum_{k=1}^{N}\sum_{j=1}^{d} b_{jk}(t)B_{jk}$ with

$$b_{jk}(t)h = \sum_{n=1}^{\infty}\sigma_{jkn}(t)h_n,\quad \text{and}\quad (B_{jk}\phi)_\ell = \delta_{k\ell}\partial_j\phi_k,$$

where $\delta_{k\ell}$ is the Kronecker symbol. Then B_{jk} generates a translation group on $L^q(\mathbb{R}^d;\mathbb{C}^N)$ given by $(e^{tB_{jk}}u(x))_\ell = u_\ell(x+\delta_{k\ell}te_j)$, where e_j denotes the j-th unit vector in $\mathbb{R}^d$. Then Assumption is fulfilled, thanks to the fact that the coefficients are x-independent. Moreover,

$$([B(t),B(t)]u)_\ell = \sum_{i,j=1}^{d}\sum_{n\geqslant 1}\sigma_{i\ell n}(t)\sigma_{j\ell n}(t)\partial_i\partial_j u_\ell.$$

Letting $\widetilde{A}(t) = A(t) + \frac{1}{2}[B(t),B(t)]$, gives that $(A,B)\in\mathrm{SMR}(p,\alpha,T)$ if and only if $(\widetilde{A},0)\in\mathrm{SMR}(p,\alpha,T)$. However, by Assumption 2.1 the operator $\widetilde{A}$ fulfils Assumption 1.1 (with $m=1$), and therefore, as in the proof of Theorem 1.1, we find that $(\widetilde{A},0)\in\mathrm{SMR}(p,\alpha,T)$. □

Lemma 2.1. *(Freezing lemma) Let $T \in (0,\infty)$, $p \in [2,\infty)$ and $\alpha \in [0, \frac{p}{2} - 1)$ (where $\alpha = 0$ if $p = 2$). Set $\delta = 1 - \frac{1+\alpha}{p}$ and $I = (0,T)$. Suppose Assumption 2.1 holds. Let $f \in L^p_{\mathcal{F}}(\Omega \times I, w_\alpha; L^p(\mathbb{R}^d;\mathbb{C}^N))$ and $g \in L^p_{\mathcal{F}}(\Omega \times I, w_\alpha; H^{1,p}(\mathbb{R}^d;\ell^2))$. Assume U is a strong solution of*

$$\begin{cases} dU(t) + A(t)U(t)dt &= f(t)dt + \sum_{n\geqslant 1}(b_n(t)U(t) + g_n(t))dw_n(t), \\ U(0) &= 0, \end{cases} \tag{2.2}$$

There exists an $\varepsilon = \varepsilon(p,q,\alpha,T,N,d,K,\mu)$ such that if U has support in $B_\varepsilon(x_0) = \{x \in \mathbb{R}^d : |x - x_0| < \varepsilon\}$ for some x_0, then for each $\theta \in [0, \frac{1}{2})$ there is a constant C such that

$$\begin{aligned} &\|U\|_{L^p(\Omega;H^{\theta,p}(I,w_\alpha;H^{2(1-\theta),p}(\mathbb{R}^d)))} \\ &\quad\leqslant C\|f\|_{L^p(\Omega;L^p(I,w_\alpha;L^p(\mathbb{R}^d;\mathbb{C}^N)))} + C\|(g_n)_{n\in\mathbb{N}}\|_{L^p(\Omega;L^p(I,w_\alpha;H^{1,p}(\mathbb{R}^d;\ell^2)))}. \end{aligned} \tag{2.3}$$

Proof. Without loss of generality we can assume $x_0 = 0$. In order to simplify the notation let

$$\begin{aligned} Y_{\theta,\eta,t} &= L^p(\Omega; H^{\theta,p}((0,t), w_\alpha; H^{2\eta,p}(\mathbb{R}^d;\mathbb{C}^N))) \\ Y_{\theta,\eta,t}(\ell^2) &= L^p(\Omega; H^{\theta,p}((0,t), w_\alpha; H^{2(1-\eta),p}(\mathbb{R}^d;\ell^2(\mathbb{C}^N)))). \end{aligned}$$

Let $\widetilde{A}(t)$ and $\widetilde{b_n}(t)$ be given by

$$\begin{aligned} \widetilde{A}(t)\phi &= -\sum_{i,j=1}^{d} a_{ij}(t,\omega,0)\partial_j\partial_k\phi(x), \\ (\widetilde{b}_n(t,\omega)\phi)(x) &= \Big(\sum_{j=1}^{d}\sigma_{jkn}(t,\omega,0)\partial_j\phi_k(x)\Big)_{k=1}^{N}. \end{aligned}$$

Furthermore, let

$$\widetilde{f}(\cdot,U) := f(\cdot,U) + (\widetilde{A} - A)U, \text{ and } \widetilde{g}_n(\cdot,U) := g_n(\cdot,U) + (\widetilde{b}_n - b_n)U.$$

Clearly, U satisfies

$$dU(t) + \widetilde{A}(t)U(t)dt = \widetilde{f}(t,U(t)))dt + \sum_{n\geqslant 1}(\widetilde{b}_n(t)U(t) + \widetilde{g}_n(t,U(t)))dw_n(t).$$

Therefore, by Theorem 2.1

$$\|U\|_{Y_{\theta,(1-\theta),T}} \leqslant C_\theta K_{0,\widetilde{f},\widetilde{g}} \leqslant C_\theta K_{0,f,g} + C\|(\widetilde{A} - A)U\|_{Y_{0,0,T}} + C\|((\widetilde{b}_n - b_n)U)_{n\geqslant 1}\|_{Y_{0,1,T}}.$$

To estimate the latter note that by Assumption 2.1 and the support condition on U, we have

$$\|(\widetilde{A}-A)U\|_{Y_{0,0,T}} \leqslant \zeta(\varepsilon)\|U\|_{Y_{0,1,T}}.$$

Similarly, for the b_n-term, by the product rule and Assumption 2.1, we obtain (with K as in Assumption 2.1) that for all $t \in [0,T]$,

$$\begin{aligned}\|((\widetilde{b}_n - b_n)U(t))_{n\geqslant 1}\|_{W^{1,p}(\mathbb{R}^d;\ell^2(\mathbb{C}^N)))} &\leqslant K\|U(t)\|_{W^{1,p}(\mathbb{R}^d;\mathbb{C}^N)} + \zeta(\varepsilon)\|U(t)\|_{W^{2,p}(\mathbb{R}^d;\mathbb{C}^N)}\\ &\leqslant C_\varepsilon\|U(t)\|_{L^p(\mathbb{R}^d;\mathbb{C}^N)} + (\zeta(\varepsilon)+\varepsilon)\|U(t)\|_{W^{2,p}(\mathbb{R}^d;\mathbb{C}^N)},\end{aligned}$$

where in the last step we used. We can conclude that

$$\|U\|_{Y_{\theta,(1-\theta),T}} \leqslant C_\theta K_{0,f,g} + C(2\zeta(\varepsilon)+\varepsilon)\|U\|_{Y_{0,1,T}} + C_\varepsilon\|U\|_{Y_{0,0,T}}. \tag{2.4}$$

Now let $\theta = 0$ and choose $\varepsilon > 0$ such that $C(2\zeta(\varepsilon)+\varepsilon) \leqslant \dfrac{1}{2}$. Then we obtain

$$\|U\|_{Y_{0,1,T}} \leqslant CK_{0,f,g} + C\|U\|_{Y_{0,0,T}}. \tag{2.5}$$

The same estimate holds with T replaced by t.

Since U is a strong solution of (2.2), the properties of A and b_n, and (2.5) give that, for all $t \in [0,T]$,

$$\begin{aligned}\|U(t)\|_{L^p(\Omega;L^p(\mathbb{R}^d;\mathbb{C}^N))} &\leqslant C\|f\|_{Y_{0,0,t}} + C\|g\|_{Y_{0,\frac{1}{2},t}(\ell^2)} + C\|U\|_{Y_{0,1,t}}\\ &\leqslant C\|f\|_{Y_{0,0,t}} + C\|g\|_{Y_{0,\frac{1}{2},t}(\ell^2)} + C\|U\|_{Y_{0,0,t}}.\end{aligned}$$

Therefore, Gronwall's lemma gives that for all $t \in [0,T]$,

$$\|U(t)\|_{L^p(\Omega;L^p(\mathbb{R}^d;\mathbb{C}^N))} \leqslant C\|f\|_{Y_{0,0,T}} + C\|g\|_{Y_{0,\frac{1}{2},T}(\ell^2)}.$$

and thus $\|U\|_{Y_{0,0,T}} \leqslant 2CK_{0,f,g}$. Substituting the latter estimate in (2.4) and (2.5) we find that for all $\theta \in [0, \dfrac{1}{2})$,

$$\|U\|_{Y_{\theta,(1-\theta),T}} \leqslant C_\theta K_{0,f,g}.$$

□

Lemma 2.2. *Let $\theta \in [0,1]$, $p \in (1,\infty)$ and $w \in A_p$. Let $I = (0,T)$ for some $T \in (0,\infty]$. Let $\phi \in L^p(\mathbb{R}^d)$ be such that $\|\phi\|_{L^p(\mathbb{R}^d)} = 1$. We have that*

$$\|f\|_{H^{\theta,p}(I,w;L^p(\mathbb{R}^d;\mathbb{C}^N))} \eqsim \Big(\int_{\mathbb{R}^d} \|(t,x)\mapsto \phi(x-\xi)f(t,x)\|^p_{H^{\theta,p}(I,w;L^p(\mathbb{R}^d;\mathbb{C}^N))} d\xi\Big)^{\frac{1}{p}},$$

for all $f \in H^{\theta,p}(I,w;L^p(\mathbb{R}^d))$.

Proof. Let $\psi \in L^{p'}(\mathbb{R}^d)$ of norm one be such that $\langle \phi, \psi \rangle = 1$. Consider the operators defined by

$$Pf(t,x,\xi) = \phi(x-\xi)f(t,x) \quad t \in I, \quad x,\xi \in \mathbb{R}^d,$$
$$QF(t,x) = \int_{\mathbb{R}^d} \psi(x-\xi)F(t,x,\xi)d\xi \quad t \in I, \quad x \in \mathbb{R}^d,$$

for $f \in L^p(I,w;L^p(\mathbb{R}^d;\mathbb{C}^N))$ and $F \in L^p(\mathbb{R}^d;L^p(I,w;L^p(\mathbb{R}^d;\mathbb{C}^N)))$. Note that $QPf = f$. By complex interpolation, it is thus enough to show that

$$P : W^{\theta,p}(I,w;L^p(\mathbb{R}^d;\mathbb{C}^N)) \to L^p(\mathbb{R}^d;W^{\theta,p}(I,w;L^p(\mathbb{R}^d;\mathbb{C}^N))),$$
$$Q : L^p(\mathbb{R}^d;W^{\theta,p}(I,w;L^p(\mathbb{R}^d;\mathbb{C}^N))) \to W^{\theta,p}(I,w;L^p(\mathbb{R}^d;\mathbb{C}^N)),$$

for $\theta = 0, 1$. Let us consider $\theta = 0$ first. For Q, by Hölder's inequality we have that

$$\|QF\|_{L^p(I,w;L^p(\mathbb{R}^d;\mathbb{C}^N))} = \Big(\int_{I\times\mathbb{R}^d} \Big| \int_{\mathbb{R}^d} \psi(x-\xi)F(t,x,\xi)d\xi \Big|^p w(t)dtdx \Big)^{\frac{1}{p}}$$
$$\leqslant \|F\|_{L^p(\mathbb{R}^d;L^p(I,w;L^p(\mathbb{R}^d;\mathbb{C}^N)))}.$$

and hence $\|Q\| \leqslant 1$ for $\theta = 0$. The above inequalities with f replaced by $\partial_t f$, and F replaced by $\partial_t F$, then gives $\|Q\| \leqslant 1$ for $\theta = 1$. By Fubini it is straightforward to check that P is an isometry for $\theta = 0, 1$. □

3. Conclusion

In this paper, we discussed about Parabolic systems of Semi linear Parabolic Differential Equation of Second order.

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A STUDY ON FUZZY PARA-LINDELOF SPACES

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Abstract

In this paper we introduce the concept of Para-Lindelof spaces in L-Topological spaces by means of locally countable families of L-fuzzy sets. Further some characterizations of fuzzy para-Lindelofness in the weakly induced L-topological spaces are obtained.

Keywords: L-Topological Spaces , Fuzzy Topological Spaces, Fuzzy Lindelof Spaces, Fuzzy Para-Lindelof Spaces.

1. Introduction

As a generalization of a set, the concept of fuzzy set was introduced by Zadeh. Fuzzy topology comes as the generalization of general topology using the concept of a fuzzy set. In 1968 Chang introduced the concept of fuzzy topology and Lowen introduced a more natural definition of fuzzy topology.

Compactness and metrizability are the heart and soul of general topology. In 1944, J. Dieudonne defined para-compactness as a natural generalization of compactness. Later several other covering properties such as meta-compactness, sub para-compactness, sub meta-compactness, para-Lindelofness etc. have naturally evolved from para compactness. The concept of para-Lindelof spaces was introduced by J. Greever in 1968 and further studies were conducted by Burke, Fleissner-Reed.

The concept of para-compactness in fuzzy topology was introduced by Luo. Authors have introduced the concept and studied some properties regarding meta-compactness, sub para-compactness, and sub meta-compactness in L-topological spaces. In this paper we define locally countable families and introduce the concept of para-Lindelof spaces in L-topological spaces.

2. Preliminaries

Definition 2.1. Let (X,τ) be an L-ts. A fuzzy point x_α is quasi coincident with $D \in L^X$ (and write $x_\alpha \prec D$) if $x_\alpha \not\leq D'$. Also D quasi coincides with E at x ($D\ q\ E$ at x) if $D(x) \not\leq E'(x)$. We say D quasi coincident with E and write $D\ q\ E$ if $D\ q\ E$ at x for some $x \in X$. Further $D\neg q\ E$ means D not quasi coincides with E. We say $U \in \tau$ is quasi coincident *nbd* of $x_\alpha (Q-nbd)$ if $x_\alpha \prec U$. The family of all $Q-nbds$ of x_α is denoted by $Q_\tau(x_\alpha)$ or $Q(x_\alpha)$.

Definition 2.2. Let (X,τ) be an L-ts, $A \in L^X$.$\Phi \subset L^X$ is called a Q-cover of A if for every $x \in \mathrm{Supp}(A)$, there exist $U \in \Phi$ such that $x_{A(x)} \prec U$. Φ is a Q-cover of (X,τ) if Φ is a Q cover of $\top$. If $\alpha \in M(L)$, then $C \in \tau$ is an α - $Q-nbd$ of A if $C \in Q(x_\alpha)$ for every $x_\alpha \leqslant A$. Φ is called an α - Q-cover of A, if for each $x_\alpha \leqslant A$, there exists $U \in \Phi$ such that $x_\alpha \prec U$. Φ is called an open α - Q-cover of A if $\Phi \subset \tau$ and Φ is an α - Q-cover of A. $\Phi_0 \subset L^X$ is called a sub α - Q-cover of A if $\Phi_0 \subset \Phi$ and Φ_0 is also an α - Q-cover of A. Φ is called an $\alpha^- - Q$ cover of A, if there exists $\gamma \in \beta^*(\alpha)$ such that Φ is γ - Q-cover of A.

Definition 2.3. Let (X,τ) be an L-ts, $D \in L^X$. D is called N-compact if for every $\alpha \in M(L)$, every open α - Q cover of D has a finite sub family which is an $\alpha^- - Q$ cover of D. (X,τ) is called N-compact if $\top$ is N-compact.

Definition 2.4. Let (X,τ) be an L-ts, $A = \{A_t : t \in T\} \subseteq L^X$, $x_\lambda \in M\left(L^X\right)$. A is called locally countable at x_λ, if there exist $U \in Q(x_\lambda)$ and a countable subset T_0 of T such that $t \in T \backslash T_0 \Rightarrow A_t \neg q U$. And A is called *-locally countable at x_λ if there exist $U \in Q(x_\lambda)$ and a countable subset T_0 of T such that $t \in T_0 \Rightarrow \chi_{(A_t)_{(\perp)}} \neg q U$. A is called locally countable (*-locally countable) for short, if A is locally countable (*-locally countable) at every molecule $x_\lambda \in M\left(L^X\right)$.

The previous notions "locally countable family" is defined for L-ts. They can be also defined for L-subsets:

Definition 2.5. Let (X,τ) be an L-ts. $A \in L^X, A = \{A_t : t \in T\} \subset L^X, x_\lambda \in M\left(L^X\right)$. A is called locally countable in A, if A is locally countable at every molecule $x_\lambda \in M(\downarrow A)$.

Definition 2.6. Let (X,τ) be an L-ts. $A = \{A_t : t \in T\} \subseteq L^X, B \in L^X$. A is called σ-locally countable in B if A is the countable union of sub families which are locally countable in B. A is called σ-locally countable for short, if A is σ-locally countable in T.

Definition 2.7. Let (X,τ) be an L-ts. Then by $[\tau]$ we denote the family of support sets of all crisp subsets in τ. $(X,[\tau])$ is a topology and it is the background space. (X,τ) is weakly induced if $U\in\tau$ is a lower semi continuous function from the background space $(X,[\tau])$ to L.

Proposition 2.1. *Let (X,τ) be an L-ts. Then the following conditions are equivalent.*

(i) *(X,τ) is weakly induced.*

(ii) *(X,τ) is weakly γ-induced for every $\gamma\in \mathrm{pr}(L)$.*

(iii) *(X,τ) is weakly α-induced for every $\alpha\in L$.*

Definition 2.8. A collection A refines a collection $B(A<B)$ if for every $A\in A$, there exists $B\in B$ such that $A\leqslant B$.

Definition 2.9. Let (X,τ) be an L-ts. $A=\{A_t : t\in T\}\subseteq L^X$ is a closure preserving collection if for every subfamily A_0 of A, $cl\left[\vee A_0\right]=\vee\left[cl\,A_0\right]$.

Proposition 2.2. *Let (X,τ) be an L-ts. $A\subset L^X$ is closure preserving. Then for every sub family $A_0=\{A_t : t\in T\}\subset A$, $\vee_{t\in T}\ cl\,A_t$ is a closed subset.*

3. Para-Lindelof Spaces

Definition 3.1. Let (X,τ) be an L-ts, $A\in L^X,\ \alpha\in M(L)$. A is called α-Lindelof if every open α - Q-cover of A has a countable subfamily which is also an $\alpha-Q$-cover of A. A is Lindelof if A is α - Lindelof for every $\alpha\in M(L)$. And (X,τ) is Lindelof if $\top$ is Lindelof.

Definition 3.2. Let (X,τ) be an L-ts, $\alpha\in M(L)$. (X,τ) is called σ-para-Lindelof if for every open α - Q-cover Φ of X, there exist an open refinement Ψ of Φ which is σ-locally countable in X and also an α - Q-cover of X.

Proposition 3.1. *Let (X,τ) be an L-ts, $A\in L^X,\alpha\in M(L)$. Then*

(i) *A is α^*-para-Lindelof $\Rightarrow$ A is α-para-Lindelof.*

(ii) *A is *-para-Lindelof $\Rightarrow$ A is para-Lindelof.*

*Para-Lindelof and *-Para-Lindelof are hereditary with respect to closed subsets.*

Theorem 3.1. *Let (X,τ) be an L-ts, $A\in L^X,\alpha\in M(L),B\in\tau'$. Then*

(i) *A is α-para-Lindelof $\Rightarrow$ $A\wedge B$ is α-para-Lindelof.*

(ii) *A is para-Lindelof $\Rightarrow$ $A\wedge B$ is para-Lindelof.*

Proof. We need to prove only (i). Suppose that U is an open $\alpha - Q-$ cover of $A \wedge B$. Take $V = U \cup \{B'\}$. Now clearly V is an open $\alpha - Q-$ cover of A. Since A is α-para-Lindelof, V has an open refinement W such that W is locally countable in A and is also an $\alpha - Q$-cover of A. Take $W_0 = \{W \in W : \exists\ U \in U, W \leqslant U\}$. Now we show that W_0 is the required locally countable refinement of V which is also an $\alpha - Q$-cover of $A \wedge B$. Clearly W_0 is a locally countable refinement. Let $x_\alpha \leqslant A \wedge B \leqslant A$, since W is an α - Q-cover of A, there exist $W \in W$ such that $x_\alpha \prec W$. Since $x_\alpha \leqslant B, B \not\leqslant B'$, i.e. $W \not\leqslant B'$. Since W is a refinement of $V = U \cup \{B'\}$, $\exists U \in U$ such that $W \leqslant U$. Thus $W \in W_0$ and hence $x_\alpha \prec W \in W_0$. □

A similar theorem holds for α^*-para-Lindelof and *-para-Lindelof spaces also.

Theorem 3.2. *Let (X, τ) be a weakly induced L-ts. Then the following conditions are equivalent*

(i) (X, τ) *is para-Lindelof;*

(ii) *There exist $\alpha \in M(L)$ such that (X, τ) is α-para-Lindelof;*

(iii) $(X, [\tau])$ *is para-Lindelof.*

Proof. (i) $\Rightarrow$ (ii) is obvious.

(ii) $\Rightarrow$ (iii): Let $U \subset [\tau]$ be an open cover of X. Now $U^* = \{\chi_U : U \in U\}$ is an open α - Q-cover of $\top$ and it has a locally countable refinement V which is also an α - Q-cover of T.

Let $W = \left\{V_{(\alpha')} : V \in V\right\}$. Clearly W is both a refinement of U and a cover of X. Since (X, τ) is weakly induced, we have $W \subset [\tau]$. Now we want to prove that W is locally countable. Let $x \in X$. Since (X, τ) is α-para-Lindelof, there exist $B \in Q(x_\alpha)$ such that B only quasi coincides with a countable number of members $V_0, V_1, V_2, \cdots$ of V. Let $O = B_{(\perp)}$. By the weakly induced property of $(X, \tau), O \in [\tau]$. For every $V \in V$, if $O \cap V_{(\alpha')} \neq \phi$, then there exist an ordinary point $y \in O \cap V_{(\alpha')}$, and hence $B(y) \not\leqslant \perp, V(y) \not\leqslant \alpha'$. Therefore $V(y)' < \alpha$ and it follows that $B(y) \not\leqslant V(y)'$ and thus $B\ q\ V$. So $V \in \{V_0, V_1, V_2, \cdots\}$ and O intersects only a countable number of members $V_{0(\alpha')}, V_{1(\alpha')}, V_{2(\alpha')}, \cdots$ of W. Hence $(X, [\tau])$ is para-Lindelof.

(iii) $\Rightarrow$ (i): Suppose that $\alpha \in M(L)$ and $U \subset \tau$ be an open α - Q-cover of $\top$. Since (X, τ) is weakly induced $U^* = \left\{U_{(\alpha')} : U \in U\right\}$ is an open cover of $(X, [\tau])$. Since $(X, [\tau])$ is para-Lindelof, there exist a refinement V of U^* which is also a locally countable cover of X. For every $V \in V$, let $U_V \in U$ such that $V \subset U_{V(\alpha')}$. Let $W = \{\chi_V \wedge U_V : V \in V\}$. Now clearly W is both a refinement of U and an α - Q-cover of $\top$. Now we will prove that W is locally countable. Let $x_\alpha \in M\left(L^X\right)$. Then since V is

locally countable, there exist a neighbourhood B of x such that B intersects with V_i for countably many $V_i \in V$. Now we have $\chi_B \in Q(x_\alpha)$. We will show that $\chi_B \, q \, \chi_{V_i} \wedge U_{V_i}$ for at most countably many i. For if possible $\chi_B \, q \, \chi_V \wedge U_V$ for uncountably many $V \in V$. Then $\chi_B \, q \, \chi_V$ or $\chi_B \, q \, U_V$ for uncountably many $V \in V$. In both cases B intersects with V for uncountably many $V \in V$, which is a contradiction and hence W is locally countable. Therefore (X, τ) is α-para-Lindelof. This completes the proof. □

Theorem 3.3. *Let (X, τ) be a weakly induced L-ts. Then the following conditions are equivalent*

(i) *(X, τ) is *-para-Lindelof;*

(ii) *There exist $\alpha \in M(L)$ such that (X, τ) is α^*-para-Lindelof;*

(iii) *$(X, [\tau])$ is para-Lindelof.*

Proof. (i) $\Rightarrow$ (ii) is obvious.

(ii) $\Rightarrow$ (iii): Let $U \subset [\tau]$ be an open cover of X. Now $U^* = \{\chi_U : U \in U\}$ is an open α - Q-cover of $\top$ and it has a locally countable refinement V which is also an α - Q-cover of $\top$.

Take $W = \left\{V_{(\alpha')} : V \in V\right\}$ then W is both a refinement of U and a cover of X. Since (X, τ) is weakly induced, we have $W \subset [\tau]$. Now we want to prove that W is locally countable. Let $x \in X$. Since (X, τ) is α^*-para-Lindelof, there exist $B \in Q(x_\alpha)$ such that $\chi_{B(\perp)}$ only quasi coincides with a countable number of members $V_0, V_1, V_2, \cdots$ of V. Then $x \in B_{(\perp)}$. By the weakly induced property of (X, τ), $B_{[\perp]} \in [\tau]$, so $B_{(\perp)}$ is a neighbourhood of x. For every $V \in V$, if $B_{(\perp)} \cap V_{(\alpha')} \neq \phi$, then there exist an ordinary point $y \in B_{(\perp)} \cap V_{(\alpha')}, V(y) \not\leq \alpha', V(y) > \perp, V(y)' < \perp$. So $\chi_{B(\perp)}(y) = \top \not\leq V(y)'$, $\chi_{B(\perp)} q \; V, V \in \{V_0, V_1, V_2, \cdots\}$. Therefore the neighbourhood $B_{(\perp)}$ is of x intersects a countable number of members $V_{0(\alpha')}, V_{1(\alpha')}, V_{2(\alpha')}, \cdots$ of W, thus W is locally countable in X. Hence $(X, [\tau])$ is para-Lindelof.

(iii) $\Rightarrow$ (i): Suppose that $\alpha \in M(L)$ and $U \subset \tau$ be an open α - Q-cover of T. Since (X, τ) is weakly induced $U^* = \left\{U_{(\alpha')} : U \in U\right\}$ is an open cover of $(X, [\tau])$. Since $(X, [\tau])$ is para-Lindelof, there exist a locally countable and open refinement V of U^* which is also a cover of X. For every $V \in V$, let $U_V \in U$ such that $V \subset U_{V(\alpha')}$. Let $W = \{\chi_V \wedge U_V : V \in V\}$. Then $W \subset \tau$ is clearly a refinement of U and an α - Q-cover of $\top$. Now we will prove that W is *-locally countable. Let $x_\alpha \in M\left(L^X\right)$ and $B \in Q(x_\alpha)$. If possible let $\chi\left(\chi_V \wedge U_V\right)_{(\perp)} qB$ for uncountably many $V \in V$. That is $\chi_V \wedge \chi_{UV(\perp)} q \, B$ for uncountably many $V \in V$. And hence $\chi_V q \, B$ or $\chi_{UV(\perp)} q \, B$ for uncountably many $V \in V$. In both cases V intersects with the neighbourhood of x for uncountably many $V \in V$

which is a contradiction that V is locally countable. Hence W is *-locally countable and this completes the proof. □

Theorem 3.4. *Let (X, τ) be an L-ts. Then the following are equivalent*

(i) *(X, τ) is para-Lindelof;*

(ii) *For every open α - Q-cover A of (X, τ), there is a locally countable refinement B such that if $x_\alpha \in M\left(L^X\right)$ then $x_\alpha \in \text{int}\left(st\left(x_\alpha, B\right)\right)$.*

Proof. (i) $\Rightarrow$ (ii) is obvious.
(ii) $\Rightarrow$ (i): Suppose $A = \{A_t : t \in T\}$ is an open $\alpha - Q$-cover of T. Let $B = \{B_t : t \in T\}$ be a locally countable refinement as given in (ii). Let C be an open $\alpha - Q$-cover of T such that every element of C intersects at most countably many elements of B. Then for every $x_\alpha \in M\left(L^X\right)$, there is a locally countable refinement D of C such that $x_\alpha \in \text{int}\left(st\left(x_\alpha, D\right)\right)$.

For each $B \in B$, take $A_B \in A$ such that $B \leqslant A_B$ and let $G_B = \text{int}(st(B, D)) \wedge A_B$. Then clearly $G = \{G_B : B \in B\}$ is an α - Q-cover of $\top$ and hence is an open refinement of A. To show G is locally countable, let $x_\alpha \in M\left(L^X\right)$ and $W \in Q\left(x_\alpha\right)$ such that W intersects only countably many elements of D. Now since each $D \in D$ intersects only countably many elements of B, it follows that W intersects only countably many elements of $\{st(B, D) : B \in B\}$. Hence G is locally countable and the theorem is proved. □

4. Conclusion

In this paper we have defined the para Lindelof spaces and L-Topological spaces by means of locally countable families of L-fuzzy sets. We have proved characterizations of fuzzy para Lindelofness.

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A STUDY ON THE CROSS PRODUCT OF FUZZY NUMBERS

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Abstract

In fuzzy arithmetic, the multiplication operation based on Zadeh's extension principle owns several unnatural properties both from theoretical and practical points of view. To overcome some of these shortcomings, a new operation called cross product has been introduced recently. We show the main properties of the cross product. We also present a comparative study of the traditional multiplication and the cross product in geological applications, especially for estimating of resources of solid mineral deposits.

Keywords: Fuzzy Number, Cross Product, Solid Mineral Deposit Estimation.

1. Introduction

In this section we study the theoretical properties of the cross product of fuzzy numbers. Let $R_F^* = \{u \in R_F : u \text{ is positive or negative}\}$. Firstly we begin with a theorem which was obtained by using the stacking theorem.

Theorem 1.1. *If u and v are positive fuzzy numbers then $w = u \odot v$ defined by $[w]^r = [\underline{w}^r, \overline{w}^r]$, where $\underline{w}^r = \underline{u}^r\underline{v}^1 + \underline{u}^1\underline{v}^r - \underline{u}^1\underline{v}^1$ and $\overline{w}^r = \overline{u}^r\overline{u}^1 + \overline{u}^1\overline{v}^r - \overline{u}^1\overline{v}^1$, for every $r \in [0,1]$, is a positive fuzzy number.*

Corollary 1.1. Let u and v be two fuzzy numbers.

(i) If u is positive and v is negative then $u \odot v = -(u \odot (-v))$ is a negative fuzzy number;

(ii) If u is negative and v is positive then $u \odot v = -((-u) \odot v)$ is a negative fuzzy number;

(iii) If u and v are negative then $u \odot v = (-u) \odot (-v)$ is a positive fuzzy number.

Definition 1.1. *The binary operation $\odot$ on R_F^* introduced by Theorem 1.1 and Corollary 1.1 is called cross product of fuzzy numbers.*

Remark 1.1. 1. The cross product is defined for any fuzzy numbers in $R_F^{\wedge} = \{u \in R_F^*;$ there exists an unique $x_0 \in \mathbf{R}$ such that $u(x_0) = 1\}$, so implicitly for any triangular fuzzy number. In fact, the cross product is defined for any fuzzy number.

2. The below formulas of calculus can be easily proved $(r \in [0,1])$:
$\underline{(u \odot v)}^r = \overline{u}^r \underline{v}^1 + \overline{u}^1 \underline{v}^r - \overline{u}^1 \underline{v}^1$,
$\overline{(u \odot v)}^r = \underline{u}^r \overline{v}^1 + \underline{u}^1 \overline{v}^r - \underline{u}^1 \overline{v}^1$
If u is positive and v is negative, $\underline{(u \odot v)}^r = \underline{u}^r \overline{v}^1 + \underline{u}^1 \overline{v}^r - \underline{u}^1 \overline{v}^1$, $\overline{(u \odot v)}^r = \overline{u}^r \underline{v}^1 + \overline{u}^1 \underline{v}^r - \overline{u}^1 \underline{v}^1$
If u is negative and v is positive. In the last possibility, if u and v are negative then $\underline{(u \odot v)}^r = \overline{u}^r \overline{v}^1 + \overline{u}^1 \overline{v}^r - \overline{u}^1 \overline{v}^1$, $\overline{(u \odot v)}^r = \underline{u}^r \underline{v}^1 + \underline{u}^1 \underline{v}^r - \underline{u}^1 \underline{v}^1$.

3. The cross product extends the scalar multiplication of fuzzy numbers. Indeed, if one of operands is the real number k identified with its characteristic function then $\underline{k}^r = \overline{k}^r = k, \forall r \in [0,1]$ and following the above formulas of calculus we get the result.

The main algebraic properties of the cross product are the following.

Theorem 1.2. If $u, v, w \in R_F^*$ then
(i) $(-u) \odot v = u \odot (-v) = -(u \odot v)$;
(ii) $u \odot v = v \odot u$;
(iii) $(u \odot v) \odot w = u \odot (v \odot w)$;
(iv) If u and v have the same sign then $(u \oplus v) \odot w = (u \odot w) \oplus (v \odot w)$;
(v) $(u \odot v)^{\odot n} = u^{\odot n} \odot v^{\odot n}, \forall n \in N^*$, where $a^{\odot n} = \underbrace{a \odot \ldots \odot a}_{n \text{ times}}$ for any $a \in R_F^*$.

Remark 1.2. 1) If u is positive and v negative (or u is negative and v positive) then the property of distributivity in (iv) is not verified even if u and v are real numbers.

2) The above properties (i)-(iii) hold for the usual product "$\cdot$" based on the extension principle. The property (iv) holds in a weaker form: If u and v are on the same side of 0 then for any $w, w \prec 0$ or $0 \prec w$ we have $(u \oplus v) \cdot w = (u \cdot w) \oplus (v \cdot w)$

The so-called $L-R$ fuzzy numbers are considered important in fuzzy arithmetic. These and their particular cases triangular and trapezoidal fuzzy numbers are used almost exclusively in applications.

Definition 1.2. Let $L, R : [0, +\infty) \to [0, 1]$ be two continuous, decreasing functions fulfilling $L(0) = R(0) = 1, L(1) = R(1) = 0$, invertible on $[0, 1]$. Moreover, let a^1 be any real number and suppose $\underline{a}, \bar{a}$ be positive numbers. The fuzzy set $u : \mathbf{R} \to [0, 1]$ is an $L-R$ fuzzy number if $u(t) = \begin{cases} L\left(\dfrac{a^1 - t}{\underline{a}}\right), & \text{for } t \leqslant a^1 \\ R\left(\dfrac{t - a^1}{\bar{a}}\right), & \text{for } t > a^1 \end{cases}$.

Symbolically, we write $u = \left(a^1, \underline{a}, \bar{a}\right)_{L,R}$, where a^1 is called the mean value of u, $\underline{a}, \bar{a}$ are called the left and the right spread. If u is an $L-R$ fuzzy number then

$$[u]^r = \left[a^1 - L^{-1}(r)\underline{a}, a^1 + R^{-1}(r)\bar{a}\right]$$

Theorem 1.3. *If u and v are strict positive $L-R$ fuzzy numbers then $u \odot v$ is a strict positive $L-R$ fuzzy number.*

Since we are interested mainly in the applications of the cross product we may restrict our attention to positive fuzzy numbers, however in other cases some similar properties can be obtained.
The cross product verifies the following metric property.

Theorem 1.4. *If u, v have the same sign and $w \in R_F^*$ then*

$$D(w \odot u, w \odot v) \leqslant K_w D(u, v), \text{ where } K_w = \max\left\{\left|\overline{w}^1\right|, \left|\underline{w}^1\right|\right\} + \overline{w}^0 - \underline{w}^0.$$

Definition 1.3. *Let u be a fuzzy number. The crisp number $\Delta_L^r(u) = \underline{u}^1 - \underline{u}^r$ is called $r-$ error to left of u and the crisp number $\Delta_R^r(u) = \bar{u}^r - u^{-1}$ is called r -error to right of u, where $r \in [0, 1]$. The sum $\Delta^r(u) = \Delta_L^r(u) + \Delta_R^r(u)$ is called r -error of u.*

If u expresses the fuzzy concept A then $\Delta_L^r(u)$ and $\Delta_R^r(u)$ can be interpreted as the values of tolerance of level r from the concept A to left and to right, respectively. For example, if the triangular fuzzy number $u = (5, 7, 9)$ expresses "early morning" then $\Delta_L^{\frac{1}{2}}(u) = 1$ (one hour) is the tolerance of level $\dfrac{1}{2}$ of u towards night from the concept

of "early morning" and $\Delta_R^{\frac{1}{4}}(u) = 0.5$ (30 minutes) is the tolerance of level $\frac{1}{4}$ of u towards moon from the concept of "early morning".

A new argument in the use of addition of fuzzy numbers as extension (by Zadeh's principle) of real addition is the validity of the formula $\Delta^r(u \oplus v) = \Delta^r(u) + \Delta^r(v)$ which is consistent to the classical error theory. It is an immediate consequence of the obvious formulas $\Delta_L^r(u \oplus v) = \Delta_L^r(u) + \Delta_L^r(v)$ and $\Delta_R^r(u \oplus v) = \Delta_R^r(u) + \Delta_R^r(v)$

Now, let us study the relative error of the cross product.

Definition 1.4. *Let u be a fuzzy number such that $\underline{u}^1 \neq 0$ and $\bar{u}^1 \neq 0$. The crisp numbers $\delta_L^r(u) = \dfrac{\Delta_L^r(u)}{|\underline{u}^1|}$ and $\delta_R^r(u) = \dfrac{\Delta_R^r(u)}{|\overline{u}^1|}$ are called relative r -errors of u to left and to right. The quantity $\delta^r(u) = \delta_L^r(u) + \delta_R^r(u)$ is called relative $r-$ error of u.*

Theorem 1.5. *If u and v are strict positive or strict negative fuzzy numbers then*

$$\delta^r(u \odot v) = \delta^r(u) + \delta^r(v)$$

2. Applications of the Cross Product in Geology

Recently, fuzzy arithmetic has found several applications in geology. In the above cited work the usual (Zadeh's extension principle based) product is used for estimation of resources of solid mineral deposits. In this section we propose an alternative study of the same problem, by using the cross product. The reasons of the possible usefulness of the cross product are the following.

Firstly, in this case the shape of the result of the product is conserved, i.e. the product of triangular numbers is triangular and the product of trapezoidal numbers is trapezoidal. Secondly, the 1-level sets are better taken into account by the use of cross product. Also, the consistency of the cross product with the classical error theory motivates this study.

As we perform resource estimation on several bauxite deposits in Hungary. In the same way as with the traditional methods, the tonnage of the resources is obtained by the product of the deposit area, the average thickness and the average bulk-density of the studied ore or mineral commodity. Large deposits can be split into blocks, preferably along natural boundaries, such as tectonic lines. We present the results obtained by the usual multiplication and the results obtained by using the cross product.

Furthermore, if we defuzzify the two results obtained by the two different producttype operations we conclude that the results are different. Also, we observe that after defuzzification (by centroid method) the result of the cross product in the study of the Óbarok deposit is smaller than that of the usual product i.e. the cross product leads to a more pessimistic result than the usual multiplication in this case. So, the risks of an investment at this site can be more realistically evaluated.

3. Conclusion

In this paper, we discussed about the concept of Cross product of fuzzy numbers and the applications of cross product in Geology.

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STRUCTURAL AND OPTICAL PROPERTIES OF A GLASERITE TYPE ORTHOVANADATE $CsK_2LA[VO_4]_2$ DOPED WITH THE TRIVALENT RARE EARTH ION Sm^{3+}

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Abstract

The conventional high temperature solid-state reaction method was used to synthesise a novel phosphor type of Glaserite type Orthovanadate $CsK_2LA[VO_4]_2$ doped with the trivalent rare-earth Sm^{3+} ion, and their structural characterization and optical properties were investigated. The pure-phase glaserite structure in the synthesized compound was verified by powder X-ray diffraction (XRD) studies. The lanthanide (Ln^{3+}) substituted luminescence behaviours and the self-activated luminescence behaviours of the host lattice have been studied in detail by (PL) Photoluminescence. Interestingly, the synthesized phosphor material not only shows red emission due to the dopant Sm^{3+}, it also shows a green emission due to the host lattice. The broad-band green emission is originated from the $[VO4]^{3-}$ emission. The UV-DRS is measured to find the highest percentage of reflectance and also the band gap is calculated. The CIE chromaticity color coordinate values were calculated and it is well matches with the NTSC standards.

Keywords: Glaserite, Orthovanadate, Chromaticity Coordinates, Self-Activated.

1. Introduction

Phosphor-converted white light-emitting diodes (PC-wLEDs) have become commercially available in recent years, and much attention has been started focusing on their development due to their widespread applications in solid lighting [1,2]. PC-wLEDs have a high potential for replacing traditional lighting such as incandescent

and fluorescent lamps, with advantages such as long lifetime, low energy consumption, and environmental friendliness [3,4]. Rare earth doped vanadate phosphors have awhile back received a lot of attention because of their long-wavelength excitation properties, which allow them to be used in LEDs, fluorescent lamps, and flat panel displays (FPD) [5,6]. Vanadate compounds have broad and intense charge transfer (CT) absorption bands in the near UV, allowing them to capture emissions at a wide range of wavelengths.

A phosphor material's first and most crucial requirement is that it have a high absorption in the near-UV to blue spectral region. Because of their poor absorption in the near-UV to blue spectral region, conventional phosphors used in fluorescent lighting are not suitable for solidstate lighting (SSL). As a result, there has been a growing interest in developing new and novel phosphor families with high absorption in the near-UV to blue region. Due to their high absorptivity, rare-earth doped vanadate based materials have emerged as the best candidate phosphors for optical wavelength conversion in the development of w-LEDs for general illumination applications.

Due to their self-activated emitting properties of the $[VO_4]^{3-}$ group, sensitization from $[VO_4]^{3-}$ to rare earth ions, long wavelength excitation, and excellent chemical stabilities, vanadatebased phosphors have recently received a lot of attention [9]. The vanadate group, namely $[VO_4]^{3-}$, has broad and intense charge transfer (CT) absorption bands in the UV region, and some of them can produce intense broadband CT emission spectra from 400 to more than 700 nm related to the local structure [10-12]. When exposed to Ultraviolet light, these vanadatesor rare earth ion-doped materials could indeed convert the ultraviolet emission into white light [13]. Similar to other Scheelite-type compounds, the $[VO_4]^{3-}$ complex ion group, in which the central V metal ion is coordinated by four O_2^- ions in tetrahedral symmetry, has been identified as an efficient luminescent centre. Other vanadate complexes with the characteristic luminescence of $[VO_4]^{3-}$ ions include $Mg_3(VO_4)_2$, $LiZnVO_4$, and $NaCaVO_4$. When trivalent rare-earth ions like Eu^{3+}, Sm^{3+}, and Dy^{3+} are incorporated into a vanadate host, bright luminescence due to the dopant ions is observed due to efficient energy transfer processes from the vanadate ions. Lanthanide ions, such as Eu^{3+}, Sm^{3+}, Dy^{3+}, and Tb^{3+}, among others, are well known as luminescent activators in a wide range of host lattices due to their abundant energy levels at a wide range of wavelengths. Among these ions, Sm^{3+} is a key activator ion in the production of visible emission [14]. Special consideration is always given to Sm^{3+}, whose ionic radius is very similar to that of Eu3+, resulting in red emission. Sm^{3+} activated luminescent materials are currently receiving a lot of attention [15]. They exhibit bright emissions in the orange and red ranges due to transitions from the excited state $^4G_{5/2}$ to the ground state $^6H_{5/2}$ and the other state ^6H_J ($J = 7/2, 9/2$, and $11/2$), which can be used in high density optical storage, temperature sensors, undersea communication, various fluorescent devices, colour display, and visible solid state lasers [16].

This research is searching for a new vanadate host for (RE - trivalent rare earth ion) Sm^{3+} ions. We chose $CsK_2LA[VO_4]_2$ because, to the best of our knowledge, neither undoped nor doped glaserite orthovanadate's luminescence properties have been reported very much up to this point. $CsK_2LA[VO_4]_2$ is an orthovanadate with a glaserite (aphthitalite)-like structure. This compound's prototype is the sulphate $K_3Na(SO_4)_2$. The formula $XY_2M(TO_4)_2$ describes it, where X, Y, M, and T are cationic sites of 12, 10, 6, and 4 coordination, respectively. The formula $XY_2M(TO_4)_2$ describes it, where X, Y, M, and T are cationic sites of 12, 10, 6, and 4 coordination, respectively. Glaserite-like orthovanadates, such as $K_3Y(VO_4)_2$ doped with Dyand $K_3Y(VO_4)_2$ codoped with Dy and Bi, [17] $A_3RE(VO_4)_2$ (A = alkali metal, RE = Sc, Y, LaLu), [18] and $Na_3Ln(VO_4)_2$ (Ln = La, Nd, and Er) [19], have received attention due to luminescence material applications. We investigated the luminescence behaviour of $CsK_2La_{1-x}[VO_4]_2$ with different ${}_xSm^{3+}$ concentrations of $x = 0, 0.025, 0.05, 0.075$, and 0.1 in this study. This investigation is being conducted to investigate emissions caused by the host $[VO_4]^{3-}$ and the dopant Sm^{3+}. This could be useful in the development of new luminescent materials in orthovanadate. Although the luminescence properties of various vanadate phosphors have been reported, there have been no reports on the luminescence properties of $CsK_2La(VO_4)_2$:Sm^{3+}.

2. Experimental Section

2.1. Materials

As precursors, high-purity (99.99 percent) oxides and carbonates were used, including reagent grade Ammonium meta vanadate - NH_4VO_3, Cesium carbonate - Cs_2CO_3, potassium carbonate - K_2CO_3, and rare earth oxides such as Lanthanum oxide - La_2O_3 and the rare earth activator samarium oxide - Sm_2O_3.

2.2. Synthesis

The Sm^{3+} doped glaserite-type orthovanadate $CsK_2LA[VO_4]_2$ was synthesised using a standard high-temperature solid-state reaction. The concentrations of Sm^{3+} doping range from 0 to 0.1 percent in 0.025 increments. The corresponding precursors are weighed based on their stoichiometric ratio. Each starting tangible mixture is ground for an hour before being loaded into a high purity silica crucible and sintered in a furnace. First, the stoichiometric mixture was gradually heated to 350°C over a 7-hour period and held there for 5 hours. The powder was remixed and then heated for 5 hours at 700°C. The sample was then thoroughly mixed and heated again at a temperature of 780-800°C for 10 hours and cooled down to room temperature to obtain a white powder. This synthesis was carried out in a standard environment. The synthesized powder is characterized by its structural and optical properties.

2.3. Measurements and Instrumentations

Powder X-ray diffraction (XRD) analysis collected on an X'Pert PRO PANalytical diffractometer (45kV, 30mA) and with Bragg-Brentano geometry using Cu-Kα radiation ($\lambda = 1.5406$Å) was used to verify phase purity. The Photoluminescence - PL and PLE

spectra were recorded on a Fluoromax-4 Spectroflurometer [Horiba Scientific]. The UV-Diffuse Reflectance Spectra is measured with the JASCO model V-670 in the range of 200 – 800nm. The CIE values were obtained using the MATLAB software and a CIE calculator. The emission intensity is employed here to find the CIE x and y coordinates values, which are needed to prove that the obtained values are very close to the NTSC standard cards. All the measurements were taken at room temperature.

3. Results and Discussion

3.1. Crystal Phase Formation

The powder XRD was used to determine the crystallinity of the phosphor materials. Figure 1 depicts the XRD patterns of ${}_x\mathrm{Sm}^{3+}$ doped $\mathrm{CsK_2La}_{(1-x)}[\mathrm{VO_4}]_2$ at various concentrations ($x = 0$ to 0.1 in 0.025 steps). The reflections from all concentrations were identical, and all of the peaks shown in the graphs are well indexed to the patterns, which are in good agreement with the powder diffraction file 2 (PDF 2) card No. 89-6536 ($\mathrm{CsK_2La[VO_4]_2}$) in the International Centre for Diffraction Data (ICDD). The XRD peaks are well indexed to the monoclinic structure with a space group of P21/m. The unit cell parameters are $a = 10.094(6)$Å, $b = 5.991(3)$Å, $c = 7.858(6)$Å and $V = 475.19$Å^3 and the interfacial angles $\alpha = \gamma = 90^\circ$ and $\beta = 90.76^\circ(8)$. There is no other phase is detected, indicating that the obtained samples are single phase and Sm^{3+} ions have been successfully incorporated into the host lattice by replacing La^{3+} ions due to their similar ionic radii and charge. But the peaks exhibit a little right shift for the samples with the Sm doping level enhanced because of the crystal lattice distortion.

Due to the lanthanide contraction, the radius of Sm^{3+} (0.96) is smaller than that of La^{3+} (1.032), so the cell volume of $\mathrm{CsK_2La(VO_4)_2}$:Sm^{3+} decreases with more Sm^{3+} replacing La^{3+}, resulting in the XRD peaks shifting to a higher degree.

Even when La^{3+} in the host lattice is replaced by Sm^{3+}, the crystal structure does not change dramatically. The average crystallite size, D = 1.19nm, is calculated using the DebyeScherrer's formula ie., $D = \dfrac{k\lambda}{\beta\cos\theta}$, where β is the full-width at half-maximum (FWHM) of the corresponding XRD peak at radiant, $k-$the so-called shape factor, λ - is X-ray wavelength ($\lambda_{CuK\alpha} = 0.15406$ nm) and θ is the Bragg diffraction angle.

3.2. UV – DRS - Diffusion Reflectance Spectra

Diffuse reflectance UV–Visible spectroscopy in the 200–800 nm range was used to monitor the optical absorption of the host $\mathrm{CsK_2La(VO_4)_2}$ and Sm^{3+}-substituted phosphors.

Figure 2(a) and (b) show the DRS of the CsK2La(VO4)2 host and the Sm3+ doped phosphors. It shows a status of high reflection in the wavelength ranging from 450 to 800nm for the host and 420 to 800 nm for the doped sample, and then shows a

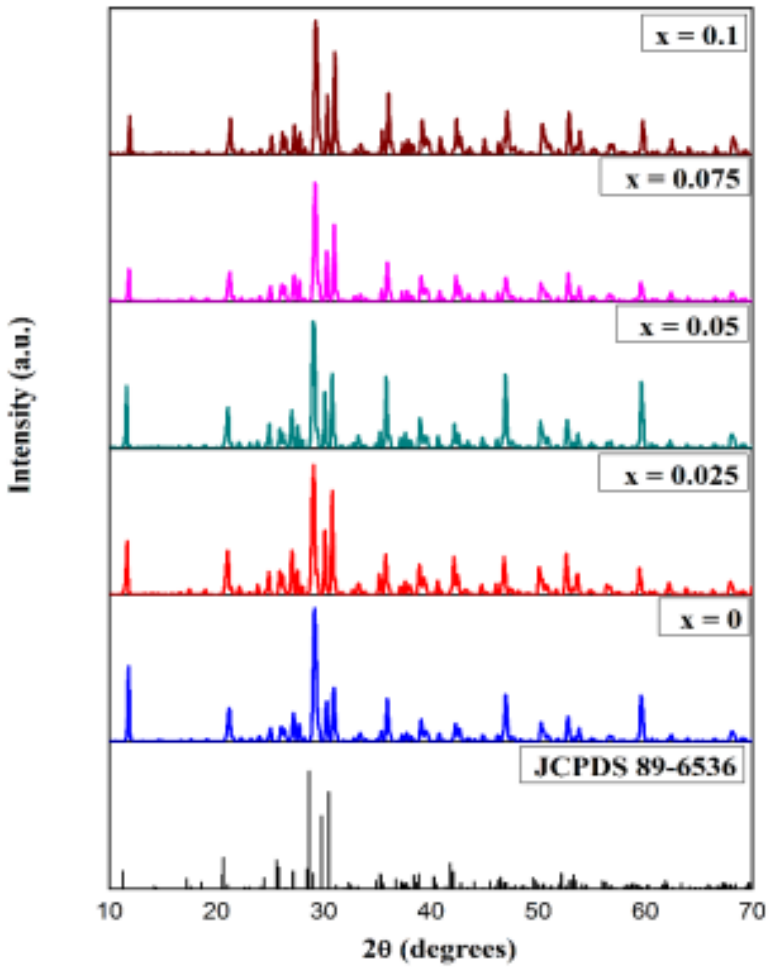

Figure 1: XRD patterns of $CsK_2La_{1-x}Sm_x[VO_4]_2$ ($x = 0$ to 0.1) orthovanadates and the corresponding PDF2 standard card No. 89-6536

remarkable drop from 450 to 300nm for the host and for the doped sample, it is from 400 to 300nm, which corresponds to the band transition. Also, it can be seen that Sm^{3+} doped has a strong absorption band in the range 300 to 350nm.

To determine the band gap of the synthesized compound, the fundamental absorption, which corresponds to the transition from the valance band to the conduction band, was used. The relation between the absorption coefficient (α) and the incident photon energy (hν) can be written as

$$\alpha h\nu = A(h\nu - E_g)^n,$$

where A is constant, α is the absorption coefficient and n depends on the type of transition having values $1/2, 2, 3/2$ and 3 corresponding to the allowed direct, allowed indirect, forbidden direct and forbidden indirectly respectively [20]. The value of the

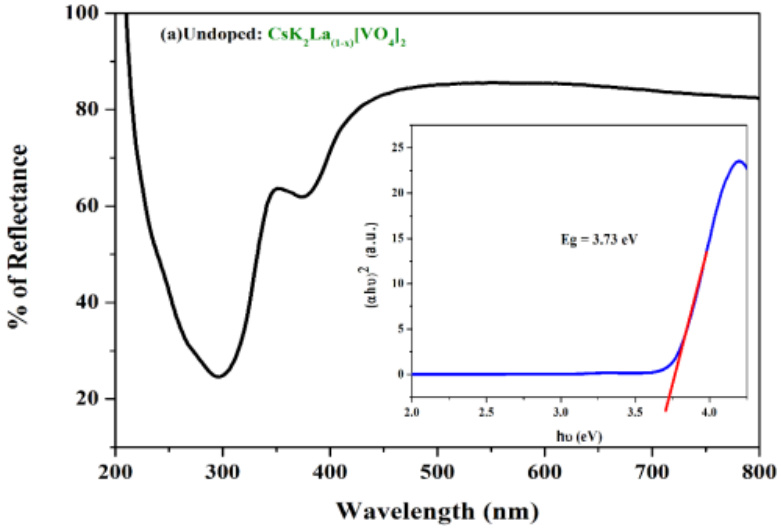

Figure 2: Diffuse reflectance spectra of the host $CsK_2La(VO_4)_2$ phosphor

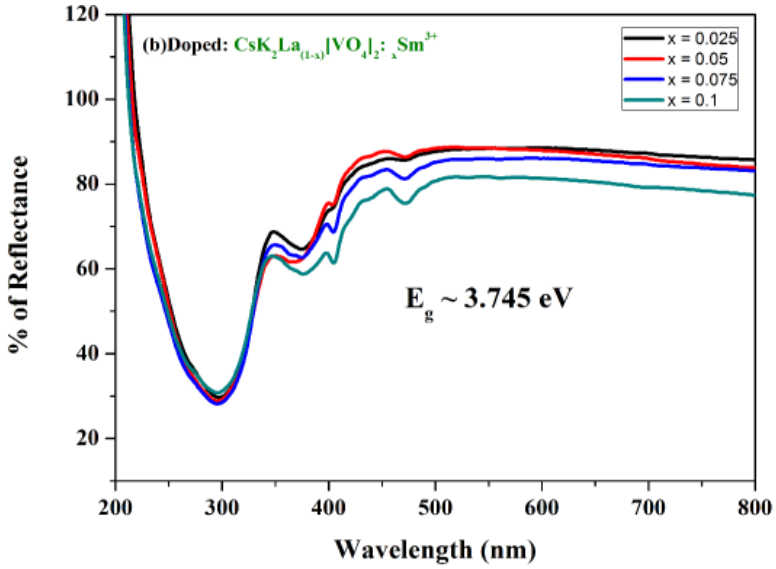

Figure 3: Diffuse reflectance spectra of $CsK_2La_{(1-x)}[VO_4]_2$: $_xSm^{3+}$ phosphor for the concentrations $x = 0.0.025, 0.05, 0.075, 0.1$

band gap has been determined by extra-plotting the straight line portion of the $(\alpha h\nu)^{1/n}$ versus $h\nu$ graph, which is shown in the inset of Fig.5(a). The band gap value has been found to be 3.73 eV for the host and for doped sample it is estimated to be 3.74 eV.

3.3. Luminescence Properties

3.3.1 PL of Rare Earth Free Self-Activated Phosphor

The PL and PLE spectra of undoped $CsK_2La(VO_4)_2$ are shown in Figure-3(a). The undoped sample shows a broad excitation spectrum band lying between the range of 250 and 400 nm with two peaks at 272 nm (Ex_1 : $^1A_1 \rightarrow {}^1T_2$) and 361 nm ($Ex_2$: $^1A_1 \rightarrow {}^1T_1$) ascribed to (CT) charge transfer transitions in $[VO_4]^{3-}$ group.

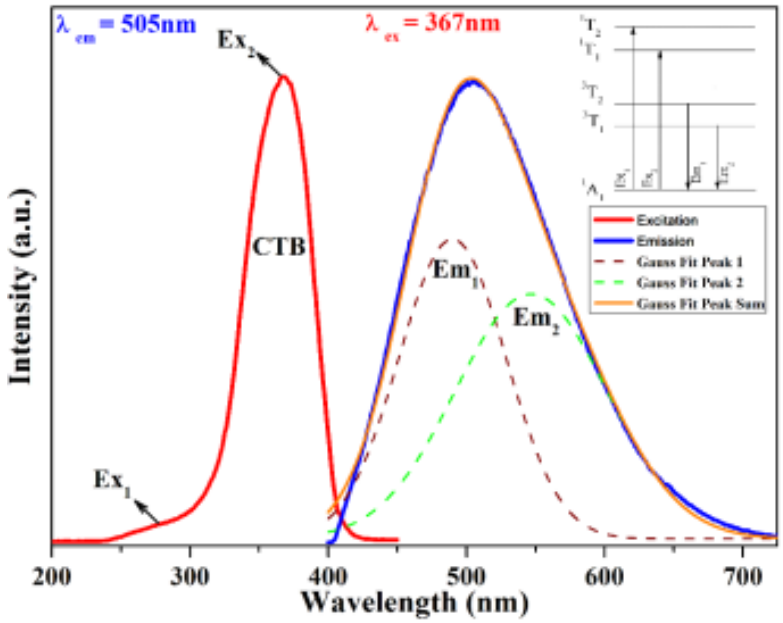

Figure 4: Photoluminescence of undoped $CsK_2La[VO_4]_2$

This shows that the phosphor absorption can perfectly match the emission spectrum of near UV-LED chips (360–400nm), which is essential for improving the efficiency of white LEDs. The nearly same excitation spectrum is in agreement with the conclusion suggested by Ronde et al., [4] that the position of the excitation bands is slightly influenced by the vanadate host. Vanadate host lattice luminescence is caused by charge transfer (CT) bands (2p orbital of O_2^- orbital of V_5^+) in ligand–metal localised tetrahedral coordinated $[VO_4]^{3-}$ groups [14,15].

According to the inset figure–3, the ground state of VO_4 is $^1A_1$, and the excited states are $^1T_1$, $^1T_2$, $^3T_1$, $^3T_2$. Electronic transitions from ground state $^1A_1$ to excited states $^1T_1$, $^1T_2$ are allowed, but emission transitions from excited states $^3T_1$, $^3T_2$ to ground state $^1A_1$ are prohibited by the spin selection rule in ideal Tetrahedral symmetry. However, because of the spin-orbit interaction, the spin-forbidden transition is partially permitted. This is due to the fact that spin orbit interaction is based not only on spin and orbital angular momentum, but also on the central field potential, and it is aided

by the distortion of the VO_4 tetrahedron [16]. Furthermore, the spin-orbit interaction allows the luminescence-process transition due to the presence of the heavy atom effect of VO_4, which contains the vanadium atom. As a result, an intense 514 nm emission (figure–3) is observed, which is caused by a CT emission transition in $[VO_4]^{3-}$.

The broad emission band of the undoped sample has a peak at 514 nm and extends from 400 to 700 nm. Therefore, taking into consideration that its emission spectrum is similar to the solar spectrum regarding the peak wavelength and broad band in the visible region, this material is useful for lighting when it is pumped with near UV-LED. Besides, the broad band is analysed using Gaussian multipeaks fitting and the results are shown in figure 3 as Em_1 and Em_2. The result shows that the broad emission band can be decomposed into two Gaussian peaks, indicating that the Gauss fit peak 2 is from the CT band $O_2^- \rightarrow V^{5+}$ at a relatively higher intensity than the Gaussian fit peak 1, which is at a relatively weak intensity.

Inset (right) in figure-3 is the schematic model to display the excitation and emission processes in VO_4 tetrahedron [14]. The molecular orbitals of the V^{5+} ion with Td symmetry are expressed as a ground $^1A_1$ state and excited $^1T_1$, $^1T_2$, $^3T_1$ and $^3T_2$ states. The absorption bands for Ex_1 and Ex_2 in the PLE spectra are correspond to the spin-allowed transitions from the ground state $^1A_1$ to the excited states $^1T_2$, $^1T_1$ levels, respectively [17].

3.3.2 PL of Sm^{3+} Activated Phosphor

The photoluminescence excitation and emission spectra of Sm^{3+} doped $CsK_2La_{(1-x)}(VO_4)_2 : {}_x Sm$ ($x = 0.075$) are shown in Figure-3(b). The broad emission, observed in an undoped compound (Figure-3a), was also observed in the Sm^{3+} doped compositions and, along with some sharp emission lines, was also observed in the Sm^{3+} doped $CsK_2La(VO_4)_2$. These sharp emission lines range from 600 to 646 nm.

The excitation spectrum of $CsK_2La_{0.025}(VO_4)_2 : {}_{0.075}Sm$ phosphor monitored at 601nm consists of a charge transfer band at 369nm and two excitation characteristics of samarium is at 406 and441 nm as depicted in Fig.5[a].The broadband around 369 nm is due to the charge transfer (CT) band of $Sm^{3+} - O^{2-}$ [21]. The other excitation peaks at 406 and 441 nm have been assigned to the 4f-4f inner shell transitions of Sm^{3+} [22]. The efficient excited wavelength range of Sm^{3+} in this phosphor covers the whole long-wavelength UV, purple, blue, and bluish-green spectral regions. This suggests that the effective excitation range well matches the output wavelengths of InGaN-based LED chips, and thus the phosphors have potential application in solid-state LED lighting.

The emission spectrum is excited by 406 nm. The characteristic emissions of this phosphor consist of five emission bands, which are attributed to the transitions from the $^4G_{5/2}$ state to $^6H_J J = 5/2, 7/2, 9/2, 11/2$ and $13/2$) states of Sm^{3+}. Among these, the strongest emission peak located at 601 nm originates from the typical transition $^4G_{5/2} \rightarrow {}^6H_{7/2}$ and the other peak at 647 nm is attributed to the transition of $^4G_{5/2} \rightarrow {}^6H_{9/2}$

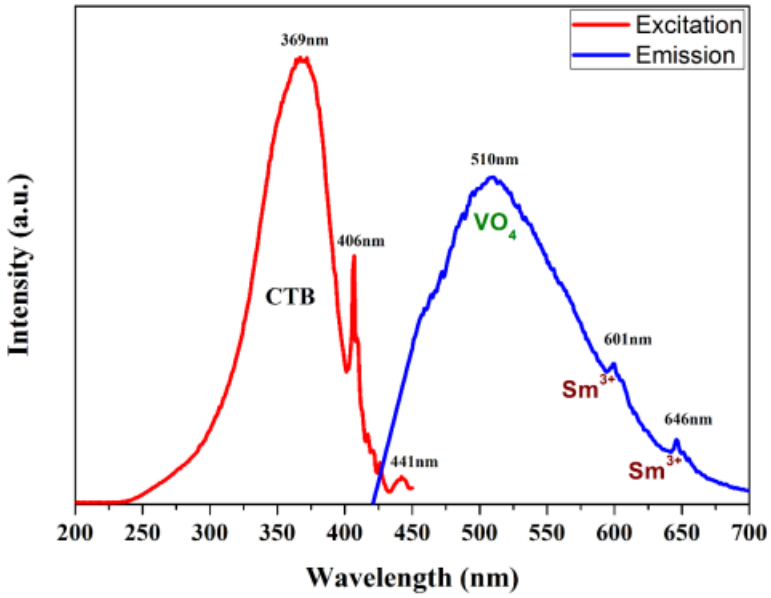

Figure 5: Shows Photoluminescence excitation and emission spectra of $CsK_2La_{(1-x)}Sm_x[VO_4]_2$; ($x = 0.075$)

respectively [23]. Among these emission peaks, the transition emission $^4G_{5/2} \rightarrow {}^6H_{7/2}$ (601 nm, orangish red) with $\triangle J = \pm 1$ is a magnetic dipole (MD) allowed one, but it is also an electric dipole (ED) dominated one, and the other transition, $^4G_{5/2} \rightarrow {}^6H_{9/2}$ (647nm, red) is purely an ED one. The MD transition does not appreciably depend on the chemical surroundings of the luminescent center and its symmetry; however, the ED transition belongs to the hypersensitive transitions. Generally, the intensity ratio of ED to MD transitions has been used to evaluate the symmetry of the local environment of the trivalent 4f ions. The greater the intensity of the ED transition, the more the asymmetry nature [24]. In this work, the emission due to the $^4G_{5/2} \rightarrow {}^6H_{9/2}$ (ED) transition of Sm^{3+} is more intense than $^4G_{5/2} \rightarrow {}^6H_{9/2}$, indicating the asymmetric nature of the host matrix. And the three main emission peaks of this phosphor always split just similarly to the reports of $LiBaBO_3 : Sm^{3+}$ [25], $Gd_2MoO_6 : Sm^{3+}$ [26] and $GdVO_4 : Sm^{3+}$ [27]. Those splits result from the crystal field effects, and the spilt extents are related to the structural characteristic of the crystal field.

3.4. Commission International del'Eclairage (CIE) Chromaticity Coordinates

The chromaticity coordinate value of emission for the host lattice $CsK_2LA[VO_4]_2$ excited at 369 nm and the Sm^{3+} activated $CsK_2LA_{(0.025)}[VO_4]_2 : {}_{0.075}Sm$ phosphor excited at 401 nm is calculated and indicated by a black cross mark in Figure-4.The CIE chromaticity coordinate values are calculated and are found to be $x = 0.247$ and $y = 0.402$ (green region) for the host lattice and $x = 0.682$ and $y = 0.316$ (red region)

for the Sm^{3+} doped phosphor.

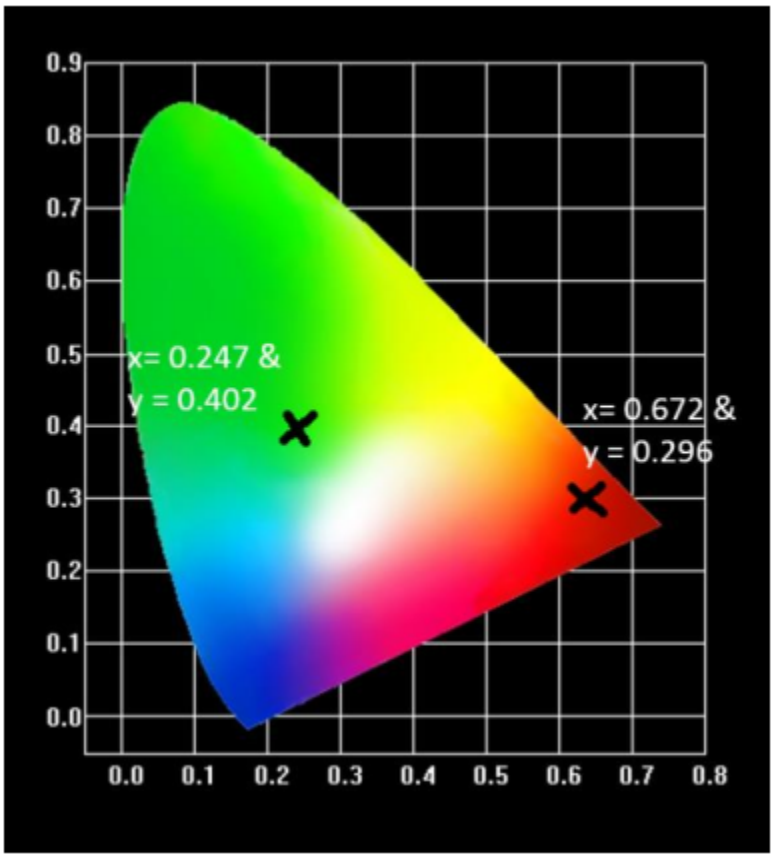

Figure 6: Shows CIE of undoped and Sm^{3+} doped phosphor compounds

The calculated CIE value is very close to the National Television Standard Committee (NTSC) standard values with ($x = 0.670$ and $y = 0.33$) [28]. Hence, this red phosphor can find potential application in white LEDs.

4. Conclusion

A new phosphor Sm^{3+} - doped glaserite-type orthovanadates $CsK_2LA[VO_4]_2$ with various concentrations of $x = 0, 0.025, 0.05, 0.075$ and 0.1 were synthesized via the conventional high temperature solid-state reaction. The XRD pattern shows that the lattice parameters of the synthesized compound exhibit a linear relationship between the host and doped XRD patterns, indicating the RE^{3+} ions are well incorporated in the host lattice. The powder XRD clearly infers that it crystalizes in a monoclinic structure. The UV-DRS shows the highest percentage in the wavelength range of 400 to 800nm which is highly essential for w-LED applications. The bandgap value is found to be 3.7

eV, which is required for solid state lighting application. The host compound material $CsK_2LA[VO_4]_2$ shows a broad emission band with a maximum at 505nm due to the CT transitions in VO_4. The self-luminescence property of orthovanadate is proved and the value lies in the green region, ie., $\lambda = 505$ nm. The trivalent rare earth Sm^{3+} doped phosphor with the host $CsK_2LA[VO_4]_2$ showed emissions in red region attributed to the transitions from the excited state $^4G_{5/2}$ to the ground state $^6H_{5/2}$ ($\lambda = 601$ nm) and the other state $^6H_{9/2}$ ($\lambda = 646$ nm) transition levels in Sm^{3+}. It can be found that $CsK_2La_{(1-x)}[VO_4]_2 : {}_xSm^{3+}$ have efficient absorption in the region of near-UV wavelengths or blue wavelength regions. This can well match with the light from UV-LED (360-400nm) or blue LED chips (450 - 480nm) based on GaN semiconductor. The calculated CIE value also shows that the x and y coordinates are very much close to the NTSC standard card. Hence, these results prove that phosphors can be suggested as suitable candidates for the application of near-UV white LED phosphors.

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STRUCTURAL CHARACTERIZATION, VIBRATIONAL AND DFT STUDIES OF BENZOXAZOLE DERIVATE AS ANTIMICROBIAL AGENT

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Abstract

The structural characterization of 2-(6-aminobenzoxazole-2yl)-5-diethylamino phenol has been reported and characterized by FT-IR spectra in the region of 4000-400 cm^{-1}. The theoretical studies such as structure parameter, vibrational frequencies, electronic absorption spectra has been investigated using gas and solvent phase, HOMO-LUMO, molecular electrostatic potential (MEP) of the title compound has been computed using the DFT- B3LYP methods with 6-31++G (d, p) basis sets. NMR Gauge Including Atomic Orbital (GIAO) chemical were calculated using 6-311++ G (d, p) basis set. HOMO-LUMO energy band gap has been determined. In addition to this the antimicrobial evaluation of was also performed against bacteria and fungi using Disc – diffusion method.

Keywords: Benzoxazole, FT-IR, UV –Vis, DFT, Anti-Microbial.

1. Introduction

The benzoxazole compounds was found to have a wide scope of natural exercises and it is right now one the main platforms viewed as in pharmacologically dynamic compounds. As a result of this huge interest, benzoxazole heterocycles and their organic properties have been investigated numerous times [1-4]. Besides, differently subbed benzoxazoles have been found in numerous normal items like Boxazomycin A, Calcimycin or Nakijinol B [5-7]. Enlivened by the nature, explores incorporated an assortment of benzoxazole subsidiaries and evaluated for their organic movement which brought about a few advertised medications [8-12].

2-Aminobenzoxazoles and their substituted compounds play an important role in medicinal chemistry and chemical biology [13-15]. Their analogues have attracted great attention. Considering the above facts, the intent of this study is to perform an experimental and computational work on benzoxazoles fusing with aryl group.

2. Materials and Methods

2.1. General

The compound 2-(6-aminobenzoxazole-2yl)-5-diethylamino phenol was purchase from the Sigma Aldrich Company with a state of 99% purity. The FT-IR spectrum of the obtained sample was taken around the 4000 - 400 cm^{-1}. The antimicrobial calculation of the title compound is carried out by disc diffusion method.

2.2. Computational Method

The theoretical calculations were performed using GAUSSIAN 9 W program and results were visualized by means of Gauss view 5.0. In the present work we have calculated the vibrational frequencies and the geometric parameters of the compound, Density functional theory using and Parr correlation functional methods (B3LYP) [17], with the standard 6-311++G(d,p) basis sets. The optimized structure is used to obtain the computation frequencies. The IEF-PCM [19] dealing with solvent effect was selected in electronic transition calculation. ^{1}H and ^{13}C NMR chemical shift values were computed by the same method in the solvent by using gauge-independent atomic orbital (GIAO). Along with this we also investigated the molecular electrostatic potential (MEP) and HOMO-LUMO pictures were also obtained.

3. Result and Discussion

3.1. Geometry Optimization

The geometry optimization of the 2-(6-aminobenzoxazole-2yl)-5-diethylamino phenol have been done with DFT -B3LYP methods with the 6-311++G (d, p) basis set. The optimized geometric structure is shown in Fig. 1, while the optimized structural parameters are presented in the Table 1. The bond length of N5-H40 and N5-H41 are almost equal to 1.0034 Å. The intra molecular highest bond lengths of C7-C15 is 1.5337 Å. The lowest is among the O2-H39 which is of 0.9632 Å. The maximum bond angle was observed for N4-C18-C20 as 132.58°. The smallest bond angle of 104.45° was obtained for C4-O1-C17. The dihedral angle C18-C20-C22-H38 and H37-C20-C22-C21 are 179.99 and −179.99, respectively.

Table 1: Geometric Bond Length, Angle, Dihedral Angle of 2-(6-aminobenzoxazole-2yl)-5-diethylamino phenol

Parameters	B3LYP/6-311++G(d,p)
Bond length (Å)	
O1-C14	1.4013

O1-C17	1.3652
O2-C12	1.3593
O2-H39	0.9632
N3-C6	1.3813
N3-C7	1.4615
N3-C8	1.4601
N4-C14	1.2957
N4-C18	1.391
N5-C21	1.3839
N5-H40	1.0034
N5-H41	1.0034
C6-C10	1.4100
C6-C11	1.4157
C7-C15	1.5337
C7-H23	1.0940
C7-H24	1.0923
C8-H16	1.5342
Bond angle (°)	
N4-C18-C20	132.11
N4-C14-C9	131.00
O1-C17-C19	128.03
C18-C17-C19	124.45
C12-C9-C14	123.47
C9-C13-C11	123.18
C6-C10-C12	122.21
N3-C7-H23	107.39
H23-C7-H24	105.70
H25-C8-H26	105.60
C14-N4-C18	105.24
C14-O1-C17	104.45
Dihedral Angle (°)	
C18-C20-C22-H38	179.99
C17-C18-C20-H37	179.99
N3-C6-C11-C13	179.99
H23-C7-C15-H31	58.65
C8-N3-C7-H23	30.16
C18-C20-C22-C21	0.0051
H36-C19-C21-N5	0.0046
C17-C19-C21-C22	0.0006
C19-C21-C22-H38	−179.99
H37-C20-C22-C21	−179.99

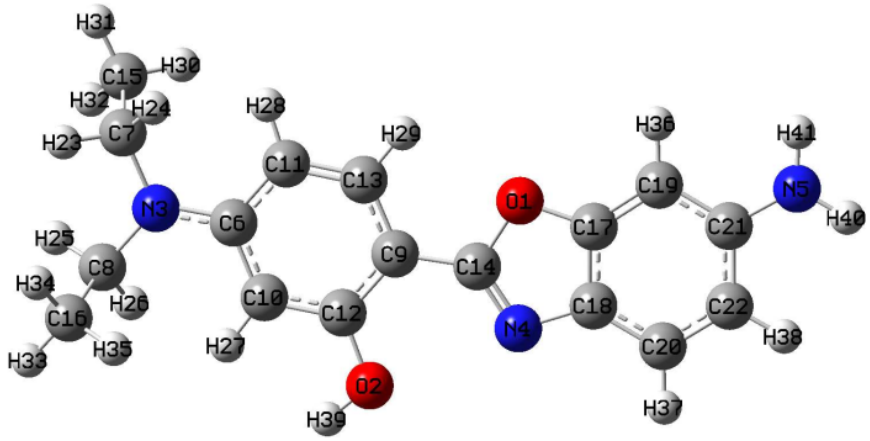

Figure 1: The Optimized Geometric Structure of the Molecule 2-(6-aminobenzoxazole-2yl)-5- diethylamino phenol

4. Vibrational Spectral Analysis

The vibrational spectral assignments of 2-(6-aminobenzoxazole-2yl)-5- diethylamino phenol were calculated on the recorded FT-IR and FT-Raman spectra based on the theoretically predicted wavenumbers by DFT using 6-311++G(d,p) basis sets are presented in Table. 2. The present title molecule has C_1 point group symmetry. The molecule has forty two (42) atoms and one hundred and seventeen (117) normal modes of fundamental vibrations. We know that *ab initio* HF and DFT potentials systematically overestimate the vibrational wavenumbers. These discrepancies are modified by introducing proper scale factors [16-17].

C-H Vibrations

The heteroaromatic structure shows the presence of C-H stretching vibrations in the region 2968-3638 cm^{-1} which is the characteristic region for ready identification of this structure [18]. These assignments are in agreement with the theoretical results at 3059, 3153 cm^{-1} calculated by B3LYP/6-311++G(d,p) method and also they are in good correlation with the earlier works [19-20]. These are pure modes and the TED results exhibit that they are around 99%.

C-C Vibrations

The weak bands observed at 1610, 1524, 1458 cm^{-1} in FT-IR and 1613, 1512 and 1455 cm^{-1} in FT-Raman are assigned to C-C stretching vibrational modes. The above assignments are in close agreement with the literature values [21-22]. The theoretically computed frequency for this vibration lies in the region 1652-1483 cm^{-1} by B3LYP/6-311++G(d,p). TED calculations also pointed out that the assignments are fairly good.

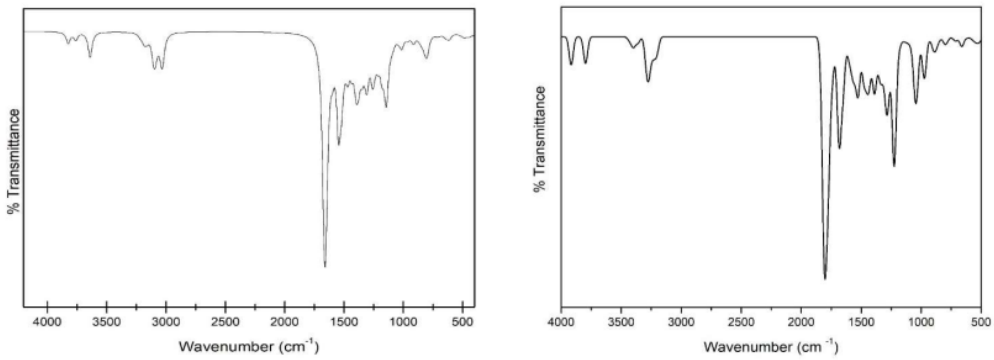

Figure 2: Exp and Theoretical FT-IR Spectra of2-(6-aminobenzoxazole-2yl)-5-diethylamino phenol

C-N Vibrations

The FT-Raman band observed at 2246 cm^{-1} (medium strong) are assigned to C-N stretching vibrations and they are in very good agreement with the literature values [23]. The assigned C-N value is around 2230 cm^{-1}.

Ring Vibrations

The ring C-C stretching vibrations happens to occur in the region 1625-1430 cm^{-1} Shimanouchi et al., [23] for five bands in the region. The theoretically computed values by B3LYP/6-311++G(d,p) at 1542, 1545, 1667, 1669 cm^{-1} show good agreement with experimental values. The aromatic stretching υ(c-c) vibrations give bands in both the observed IR and Raman spectra, covering the spectral range from 1600-1400 cm^{-1} [27-31].

Table 2: Experimental and Calculated Vibrational; Frequencies Title Compound

Mode No.	Observed Freq. (cm^{-1})		Computed at B3LYP/6-311++G(d,p)					TED (%)
	FT-IR	FT -Raman	Freq. scaled	Intensity		Red. Mass	Force Const.	
				IR	Raman			
υ_1	1124		1146	9.03	2.83	1.28	0.99	$\nu_{CC}(11\%) + \delta_{HCC}(10\%) + \delta_{HCC}(10\%) + \delta_{HCC}(15\%)$
υ_2			1179	9.32	21.64	1.49	1.22	$\delta_{HNC}(61\%) + \Gamma_{HCNC}(24\%)$
υ_3			1191	18.61	92.17	1.39	1.16	$\nu_{NH}(12\%) + \delta_{HCC}(16\%) + \delta_{HNC}(16\%) + \Gamma_{HCNC}(20\%)$
υ_4			1230	13.31	64.70	2.45	2.18	$\delta_{HCC}(14\%) + \delta_{HCC}(14\%) + \delta_{HCC}(23\%) + \delta_{HCC}(18\%)$
υ_5	1267w		1297	44.0	2.48	2.89	2.87	$\nu_{CC}(10\%) + \nu_{CC}(22\%) + \nu_{NC}(12\%) + \delta_{HNC}(14\%)$

υ_6		1503ms	1329	0.98	2.51	1.53	1.59	$\nu_{CC}(13\%) + \nu_{CC}(10\%) + \delta_{HCC}(24\%) + \delta_{HCC}(16\%)$
υ_7			1352	106.2	21.46	3.49	3.75	$\nu_{CC}(11\%) + \nu_{CC}(13\%) + \nu_{NH}(24\%)$
υ_8			1441	9.75	1.03	2.26	2.76	$\nu_{CC}(21\%) + \nu_{CC}(25\%) + \delta_{HNC}(12\%) + \Gamma_{HCNC}(10\%)$
υ_9			1471	4.36	7.66	1.24	1.58	$\delta_{HCC}(10\%) + \delta_{HNC}(35\%) + \delta_{HCC}(17\%) + \Gamma_{HCNC}(18\%)$
υ_{10}	1487ms	1582ms	1487	13.13	15.82	1.05	1.37	$\delta_{HNC}(32\%) + \Gamma_{HCCN}(62\%)$
υ_{11}			1509	1.72	9.42	1.52	2.05	$\delta_{HNC}(27\%) + \delta_{HCC}(18\%) + \Gamma_{HCNC}(11\%)$
υ_{12}			1527	8.26	25.10	1.22	1.68	$\delta_{HNH}(64\%)$
υ_{13}	1560w		1553	267.0	14.80	2.59	3.68	$\nu_{NH}(18\%) + \delta_{HNC}(10\%) + \delta_{HCC}(12\%) + \delta_{HNC}(13\%)$
υ_{14}			1610	2.87	0.73	4.45	6.80	$\nu_{CC}(19\%) + \nu_{CC}(26\%) + \delta_{HNC}(16\%) + \delta_{CCC}(10\%)$
υ_{15}	1643s	2198vs	1654	187.0	350.2	5.79	9.33	$\nu_{CC}(26\%) + \nu_{CC}(21\%) + \delta_{CCC}(10\%)$
υ_{16}		2364w	2185	118.5	1099.5	13.0	36.5	$\nu_{NH}(90\%)$
υ_{17}		2537w	2986	82.07	225.6	1.05	5.52	$\nu_{CH}(77\%) + \nu_{CH}(16\%)$
υ_{18}	2967s	2889w	3051	33.96	101.3	1.09	5.98	$\nu_{CH}(19\%) + \nu_{CH}(76\%)$
υ_{19}			3118	18.55	119.6	1.09	6.28	$\nu_{CH}(90\%)$
υ_{20}			3162	12.51	85.33	1.08	6.41	$\nu_{CH}(89\%)$
υ_{21}			3190	5.09	50.66	1.08	6.52	$\nu_{CH}(27\%) + \nu_{CH}(67\%)$
υ_{22}			3197	3.27	89.28	1.09	6.58	$\nu_{CH}(86\%)$
υ_{23}	3281s		3206	4.54	124.0	1.09	6.62	$\nu_{CH}(72\%) + \nu_{CH}(26\%)$
υ_{24}	3427s		3638	40.44	143.0	1.07	8.39	$\nu_{CH}(100\%)$

4.1. ^{1}H and ^{13}C NMR Spectra Analysis

In order to provide an of ^{1}H and ^{13}C NMR spectra of the 2-(6-aminobenzoxazole-2yl)-5-diethylamino phenol compound, we consider series of NMR calculation using GIAO method [24]. The results are tabulated in the Table 3. The IEF-PCM provided by Gaussian 9W is considered to describe the influence exerted by the solvents (DMSO, Ethanol, Chloroform) on the NMR spectra of the given compound values are also listed in the Table 3.

The ^{1}H NMR calculated chemical shifts of tittle compound were 0.86-8.14 ppm as shown in Table 3. The C^{13} chemical shifts of tittle compound were found to be 94.43-157.03 ppm. It was observed that the chemical shifts in all solvents doesn't show any difference they are almost equal. The solvents DMSO and chloroform calculated values doesn't have any difference this shows that the solvent effect is negligible.

Table 3: Computed ^{1}H and ^{13}C NMR Chemical Shifts of 2-(6-aminobenzoxazole-2yl)-5-diethylamino phenol

Atom	**B3LYP/6-311++G(d,p)**

^{1}H	DMSO	Ethanol	Chloroform
23-H	3.1834	3.1796	3.1834
24-H	3.4663	3.4659	3.4663
25-H	3.1919	3.1885	3.1919
26-H	3.3737	3.3722	3.3737
27-H	6.0759	6.0674	6.0759
28-H	6.4347	6.4297	6.4347
29-H	8.147	8.1467	8.147
30-H	1.2519	1.2545	1.2519
31-H	0.8571	0.8547	0.8571
32-H	1.1381	1.1369	1.1381
33-H	0.8614	0.8588	0.8614
34-H	1.139	1.1385	1.139
35-H	1.2647	1.2663	1.2647
36-H	6.8154	6.8102	6.8154
37-H	7.3912	7.3916	7.3912
38-H	6.5979	6.5929	6.5979
39-H	4.9711	4.9579	4.9711
40-H	3.887	3.8786	3.887
41-H	3.8423	3.8331	3.8423
9-C	105.2828	105.3406	105.2828
6-C	155.9873	155.9557	155.9873
22-C	111.4889	164.597	111.4889
21-C	149.732	111.4725	149.732
20-C	123.4712	149.698	123.4712
19-C	94.4362	123.5021	94.4362
18-C	139.1031	94.4222	139.1031
17-C	157.0353	139.1423	157.0353
13-C	135.6971	135.7149	135.6971
12-C	163.7887	163.8033	163.7887
11-C	107.0453	107.0145	107.0453
10-C	99.9028	99.8756	99.9028

4.2. Electronic Transition

The electronic transition of title compound was performed by B3LYP/6-311++G (d, p) basis set of DFT in gas phase and in solvents such as DMSO and water. The effect of solvent was evaluated by IEF-PCM. The UV-vis spectral data for both the gas and solvents phase were illustrated in Table 5, by using Gausssum3.0 [25]. According to calculated electronic data shows the absorption band in the range of 290-360 nm in all

the states. The longest wavelength corresponding to electronic transition from HOMO to the LUMO (99%) and the wavelength is 360 nm for all the phases. The large λ shows that further electrons are pushed into the benzoxazole structure.

The energies of the frontier molecular orbitals (FMOs) for the tittle compound are shown in Figure 3. The following properties of the title compound calculated include energy gap ($\triangle$E), the ionization potential (I), the electron affinity (A), the electronegativity (χ), global hardness (η), global softness (S) and the Electrophilicity index (ω). The HOMO-LUMO energy gap is very important parameter for the stability of the structure [26] and also it imitates the biological activity of the compound. The energy value of the band gap is 0.14319 a.u. for HOMO to LUMO. The global hardness of the tittle compound is 1.9482 as shown in Table 6, which indicates the good chemical stability. The global softness value 0.2566 displays that it is nontoxic.

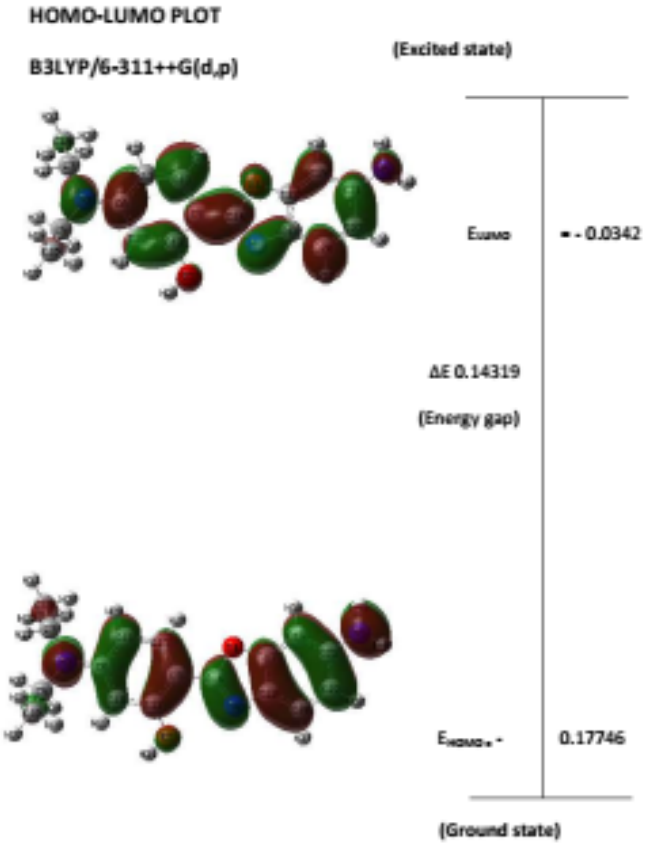

Figure 3: The Atomic Orbital Compositions of the Frontier Molecular Orbital for 2-(6-aminobenzoxazole-2yl)-5-diethylamino phenol

Table 4: Electronic Transition States for the Title Compound 2-(6-aminobenzoxazole-2yl)-5-diethyl aminophenol with the TD-DFT/IEF-PCM Method

DFT/B3LYP with 6-311 ++ G(d,p)					
	λ (nm)	Band gap (eV)	Osc. Strength	Major contributions	Minor contributions
Gas	359	3.45	1.2735	HOMO→LUMO (99%)	
	305	4.06	0.0087	HOMO→L+1 (87%)	H-2→LUMO (5%), H-1→LUMO (3%)
	291	4.26	0.021	H-1→LUMO (42%), HOMO→L+3 (46%)	
				H-1→LUMO (42%), HOMO→L+3 (46%)	
DMSO	360	3.44	1.2991	HOMO→LUMO (99%)	H-2→LUMO (5%), H-1→LUMO (2%)
	305	4.06	0.0087	HOMO→L+1 (88%)	H-2→LUMO (5%)
	291	4.26	0.0223	H-1→LUMO (43%), HOMO→L+3 (45%)	
WATER	359	3.45	1.2735	HOMO→LUMO (99%)	H-2→LUMO (5%), H-1→LUMO (3%)
	305	4.06	0.0087	HOMO→L+1 (87%)	H-2→LUMO (6%)
	291	4.26	0.021	H-1→LUMO (42%), HOMO→L+3 (46%)	

4.3. Molecular Electrostatic Potential (MEP)

Molecular electrostatic potential is a very important descriptor to know the electrophilic and nucleophilic reactions as well as hydrogen bonding interactions [27-28]. The negative region represents the electrophilic reactivity which is red in color and the positive region which is in blue represents the nucleophilic reactivity as shown in Fig 4. It can be seen from the figure negative region is localized near the hydrogen atom of the amino benzoxazole ring and the maximum nucleophilic region is associated with the N atoms of the benzoxazole indicating possible site for nucleophilic attack. This result extends information about the molecular interaction and bonding.

5. Anti-Microbial Activity

The antimicrobial activities were carried for bacterial strains viz. *Staphylococcus aureus*, as Gram –positive bacteria and *Escherichia coli* as Gram-negative bacteria and

Table 5: Global Chemical Reactivity Description

Properties	B3LYP/6-311++G(d,p)
EHUMO(eV)	−4.8289
ELUMO(eV)	−0.9325
Ionisation potential (I)	4.8289
Electron affinity(A)	0.9325
Chemical potential (μ)	−2.8807
Electronegativity (χ)	2.8807
Global hardness(η)	1.9482
Global softness (S)	0.2566
Electrophilicity index (ω)	2.1298

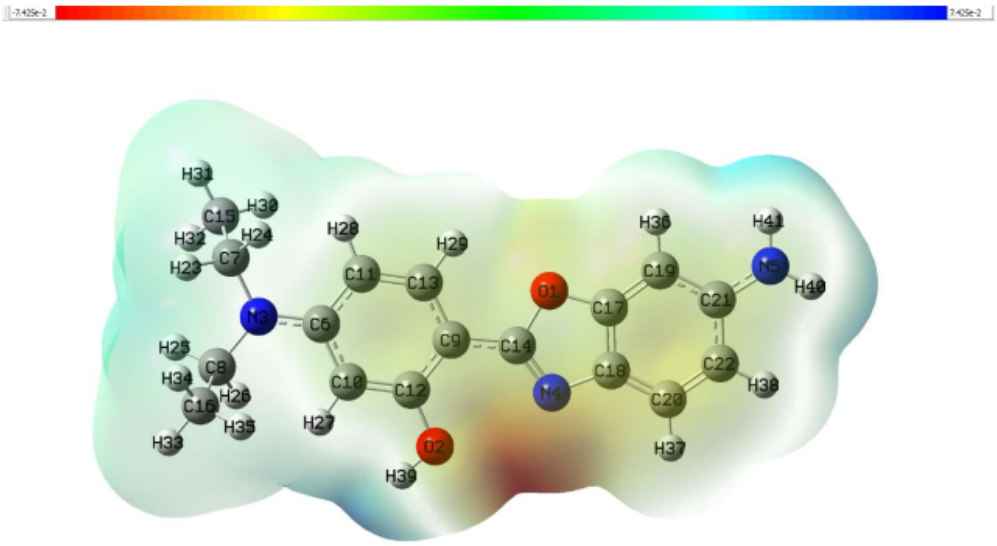

Figure 4: The MESP of the 2-(6-aminobenzoxazole-2yl)-5- diethylamino phenol Compound using DFT/B3LYP with 6-311++G (d, p)

fungal strains viz. *Candida albicans* were used in the investigation. They were chosen based on their clinical and pharmacological importance. Antimicrobial activity of title compound was determined by Disc diffusion method according to Clinical Laboratory Standards. Subsequently, discs of 6 mm diameter were kept into the agar medium and filled with 20 μl (25, 50, 100 μg/ml) of compound and allowed to diffuse at room temperature for 2 h. The plates were then incubated in the upright position at 37°C for 24 h for bacterial culture and 28°C for 48 h for fungal culture. Disc containing the same volume of ethanol served as negative controls while standard antibiotic Ciprofloxacin

Table 6: The Antimicrobial Activity of the 2-(6-aminobenzoxazole-2yl)-5-diethylamino phenol by using Disc Diffusion Method

Microbial Compound	**Diameter of the Inhibition Zone (mm)**			
Gram Positive	25 μg/ml	50 μg/ml	100 μg/ml	Ciprofloxacin 25μg/ml
Staphylococcus aureus	13	21	29	30
Gram -Negative				
Escherichia coli	14	23	32	33
Fungal Compound				
Candida albicans(MTCC 227)	38	47	53	40

25 mg (50 μl) were used as the positive controls. After incubation, the growth inhibition zones were measured in mm dia.

It showed moderate inhibitory action in both anti-bacterial and anti-fungal due to structure activity relationship of benzoxazole. Electron donating and electron withdrawing groups on benzoxazole do not affect the growth inhibitory activity. It is clear that some more exploration of changing the derivatives at different position will lead to some important antimicrobial activity.

6. Conclusion

In this study the title compound. We thoroughly investigated the FT-IR spectra, UV-vis, NMR, and Molecular Electrostatic Potential (MEP) in detail manner for the first time . A FT-IR spectra analysis shows that the calculated vibrational frequencies are in good agreement with the experimental values. The chemical shifts values for different solutions were determined for NMR.

The energy gap of the present compound and electrophilicity index value was determined using HOMO and LUMO energies. MEP give us the information about the intermolecular interaction and the metallic bonding of the title compound. The compound is studied for antimicrobial activity against the bacteria and fungi.

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MATERIAL SYNTHESIS, SPECTRAL, OPTICAL ABSORPTION AND PHYSICOCHEMICAL PROPERTIES OF L-THREONINE COBALT NITRATE (LTCN)

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Abstract

AR grade of L-Threonine and Cobalt nitrate were dissolved in double distilled water using solution growth technique at room temperature. Single crystal X-ray diffraction study shows that LTCN crystal belongs to orthorhombic crystal system. In the powder XRD pattern well defined Bragg's peaks are observed which reveals that the grown crystal has good crystalline nature. FT-IR analysis was carried out to identity the various functional groups present in LTCN crystal. The optical transmission spectrum was recorded; the UV transmission is high in the entire visible region from 234 nm to 1000 nm. The mechanical behavior of LTCN crystal was analyzed by microhardness study which reveals that the grown crystal has high mechanical strength.

Keywords: XRD, FTIR, UV, NLO.

1. Introduction

A semiconducting materials possess wide range of applications in the field of telecommunication, high density optical recording, color display, medical diagnostics, etc., [1-3]. The overwhelming success of molecular engineering in controlling NLO properties has promoted the growth and characterization of a variety of new types of NLO materials [4]. The search for new frequency conversion materials over the fast decades has concentrated primarily on organic and inorganic compounds. Organic-inorganic hybride materials have received extensive attention in recent years owing to their great fundamental and practical interest such as second order nonlinear optical (NLO) responses. These applications depends upon the various properties of the materials, such as transparency, bifriengence, refractive index, dielectric constant, thermal, photochemical and chemical stability [5].

Organic crystals have large nonlinear susceptibilities compared to inorganic crystals. In last several years, considerable interest in growth and characterization of non linear optical materials (NLO) due to their important contribution in areas of optical modulation, Optical switching, optical logic, optical data storage and frequency. Several attempts have been made for exploration of nonlinear optical materials which found various applications in optoelectronics [6-11].

Inorganic materials have advantages over organic materials, such as architectural flexibility for molecular design and morphology, high mechanical strength and good environmental stability with non toxicity and usability in high power applications. Molecular hyper polarizability of inorganic nonlinear optical crystal are used in optical switching (modulation), frequency conversion (SHG,wave mixing) and electro-optic applications especially in EO modulation [12].

Some complexes of the amino acids with simple organic and inorganic salts appear to be promising for optical second harmonic generation (SHG).

This research is extended to semi-organic NLO material crystal so as to obtain superior NLO crystal by combining the advantages of organic and inorganic materials [13]. Hence Semi-organic single crystals are attracting great attention in the field of non linear optics because of their high optical nonlinearity, chemical flexibility of ions, high mechanical strength, thermal stability and excellent transmittance in the UV-Vis region [14-17].

In this present investigation we concentrate on growth of semiorganic L-Threonine Cobalt nitrate (LTCN) by solution growth technique. The grown crystals were characterized using single crystal XRD and powder X-ray diffraction analysis, Fourier transform infrared (FT-IR) analysis, UV-Vis spectroscopy studies, SHG efficiency and microhardness studies were carried out and discussed.

2. Experimental Procedure

2.1. Synthesis of L-Threonine Cobalt Nitrate (LTCN)

LTCN crystal is synthesized by dissolving L-Threonine (AR grade) and Cobalt nitrate (AR grade) in the equimolar ratio 1:1 using double distilled water as a solvent. The saturated solution of Cobalt nitrate was slowly added to the saturated solution of L-Threonine. This was stirred well to get a clear solution. Pure LTCN was synthesized according to the reaction.

$$C_4H_9NO_3 + Co(NO_3)_2 \rightarrow Co(C_4H_9NO_3) \cdot (NO_3)_2.$$

The saturated solution of LTCN was prepared in double distilled water solvent at room temperature. The solution stirred well for more than 9 hours using magnetic stirrer to obtain a homogenous mixture.

2.2. Growth of L-Threonine Cobalt Nitrate (LTCN)

The saturated solution was filtered using ashless whatman filter paper. The filtered solution was taken in a beaker and covered with good quality perforated polythene cover to restrict the fast evaporation it is kept at room temperature in a dust free compartment for slow evaporation. After the period of 45 days, colorless crystals with dimension $6 \times 6 \times 3$ mm^3 were obtained and it is shown in the figure 1. The crystal is non-hygroscopic and optically transparent. After a span of 45 days, well-faced good quality crystals of LTCN were harvested and details of growing condition given in Table 1.

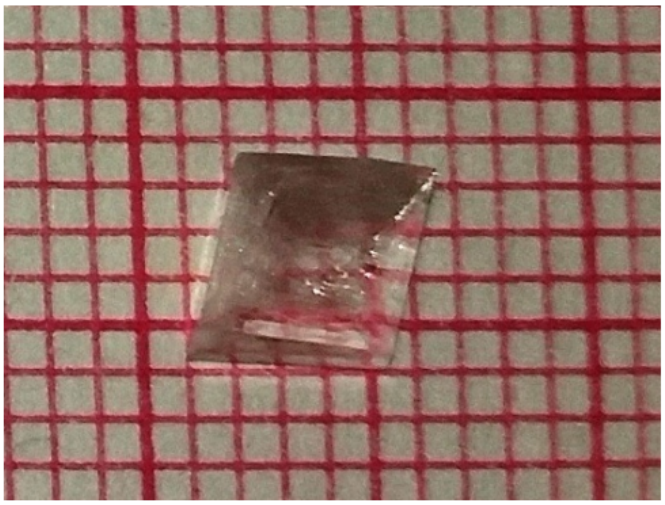

Figure 1: As Grown Crystal of LTCN

Table 1: Growth of LTCN Crystal

Method of Growth	Slow Evaporation
Solvent used	Double Distilled (2D) Water
Molar ratio 1:1	L-Threonine and Cobalt nitrate
Operating Temperature	Room temperature
Period of growth	45 days
Dimension	$6 \times 6 \times 3$ mm^3

3. Characterization of LTCN Crystal

The grown crystals have been analyzed by different characterization techniques. LTCN single crystal was subjected to single crystal X-ray diffraction analysis using ENRAF

NONIUS CAD4-F X-ray diffract meter with Mo Kα ($\lambda = 0.7170A^{\circ}$) radiation. The crystalline nature of LTCN was confirmed by powder X-ray diffraction analysis using BRUKER, Germany (model D8 Advance) X-ray diffract meter.

Also the formation and quality of compounds were checked by x-ray powder diffraction (PXRD) spectrum. The functional groups were identified by using BRUKER Fourier Transform infrared spectrometer in the range of 450 - 4000 cm^{-1}. The optical transmission spectrum of LTCN crystal was taken in the wavelength range 200 – 1000 nm by Varian Cary 5E model spectra photometer.

A Q-switched mode locked Nd:YAGlaser, used to generate about 6.2 mJ/pulse at the 1064 nm fundamental radiation, was used for SHG efficiency measurements. The microhardness study was carried out for the grown crystal using Vicker's microhardness tester.

3.1. Single Crystal XRD Analysis of LTCN Crystal

The single crystal XRD study of LTCN crystal indicate that the unit cell parameters are $a = 5.97A^{\circ}, b = 12.86A^{\circ}, c = 4.89A^{\circ}, \alpha = \beta = \gamma = 90^{\circ}$ and volume = $375.42A^{\circ 3}$. Hence, the LTCN crystal is an orthorhombic crystal structure.

3.2. Powder XRD Analysis of LTCN Crystal

The powdered sample was scanned in the range 10-80°C at a scan rate of 1° per minute. In the powder XRD pattern well defined Bragg's peaks are observed which reveals that the grown crystal has highly crystalline nature. The recorded indexed powder XRD patterns of the grown l-threonine cobalt nitrate (LTCN) were shown in Figure 2.

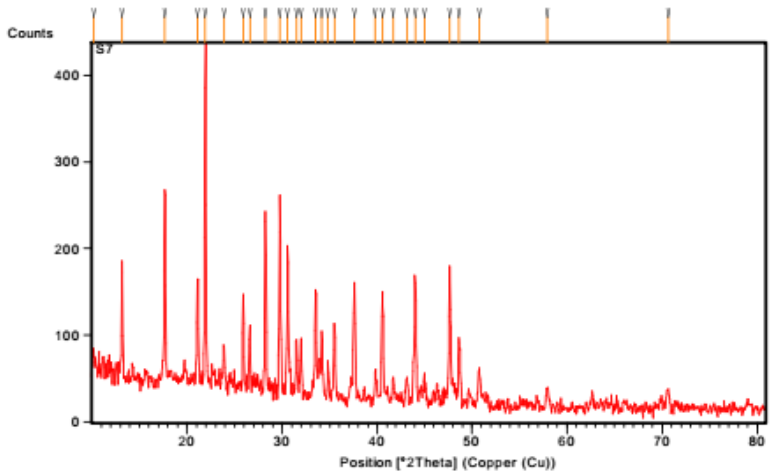

Figure 2: Powder XRD Pattern of LTCN

3.3. FTIR Spectral Studies of LTCN Crystal

In order to identity the presence of functional groups and chemical composition, the FTIR spectrum was taken using BRUKER IFS 66v spectrometer by Kerr pellet techniques. Figure 3 shows FTIR spectrum of the grown l-threonine cobalt nitrate (LTCN) crystal. The observed peak at 3749 cm^{-1} to 3624 cm^{-1} belongs to OH stretching, observed peak at 3498 cm^{-1} for NH stretching. The absorption peak at 2285 cm^{-1} for N $\equiv$ N Stretch (aq.sol). C $\equiv$ N Stretching at the peak 2222 cm^{-1}. Observed peak at 2049 cm^{-1} belongs to C $\equiv$ C Stretching. The absorption peak at 1216 cm^{-1} for C-O-C antisymmetric stretching. Out of plane deformation at the peak 976 cm^{-1} and 787 cm^{-1}. The addition of Cl conformed by the observed peak at 527 cm^{-1}. The wave number corresponding assignments are given in the Table 2.

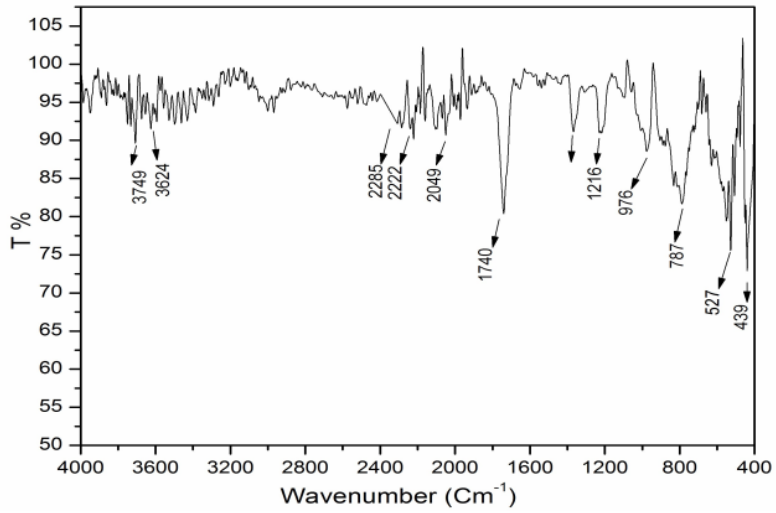

Figure 3: FTIR Spectrum of LTCN Crystal

3.4. UV-Visible NIR Spectrum Analysis of LTCN Crystal

The selective electronic absorption spectrum of LTCN crystal was recorded in the range 200-1000 nm. Optically polished single crystal of thickness 2mm was used for this study. The recorded spectrum gives limited introduction about the structure of the molecule, because the absorption of UV and visible light involves promotion of the electron in the σ and π orbital from ground state to higher energy state. The absorption spectrum of grown crystal of LTCN is shown in the figure 4.

UV-visible spectral analysis shows that crystal is transparent in the entire visible region. The UV cut off wavelength occur at 234 nm. It is well known that an efficient NLO crystal has an optical transparency lower cut off wavelength between 200-400 nm. The large transmittance window in the visible and NIR region enable very good optical transmission of the second harmonic frequencies of Nd:YAG laser.

Table 2: Wave Number Assignments of LTCN Crystal

Wavenumber cm^{-1}	**Assignment**
3749-3624	OH Stretching
3498	NH Stretching
2285	N=O stretching
2222	C=O stretching
2049	C $\equiv$ C Stretching
1740	C=O Stretching
1216	C-O-C Antisymmetric Stretching
976,787	=CH Out of plane deformation
527, 439	NO_2 scissoring

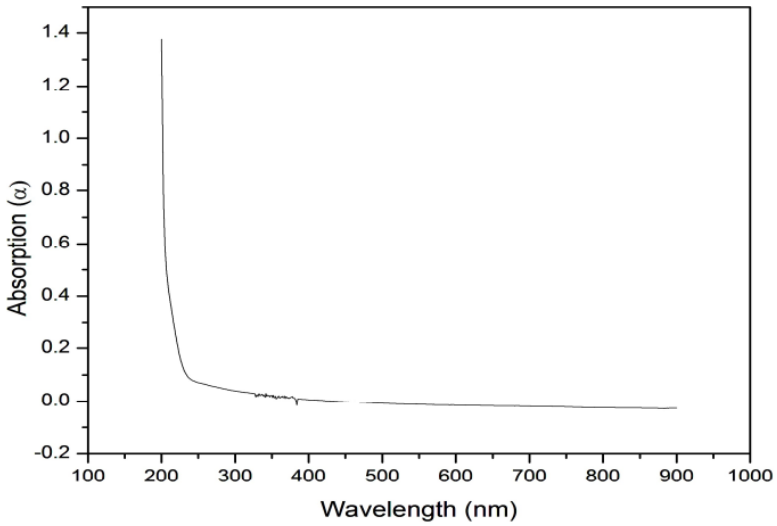

Figure 4: Optical Absorption Spectrum of LTCN Crystal

3.5. SHG Efficiency Measurement of LTCN Crystal

In order to confirm the nonlinear optical property, powdered sample of LTCN was subjected to Kurtz and Perry techniques, which remains a powerful tool for initial screening of materials for SHG [18]. The fundamental beam of wavelength 1064 nm from Q switched mode locked Nd:YAG laser was made to fall normally on the powder from of grind sample which was made available between two transparent glass slides. Pulse energy 2.9 mJ/pulse and pulse width 8 ns with a repetition rate of 10 Hz were used. The photo multiplier tube (Hamamatsu R2059) was used as a detector and 90 degree geometry was employed. The SHG signal generated in the sample was confirmed from

emission of bright green (532 nm) radiation from the sample. The measured amplitude of second harmonic generation for LTCN crystal is 6.32 mJ. It shows a powder SHG efficiency of LTCN crystal is about 0.71 times than that of KDP.

3.6. Vicker's Microhardness Test for LTCN Crystal

The Vickers hardness indentations were made on the cut and polished samples of LTCN crystals grown by slow evaporation method. At room temperature, the load was varied as, 25, 50 and 100 g and the several indentations were made for each load and the diagonal lengths (d) of the indented impressions were measured using Vickers hardness tester (LEITZ WETZLER) fitted with Vickers diamond indenter and attached to an incident light. Vicker's hardness number was determined using the formula Hv=1.8544 P/d2 Kg/mm^2. The variation of hardness Hv with load P for LTCN crystal are shown in Figure 5. A plot between log p vs. log d for the grown crystal is shown in Figure 6. The work hardening index n, which is found to be n = 1.1 for grown LTCN crystal. According to Onitsch [19], the value of 'n' should lie below 1.6 for comparatively hard materials, whereas it is above 1.6 for softer ones. The microhardness study indicates that the crystal belongs to the class of hard materials.

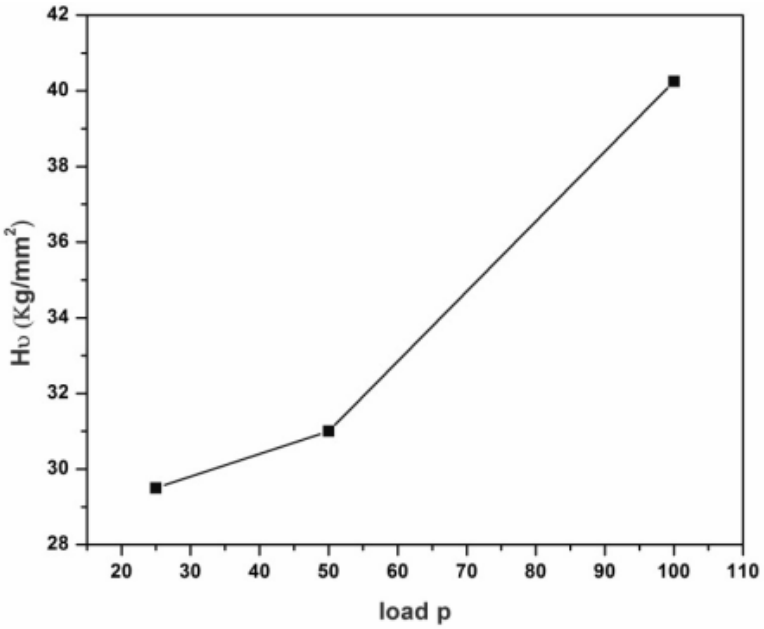

Figure 5: The Variation of Hardness Hv with Load P for LTCN Crystal

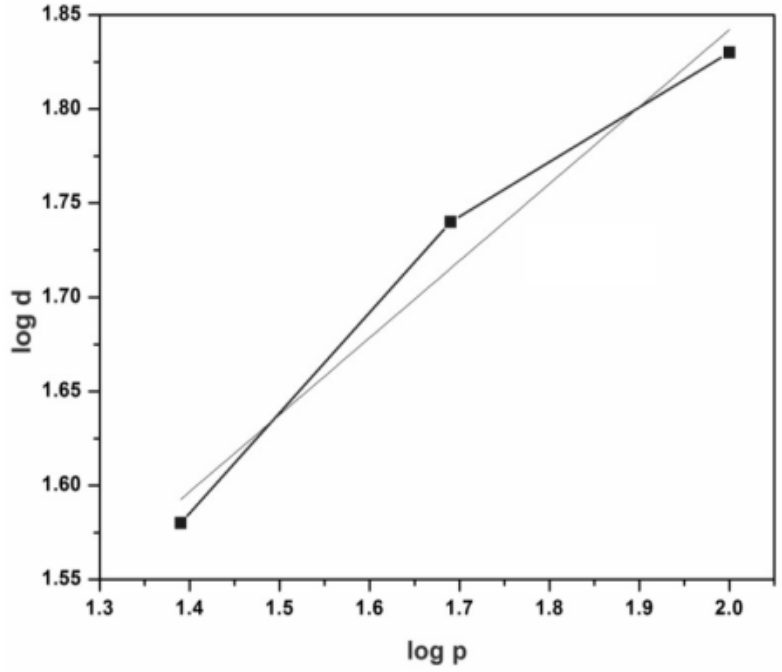

Figure 6: A Plot between log p vs. log d for LTCN Crystal

4. Conclusion

A new NLO material L-Threonine Cobalt Nitrate (LTCN) has been synthesized and crystals were grown by slow evaporation method. The lattice parameter values have been evaluated by single crystal XRD analysis and shows orthorhombic crystal system. From the powder XRD analysis, the sharp well defined Bragg's peaks confirmed the good crystalline nature of the LTCN crystal. The presence of various functional groups in LTCN sample were identified by FT-IR analysis. UV-visible-NIR spectral analysis was carried out to determine the lower cut off wavelength at 234 nm for the grown sample. The NLO property was confirmed from the emission of bright green radiation (532nm) from the LTCN. So it is a good NLO material for optical applications. The work hardening coefficient value was found to be n = 1.1, which shows that a grown LTCN belong to hard category of material. A good physicochemical property of LTCN crystal evident that they have various NLO applications.

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DIELECTRIC RELAXATION STUDIES OF ACETATES WITH PHENOL DERIVATIVES IN Ccl4

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Abstract

The dielectric relaxation of acetates (methyl acetate, ethyl acetate and butyl acetate) with phenol derivatives ($p-$chlorophenol, $p-$iodophenol and $p-$nitrophenol) has been measured at microwave frequency 9.37 GHz in dilute solution of carbon tetrachloride at 308K. Different dielectric parameters like dielectric constant (ε') and dielectric loss (ε'') at microwave frequency, static dielectric constant (ε_0) and dielectric constant (ε_∞) at optical frequency have been determined. The relaxation time and activation energy have been determined using the measured dielectric data. The results show a linear dependence of relaxation time on alkyl chain length of acrylic esters and acidity of phenol derivatives. The result shows that, 1:1 complex is predominant in these systems. The strength of hydrogen bond (O–H: C=O) formation shows a significant dependence of on alkyl chain length of both phenols and acetates and the nature of the solvent used.

Keywords: Acetates, Phenol Derivatives, Dielectric Relaxation, Hydrogen Bonding.

1. Introduction

Acetates are useful for many industrial applications. Biodiesel fuels typically contain mono alkyl, often methyl and ethyl, esters of long–chain fatty acids, derivedfrom vegetable oils or animal fats. In general combustion performance is influenced by the chemical nature of the esters in thebiodiesel fuel and their structural features, including the length of the alkyl chain and their degree of saturation and branching [1-4]. Dielectric relaxation investigations, especially in polar liquids dissolved in non-polar solvents have always been of considerable interest to a large number of researchers [5-8]. Such studies are very useful in understanding the molecular structure and the molecular forces. Syamalamba and Premaswarup [9] have studied the dielectric properties of methyl, ethyl and butyl acrylate in the microwave region at different temperature and compared their result with those on butyrates. Sivagurunathan et

al. [10] reported the dielectric parameters of methyl acrylate, ethyl acrylateand butyl acrylate with primary alcohols using time domain reflectometry. Dielectric parameters for dimethyl formamide-phenol and dimethyl acetamide-phenol were reported by Tuckeretal [11]. Recently, our research group [12 - 17] has investigated the complex formation of acrylic esters with proton donors (alcohols) in non-polar solvents using FTIR spectroscopic method. The study of H-bonds of the type O - H .... O=C is of considerable importance as itrelates to the study of biopolymers. Thus, the study ofdielectric properties of the ternary mixtures of acrylic esters with polar associating liquids in non-polar solvents provides useful and vital process parametersfor efficient design of transesterification process of industrial interest. Keeping both the industrial and scientific interests in mind, an attempt has been madein the present work to study the hydrogen bonding between free hydroxyl groups of phenol and the carboxyl group of ester using dielectric method. This study provides a better understanding of the nature of the molecular orientation process.

2. Experimental Details

The static dielectric constants were measured by heterodyne beat method at 308 K using a commercial instrument, Dipole meter DM01 supplied by Wissenschaijftlich Technische Werkstatter, Germany, operated at 220V.The refractive indices were measured by Abbe's refractometer with an accuracy of ± 0.0005. The percentage of deviation in the measurement of refractive index is about $\pm 0.02\%$. The measurements of dielectric constant at an angular frequency (ε') and dielectric loss (ε''),were carried out in the X-band microwave frequency of 9.37 GHz. The viscosities were measured with the help of Oswald's viscometer. The temperatures of all these measurements were maintained at $35 \pm 0.1^{\circ}$C using water circulating thermostat. Merck varieties of methyl acetate, ethyl acetate and butyl acetate and p- chlorophenol, p-iodophenol as well as p-nitrophenol were used. The physical parameters of all the chemicals used here have been checked against their literature values.

The proton donors (phenol derivatives) and the acceptors (acetates) under study were separately dissolved at the same molar concentration (0.3 mol^{-1}) in the solvent carbon tetrachloride. Their dielectric constants were measured separately. Then, two solutions were mixed in different proportions but with the total concentration kept at a fixed value and subjected to the dielectric constant measurements. Asthe maximum deviation of dielectric constant for all the systems studied occurs at equimolar ratio of the solutes, it is presumed that the deviation is due to the formation of 1:1 complex alone.

3. Dielectric Parameters

According to Higasi's method [18], the average relaxation time $\tau_{(1)}$ is described by

$$\tau_{(1)} = \frac{a''}{\omega(a' - a_{infty})}.$$

While the overall dielectric relaxation $\tau_{(2)}$ is given by

$$\tau_{(2)} = \frac{a_0 - a'}{\omega a''}$$
$$\tau_{(0)} = \sqrt{\tau_{(1)}\tau_{(2)}}$$

$\tau_{(0)}$ may be called the mean relaxation time, where ω is the angular frequency $\varepsilon_0, \varepsilon', \varepsilon''$ and ε_∞ are defined by equation

$$\varepsilon_0 = \varepsilon_{01} + a_0 w_2$$
$$\varepsilon' = \varepsilon'_1 + a' w_2$$
$$\varepsilon_\infty = \varepsilon_{\infty 1} + a_\infty w_2.$$

In which subscript refer to the solvent and 2 refers to the solute, 0 refers to the static frequency and ∞ refers to the infinite or optical frequency measurements and w_2 is the weight fraction of the solute.

The molar free energies have been calculated using the Eyring's equation.

$$\tau = \frac{h}{KT} \exp\left(\frac{\triangle F_\tau}{RT}\right)$$
$$\eta = \frac{Nh}{V} \exp\left(\frac{\triangle F_\eta}{RT}\right),$$

where h-Planck's constant, k-Boltzmann's constant, N-Avagadro number and V- the molar volume.

4. Result and Discussion

The value of static dielectric constant(ε_0), dielectric constant at angular frequency(ε_∞), dielectric constant (ε'), dielectric loss (ε''), relaxation time (τ_1), due to the individual rotation of the molecule, relaxation time (τ_2) due to the whole or overall rotation of the molecule, the most probable relaxation time (τ_0), activation energy $\triangle F_\tau$ due to dielectric relaxation process and activation energy $\triangle F_\eta$ due to viscous flow of proton donors ($p-$chlorophenol, $p-$iodophenol and $p-$nitrophenol) with proton acceptors (MA, EA and BA) in carbon tetrachloride at 308 K have been presented in Tables 1-3. The value of relaxation time increases with increasing chain length of acetates and acidity of phenol derivatives [11, 19] (Tables 1-3). The increase in relaxation time may be due to the increase in effective radius of the rotating unit. The observed higher value of butyl acetate (BA) can be attributed to the larger size of BA molecule in comparison to methyl acetate (MA) molecules. Figs1-3 shows the variation of mean relaxation time with ratio of phenol derivatives and acrylic esters.

Tables 1-3 presents the values of τ_2 which are significantly higher than τ_1 and τ_0 for all the systems. Higher values of τ_2 indicate that the contribution of intermolecular

or overall molecular relaxation is larger in comparison to intramolecular or individual molecular relaxation in the systems [20]. In the present systems, it has been observed that the relaxation time of ternary mixtures (phenol derivatives with acetates in solvent) are much greater than the binary mixture (phenol derivatives with solvent or acetates with solvent). This result indicates that there is a hydrogenbond formation between the hydrogen atom in O-H group of phenol derivatives and the oxygen atom in C=O group of esters and is shown in Fig.1

In these systems, the complex formation is likely to occur between $H^{\delta+}$ of phenols and $O^{\delta-}$ of C=O group of esters. Oxygen atom is sp^3 hybridized and in the ester structure many voids are available for O-H to penetrate and enter into complexation components.

Carbon tetrachloride is a symmetrical and non-polar molecule. But each chlorine atom in this molecule is highly polarizable due to its three lone pair of electrons and therefore, it can function as an electron donor. There is, therefore, a possibility of interaction between the positive hydrogen of hydroxyl group and a chlorine atom of the carbon tetrachloride molecule. The potential hydrogen bonding nature of the carbon tetra chloride molecule may, therefore, contribute to increasing the relaxation time [21].

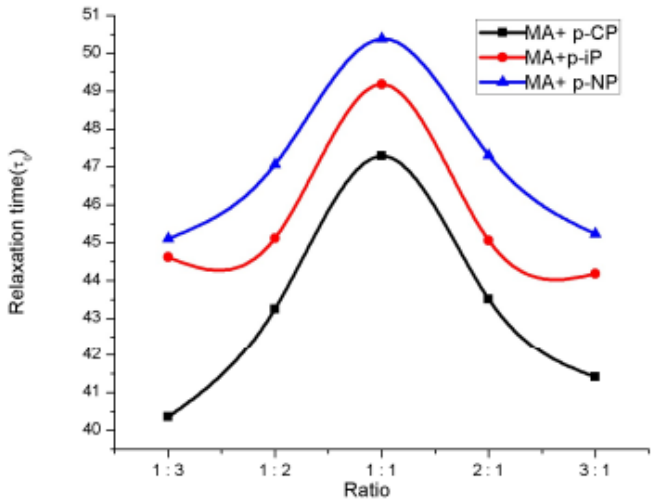

Figure 1: Variation of Mean Relaxation Time with Ratio of Phenol Derivatives and Methyl Acetate

The dielectric relaxation time shows continuous increase with chain length of esters and acidity of proton donors (phenols) and offers hindrance to the rotation of the molecule. The increase in relaxation time with chain length is expected as hydroxyl group reorientation depends to some extent on the length of the alkyl group, and the viscosity of the liquid. Our results show that the relaxation time is larger at 1:1 mole ratio of acetates with phenols. Fig 1-3 show the variation of mean relaxation time with

Table 1: Values of Dielectric Constants and Relaxation Times for Various Weight Fractions, Methyl Acetate with Phenol Derivatives in Carbon Tetrachloride

Ratio of Phenol: Acetates	Weight Fraction W2	ε'	ε''	ε_0	ε_∞	Relaxation time(ps) using Higasi's method		
						τ_1	τ_2	τ_0
p-chlorophenol								
1: 3	0.015	2.3011	0.1636	2.7612	2.2342	39.62	43.14	40.35
1: 2	0.017	2.3576	0.2051	2.8118	2.2391	42.54	46.27	43.25
1: 1	0.019	2.3918	0.2366	2.9676	2.2421	45.19	51.21	47.29
2: 1	0.021	2.3512	0.2012	2.8011	2.2391	42.16	46.95	43.51
3: 1	0.023	2.3113	0.1678	2.7566	2.2338	39.59	43.37	41.41
p-iodophenol								
1: 3	0.020	2.2716	0.1436	2.6573	2.2378	43.18	45.19	44.62
1: 2	0.022	2.2962	0.1846	2.7419	2.2418	45.72	47.18	45.12
1: 1	0.025	2.3218	0.2013	2.9185	2.2456	47.13	50.26	49.18
2: 1	0.027	2.2908	0.1878	2.7406	2.2412	45.07	47.18	45.06
3: 1	0.029	2.2756	0.1413	2.6522	2.2396	43.28	45.36	44.18
p-nitrophenol								
1 :3	0.022	2.2363	0.1251	2.5269	2.2391	45.12	47.62	45.11
1: 2	0.025	2.2447	0.1744	2.5927	2.2476	45.08	49.23	47.06
1: 1	0.027	2.2685	0.1643	2.8019	2.2512	49.11	52.66	50.38
2: 1	0.029	2.2391	0.1751	2.6302	2.2466	48.23	49.16	47.31
3 :1	0.031	2.2305	0.1243	2.5227	2.2369	45.66	47.38	45.23

ratio of phenol derivatives and acrylic esters and maximum variation is observed at 1:1 ratio for all phenol derivatives with individual acrylic ester.

The relaxation time decreases for the other mole ratios but is higher than either of the components. Saxena et al [22]. Studied the H-bonding in some carbonyl + phenol derivative systems in different compositions. They also observed that the relaxation time of ternary mixtures is always much greater than either of the polar solutes in the inert solvent. The relaxation time for dilute solution of $p-$chlorophenol, $p-$iodophenol and $p-$nitrophenol observed in the present study ranges between 14 and 35ps. With excess of phenols, the relaxation time of acetate +phenol system shows the slight increase. This result is in agreement with Tucker et al11. The result also showsthat the molecular

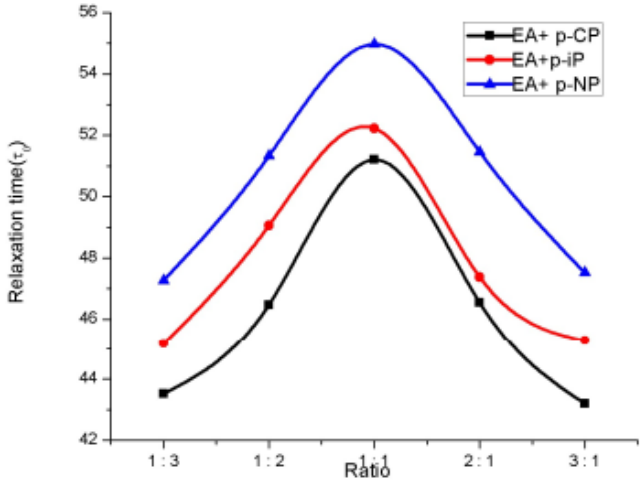

Figure 2: Variation of Mean Relaxation Time with Ratio of Phenol Derivatives and Ethyl Acetate

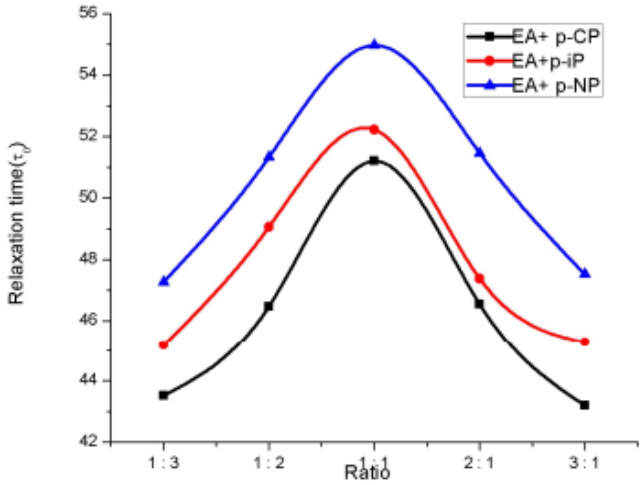

Figure 3: Variation of Mean Relaxation Time with Ratio of Phenol Derivatives and Butyl Acetate

association between acetates and phenols is maximum at 50:50 mol% ratio and then decreases at other mol %. From this, we conclude that the1:1 complex is dominant in acetate-phenol systems.

The relaxation time τ increases with increasing acidity of proton donor in complex systems. The relaxation time of acetates with $p-$chlorophenol is less than that of acetate with other phenol complexes due to steric hindrance and inductive effect. At high concentration of phenol in the mixtures, there is large number of alcohol molecules

Table 2: Values of Dielectric Constants and Relaxation Times for Various Weight Fractions, Ethyl Acetate with Phenol Derivatives in Carbon Tetrachloride

Ratio of Phenol: Acetates	Weight Fraction W2	ε'	ε''	ε_0	ε_∞	Relaxation time(ps) using Higasi's method		
						τ_1	τ_2	τ_0
p-chlorophenol								
1: 3	0.018	2.2518	0.1376	2.6172	2.2365	40.26	47.22	43.52
1: 2	0.020	2.2732	0.1448	2.7316	2.2408	44.29	51.29	46.47
1: 1	0.022	2.3211	0.1673	2.8213	2.2513	49.35	55.23	51.21
2: 1	0.023	2.2718	0.1452	2.7338	2.2384	44.98	51.15	46.54
1: 3	0.025	2.2521	0.1392	2.6212	2.2359	41.08	47.39	43.19
p-iodophenol								
1: 3	0.022	2.2462	0.1214	2.5262	2.2387	44.16	47.23	45.18
1: 2	0.025	2.2618	0.1336	2.6215	2.2472	46.23	48.07	49.06
1: 1	0.027	2.3162	0.1552	2.7452	2.2521	48.36	52.12	52.23
2: 1	0.028	2.2622	0.1338	2.6311	2.2392	46.18	48.18	47.38
1: 3	0.030	2.2478	0.1296	2.5203	2.2378	44.22	47.33	45.29
p-nitrophenol								
1: 3	0.027	2.1974	0.0757	2.2815	2.2418	49.16	50.76	47.26
1: 2	0.029	2.1443	0.1273	2.4613	2.2526	50.23	53.42	51.33
1: 1	0.032	2.1649	0.1348	2.5513	2.2613	53.56	55.18	54.97
2: 1	0.035	2.1468	0.1233	2.4697	2.2526	50.18	53.36	51.46
1: 3	0.037	2.1925	0.0762	2.2937	2.2519	49.23	50.49	47.52

surrounding the ester molecules. The associative alcohol molecules act as proton donors enabling hydrogen bonding with ester molecules. Thus dipole-dipole interaction occurs in such ways that the effective dipole moment gets increased and linear $\alpha-$multimers are formed [23]. The dipole-dipole interaction is the interaction of the –OH group of alcohol with C=O of ester.

At low concentration of alcohol in the mixtures, there are only a small number of alcohol molecules to enable dipole-dipole interaction through hydrogen bonding with the non-associative ester molecules. As a result, intermolecular interaction is weak. The relaxation time increases with increasing alkyl chain length of ester and acidity of proton donor (phenols), indicating that the degree of cooperatively for reorientation of the

Table 3: Values of Dielectric Constants and Relaxation Times for Various Weight Fractions, Butyl Acetate with Phenol Derivatives in Carbon Tetrachloride

Ratio of Phenol: Acetates	Weight Fraction W2	ε'	ε''	ε_0	ε_∞	Relaxation time(ps) using Higasi's method		
						τ_1	τ_2	τ_0
p-chlorophenol								
1: 3	0.021	2.2307	0.0918	2.5172	2.2378	42.14	49.23	45.33
1: 2	0.023	2.2512	0.1276	2.5946	2.2507	45.28	53.35	49.31
1: 1	0.026	2.2766	0.1339	2.7632	2.2623	50.19	57.67	53.12
2: 1	0.027	2.2508	0.1267	2.5918	2.2497	45.32	53.43	49.12
1: 3	0.029	2.2339	0.0926	2.5066	2.2365	42.33	49.36	45.88
p-iodophenol								
1: 3	0.025	2.2112	0.0817	2.4672	2.2393	45.93	49.78	46.12
1: 2	0.027	2.2463	0.1123	2.5719	2.2572	48.16	50.18	49.92
1: 1	0.028	2.2682	0.1276	2.7212	2.2761	50.72	54.22	54.18
2: 1	0.031	2.2413	0.1093	2.5915	2.2586	48.26	50.07	49.56
3: 1	0.035	2.2296	0.0833	2.4816	2.2356	45.12	49.11	46.08
p-nitrophenol								
1: 3	0.030	2.0892	0.0633	2.2451	2.2772	50.83	55.39	49.76
1: 2	0.032	2.1806	0.0721	2.2966	2.3152	51.26	57.92	50.18
1: 1	0.035	2.2243	0.0922	2.4256	2.3378	53.73	59.43	55.31
2: 1	0.037	2.1829	0.0749	2.3034	2.3156	51.09	57.83	50.26
3: 1	0.038	2.0887	0.0686	2.2407	2.2812	50.96	55.42	49.66

molecules increases with increasing length and the bulk of cluster increases.This is due to two effects: (i) the increase of viscosity as chain length increases and (ii) the increase of molecular size as the chain length increases [24-25]. The higher values of relaxation time observed for $p-$nitrophenol with alkyl acetate; suggest that $p-$nitrophenol is more acidic than other phenol derivatives.

The molar free energy of activation for viscous flow $\triangle F_\eta$ and the free energy $\triangle F_\tau$ are calculated for alkyl acetate(methyl acetate, ethyl acetate and butyl acetate)with phenols($p-$chlorophenol, $p-$iodophenol and $p-$nitrophenol) in carbon tetrachloride and presented in Table 4. It is evident from our data that the $\triangle F_\eta$ is $\triangle F_\tau$. This is in agreement with the fact that the process of viscous flow, which involves both the

Table 4: Activation Energies at 1:1 Stoichiometric Ratios of Acetates + Phenol Derivatives Systems in Carbon Tetrachloride

System	Activation energy (KJ mol^{-1})	
	$\triangle F_\tau$	$\triangle F_\tau$
Methylacetate+p-chlorophenol	15.51	17.43
Methylacetate+p-iodophenol	15.68	17.51
Methylacetate+p-nitrophenol	15.81	17.69
Ethylacetate+p-chlorophenol	15.69	17.53
Ethylacetate+p-iodophenol	15.87	17.63
Ethylacetate +p-nitrophenol	15.92	17.73
Butylacetate+p-chlorophenol	15.82	17.62
Butylacetate+p-iodophenol	15.92	17.78
Butylacetate+p-nitrophenol	16.01	17.91

rotational and translational forms of motion, faces greater interference from neighbours than dielectric relaxation, which takes place by rotation only [27]. Smyth et al [28] pointed out that the relaxation time of a proton donor increases as the acceptor ability of the solvent environment increases. Similarly, for a given proton acceptor, the relaxation time must increase with the proton donor ability of the donor solute. Our results are in accordance with this conclusion.

5. Conclusion

The hydrogen bonded complexes of phenol derivatives ($p-$chlorophenol, $p-$iodophenol and $p-$nitrophenol) and acetates (methyl acetate, ethyl acetate and butyl acetate) have been studied in dilute solution of carbon tetrachloride using dielectric method. The dielectric parameters show significant changes with concentration, the alkyl chain length of the esters and the acidities of the phenol derivatives. From these studies, it may be concluded that the alkyl chain length of esters and acidity of phenol derivatives play an important role in the determination of strength of the hydrogen bond formation in phenols and acetates.

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Part B:

HUMANITIES

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A STUDY ON DIGITAL BANKING SERVICES IN INDIAN BANKING SECTOR

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Abstract

The banking & Finance plays essential role in the development of one country's economy. The developing of banking sector depends upon the services provided by them to the customers in various aspects. The growing trend of banking services is found significant after the new economic reforms in India. Today, India has a fairly well developed banking system with different classes of banks – public sector banks, foreign banks, private sector banks – both old and new generation, regional rural banks and co-operative banks with the Reserve Bank of India as the fountain Head of the system. Nowadays banking sector acts as a backbone of Indian economy which reflects as a supporter during the period of boom and recession. Today, we live in the digital era where technology is driving change in almost every industry, whether it is the use of smartphones, automation to improve operations, cloud computing to collaborates, data analysis to extract insight that defines what a new generation bank/banking is and how they can be operated in a sustainable manner not just witnessing profit. But, witnessing existence on a long run for a better tomorrow. This paper enlightens the knowledge light on Technologies banking and its innovative products and channels.

Keywords: Banking Sector, Banking products, Services and Technologies.

1. Introduction

The Indian banking system is unique in the banking history of any country in the world.it is very interesting to studying the evolution of Indian banking, in term of organization, functions, financial matter, socio-economic role, different problems and solutions. Financial innovation in India is key to making growth inclusive by connecting hundreds of millions to the banking system, said panelists at the World Economic Forum's

India Economic Summit. The deregulation of financial service industry and increased competition with in investment banking undoubtedly led to increased emphasis on the ability to design new products, develop better process, and implement more effective solution for increasingly complex financial problems. These financial innovations are a result of number of Government regulations, tax policies, globalization, liberalization, privatization, integration with the international financial market and increasing risk in the domestic financial market. Financial innovation is the process through which finance managers or intermediary institutions in financial markets add value to existing plain vanilla products that satisfy the user needs. According to John Finnerty, "Financial Innovation involves the design, the development, and the implementation of innovative financial instruments and processes, and the formulation of creative solutions to problems in finance". The various innovations in banking and financial sector are ECS, RTGS, EFT, NEFT, ATM, Retail Banking, Debit & Credit cards, free advisory services, implementation of standing instructions of customers, payments of utility bills, fund transfers, internet banking, telephone banking, mobile banking, selling insurance products, issue of free cheque books, travel cheques and many more value added services.

Today, we are having a fairly well developed banking system with different classes of banks – public sector banks, foreign banks, private sector banks, regional rural banks and cooperative banks. The Reserve Bank of India (RBI) is at the paramount of all the banks. The RBI's most important goal is to maintain monetary stability (moderate and stable inflation) in India. The RBI uses monetary policy to maintain price stability and an adequate flow of credit. The rates used by RBI to achieve this are the bank rate, repo rate, reverse repo rate and the cash reserve ratio. Reducing inflation has been one of the most important goals for some time.

2. Review of Literature

Financial innovations lower cost of capital, reduce financial risks, improve financial intermediation, and hence welfare enhancing. The primary function of financial system is to facilitate the allocation and deployment of economic resources in an uncertain environment (Merton, 1992). E-banking is an innovation when new information technologies merge into traditional banking services. Operating costs minimization and revenue maximization are the major drivers that boost e-banking services (Sannes, 2001; Reibstein, 2002). E-banking service is basically a self-service by customers, so for banks, it requires less resources and lower transaction and production costs (Southard and Siau, 2004; Witman and Poust, 2008). A study about the e-banking over 1999–2006 shows that the application of e-banking can improve banks' performance in terms of the growth in assets, reduction in operating expenses and portfolio enhancement (Dandapani et al., 2008). Even in 1990s, Sraeel (1996) emphasises that creating virtual banking will not only create a new service delivery channel, but also lead to value creation to both banks and customers (Hwang et al., 2007; Murphy, 2007).

Financial innovation is helpful in ensuring smooth functioning and improves the overall efficiency of the system by minimizing cost and reducing risk. More generally, financial innovation has been a central force driving the financial system toward greater economic efficiency (Merton and Bodie 2005).Avasthi & Sharma (2000-01) have analyzed in their study that advances in technology are set to change the face of banking business. Technology has transformed the delivery channels by banks in retail banking. It has also impacted the markets of banks. The study also explored the challenges that banking industry and its regulator face. B. Janki (2002) analyzed that how technology is affecting the employees' productivity. There is no doubt, in India particularly public sector banks will need to use technology to improve operating efficiency and customer services. The focus on technology will increase like never before to add value to customer services, develop new products, strengthen risk management etc. the study concludes that technology is the only tool to achieve their goal Padhy, K.C. (2007) studied the impact of technology development in the banking system and he also highlights the future of banking sector. The core competencies will provide comparative advantages.

3. Objectives of the Research

- To examine the new generation banking sector and its products/services.
- To present the technological developments in Indian banking sector.
- To study the emerging trends in banking technology.
- Provide Customer/Members with a Better Branch Experience.
- Provide Customer/Members with Additional Choices on How to Engage with the Branch.

4. Research Methodology

This research is based on the analysis of the secondary data and the research proposes to throw light on the banking services of Technologies of banking in the Indian Banking Sector.

5. Recent Trends and Development in Banking Sector

Today, we are having a fairly well developed banking system with different classes of banks – public sector banks, foreign banks, private sector banks, regional rural banks and cooperative banks. The Reserve Bank of India (RBI) is at the paramount of all the banks.

The RBI's most important goal is to maintain monetary stability (moderate and stable inflation) in India. The RBI uses monetary policy to maintain price stability and an adequate flow of credit. The rates used by RBI to achieve the bank rate, repo

rate, reverse repo rate and the cash reserve ratio. Reducing inflation has been one of the most important goals for some time. Growth and diversification in banking sector has transcended limits all over the world. In 1991, the Government opened the doors for foreign banks to start their operations in India and provide their wide range of facilities, thereby providing a strong competition to the domestic banks, and helping the customers in availing the best of the services. The Reserve Bank in its bid to move towards the best international banking practices will further sharpen the prudential norms and strengthen its supervisor mechanism. There has been considerable innovation and diversification in the business of major commercial banks. Some of them have engaged in the areas of consumer credit, credit cards, merchant banking, internet and phone banking, leasing, mutual funds etc. A few banks have already set up subsidiaries for merchant banking, leasing and mutual funds and many more are in the process of doing so. Some banks have commenced factoring business.

6. New Generation or Technologies Banks

"New generation banks are not just banks who are involved in the implementing a new strategy for the sake of survival. But, banks who are involved in the process of creating a paradigm shift to overcome the ever-changing market requirements and customer preferences by the way they organize the internal and external activities, and initiatives by considering traditional human values and using modern technology. That may result in creating larger revenues by properly investing and managing the funds to create optimum profit and goodwill for the long run of the business can be considered and proved as sustainable". Similarly, ages pass on and so does time, thus organizations who are involved in creating change and surviving the change by implementing innovative and effective strategies to serve the future generations to come can be considered so. Thus, In this process the bank that excels with its innovative strategy is to be considered as a new generation bank as the those strategies used to exhibit customer service and welfare is just a marketing strategy which brings in customers but on a long run its only the internal affairs and money management strategy that helps a business retain its position in the market.

7. Development in New Generation Banks

Electronic Banking

E-banking is a blanket term used to indicate a process through which a customer is allowed to carry out, personal or commercial banking transactions using electronic and telecommunication network. E-banking .E-banking is a blanket term used to indicate a process through which a customer is allowed to carry out, personal or commercial banking transactions using electronic and telecommunication network. It is a product offered by banks which facilitates online banking, with the help of which the customer can have access to the bank account in just one click. E-banking covers facilities such

as – fund transfer, checking account statements, utility bill payments, opening of bank account, locating nearest ATM, obtain information on financial products and services, applying for loans, etc. using a personal computer, smart phone, laptop or personal digital assistant.

Electronic Payment Services - E Cheques

Nowadays we are hearing about e-governance, e-mail, e-commerce, e-tail etc. In the same manner, a new technology is being developed in US for introduction of e-cheque, which will eventually replace the conventional paper cheque. India, as harbinger to the introduction of echeque, the Negotiable Instruments Act has already been amended to include; Truncated cheque and E-cheque instruments.

Real Time Gross Settlement (RTGS)

Real Time Gross Settlement system, introduced in India since March 2004, is a system through which electronics instructions can be given by banks to transfer funds from their account to the account of another bank. The RTGS system is maintained and operated by the RBI and provides a means of efficient and faster funds transfer among banks facilitating their financial operations. As the name suggests, funds transfer between banks takes place on a 'Real Time' basis. Therefore, money can reach the beneficiary instantaneously and the beneficiary's bank has the responsibility to credit the beneficiary's account within two hours.

National Electronic Funds Transfer (NEFT)

The transfer of money from the customer remitting it to the beneficiary account usually takes place on the same day. Settlement or clearance of funds takes place in batches as specified by the guidelines by the RBI. Any amount of money can be transferred using NEFT, making it usually the best method for retail remittances. Customers with Internet banking accounts can use the NEFT facility to transfer funds nationwide on their own. Funds can also be transferred via NEFT by customers by walking into any bank branch (which is NEFT-enabled) and leaving relevant instructions for such transfer - either from their bank accounts or by payment of cash. Transfer of funds to Nepal using NEFT, is also allowed subject to limits.

Electronic Funds Transfer (EFT)

Electronic Funds Transfer (EFT) is a system whereby anyone who wants to make payment to another person/company etc. can approach his bank and make cash payment or give instructions/authorization to transfer funds directly from his own account to the bank account of the receiver/beneficiary. Complete details such as the receiver's name, bank account number, account type (savings or current account), bank name, city, branch name etc. should be furnished to the bank at the time of requesting for such

transfers so that the amount reaches the beneficiaries' account correctly and faster. RBI is the service provider of EFT.

Electronic Clearing Service (ECS)

Electronic Clearing Service is a retail payment system that can be used to make bulk payments/receipts of a similar nature especially where each individual payment is of a repetitive nature and of relatively smaller amount. This facility is meant for companies and government departments to make/receive large volumes of payments rather than for funds transfers by individuals.

Automatic Teller Machine (ATM)

Automatic Teller Machine is the most popular devise in India, which enables the customers to withdraw their money 24 hours a day 7 days a week. It is a devise that allows customer who has an ATM card to perform routine banking transactions without interacting with a human teller. In addition to cash withdrawal, ATMs can be used for payment of utility bills,funds transfer between accounts, deposit of cheques and cash into accounts, balance enquiry etc.

Internet Banking

Internet banking enables a customer to do banking transactions through the bank's website on the Internet. It is a system of accessing accounts and general information on bank products and services through a computer while sitting in its office or home. This is also called virtual banking. It is more or less bringing the bank to your computer. In traditional banking one has to approach the branch in person, to withdraw cash or deposit a cheque or request a statement of accounts etc. but internet banking has changed the way of banking. Now everyone can operate all these type of transactions on his computer through website of bank. All such transactions are encrypted; using sophisticated multi- layered security architecture, including firewalls and filters. One can be rest assured that one's transactions are secure and confidential.

Mobile Banking

Mobile banking facility is an extension of internet banking. With recent developments in handset designs and mobile software, this is a trend which has already caught focus of majority of the banks. The bank is in association with the cellular service providers offers this service. For this service, mobile phone should either be SMS or WAP enabled. These facilities are available even to those customers with only credit card accounts with the bank.

Electronic Data Interchange (EDI)

Electronic Data Interchange is the electronic exchange of business documents like purchase order, invoices, shipping notices, receiving advices etc. in a standard, computer processed, universally accepted format between trading partners. EDI can also be used to transmit financial information and payments in electronic form.

Challenges Faced by Banks, vis-à-vis, IT Implementation

It is becoming increasingly imperative for banks to assess and ascertain the benefits of technology implementation. The fruits of technology will certainly taste a lot sweeter when the returns can be measured in absolute terms but it needs precautions and the safety nets. The increasing use of technology in banks has also brought up 'security' concerns. To avoid any mishaps on this account, banks ought to have in place a well-documented security policy including network security and internal security. The passing of the Information Technology Act has come as a boon to the banking sector, and banks should now ensure to abide strictly by its covenants. An effort should also be made to cover e-business in the country's consumer laws. Some are investing in it to drive the business growth, while others are having no option but to invest, to stay in business. The choice of right channel, justification of IT investment on ROI, e-governance, customer relationship management, security concerns, technological obsolescence, mergers and acquisitions, penetration of IT in rural areas, and outsourcing of IT operations are the major challenges and issues in the use of IT in banking operations.

Future banking

Everyone today is convinced that the technology is going to hold the key to future of banking. The achievements in the banking today would not have make possible without IT revolution. Therefore, the key point is while changing to the current environment the banks has to understand properly the trigger for change and accordingly find out the suitable departure point for the change.

8. Conclusion

In the future time it is expected that banks will grow at a tremendous growth rate . through the innovation and IT system used in banking field, banks are able to better risk management system, adoption of internationally accepted accounted practices and increased disclosure and transparency. These reforms which has taken place in different phases further strengthen the system.To conclude it all, the banking sector in India is progressing with the increased growth in customer base, due to the newly improved and innovative facilities offered by banks. The economic growth of the country is an indicator for the growth of the banking sector. The onus for this lies in the capabilities

of the Reserve Bank of India as an able central regulatory authority, whose policies have shielded Indian banks from excessive leveraging and making high risk investments. By the government support and a careful re-evaluation of existing business strategies can set the stage for Indian banks to become bigger and stronger, thereby setting the stage for expansions into a global consumer base. Although this article gives a view on what new age banking and its trends what their customers expect the banks of tomorrow to look like and what they should important on, the right decision for adoption may vary from bank to bank.

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CONSUMER PURCHASE BEHAVIOUR ON WHITE GOODS – A RESEARCH INVESTIGATION

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Abstract

Consumer Purchase Behaviour issues a signal to find ways and avenues of giving away the commodities according to the tastes and preferences. Behaviour of an individual is greatly influenced by numerous factors such family, friends, income, price, quality etc. therefore, it is the duty of the marketers to identify the factors and provide the consumers with much sought after products through the best possible means. In this study, Primary Data has been collected through circulating well structured questionnaire among 200 respondents who are available in and around Pernambut Taluk while Secondary Data have been extracted from Well Reputed Journals, Weekly Magazines, Research Dissertation, Theses, Books and Websites. Deliberate Sampling Method was adopted. Empirical Research was put to use throughout the research work.

Keywords: Consumer, Behaviour, Emblem and Brand.

1. Introduction

Consumer behaviour is much more than buying things; It additionally embraces and examine approximately how certain (or now no longer having) matters influences our lives, and the way our possessions impact the manner we experience approximately ourselves and approximately every other our kingdom of being. In addition to knowledge why human beings purchase matters, we additionally try and admire how products, offerings and intake sports make a contribution to the wider social global we experience. Whether shopping, cooking, cleaning, gambling soccer or hockey, mendacity at the beach, emailing or texting friends, or maybe searching at ourselves within side the mirror, our lives are touched through the advertising and marketing system. The area of purchaser behaviour is young, dynamic and in flux. It is continuously being cross-fertilized through views from many unique disciplines. We have attempted to specific the area's spectacular variety on this text.

2. Consumer Purchase Behaviour

Consumer purchase behaviour is the study of the ways of buying and disposing of goods, services, ideas or experiences by the individuals, groups and organizations in order to satisfy their needs and wants. Buyer behaviour has been defined as "a process, which through inputs and their use though process and actions leads to satisfaction of needs and wants". Consumer buying behaviour has numerous factors as a part of it which are believed to have some level of effect on the purchasing decisions of the customers. From marketers' point of view issues specific aspects of consumer behaviour that need to be studied include the reasons behind consumers making purchases, specific factors influencing the patterns of consumer purchases, analysis of changing factors within the society and others.

3. Objective of the Study

To investigate various factors influencing Consumer Buying Behaviour on white goods and explore the competition of same products.

4. Statement of the Problem

There are many issues with respect to Consumer Buying Behaviour while buying any type of product. Consumer Buying Behaviour really matters when consumer goes for purchases of products. Sellers do not sell the commodities according to the whims and fancies of consumers rather dispose of the products according to their specifications resulting in consumer dissatisfaction. Channel of Distribution also is sometimes wrongly selected by the sellers and those platforms do not convey the message about the product properly to the consumers. Dealers are not able to fix the prices being demanded by consumers. They sell the commodities over and above the organized prices which really hampers the consumer buying behavior. Sometimes introduction of new products also deters consumer from executing the purchases of preferable products and this prevents consumers from choosing branded products thus consumer buying behavior is greatly influenced by availability of new products.

5. Significance of the Study

Consumers get the products at very cheaper prices as competition emerges everywhere across the country. They Sell the Products based on overcoming unhealthy competition. Even organizations get fabulous opportunities thereby exhibiting their potentials in exhibiting their products. They fairly use research techniques and supply the products some new and unique. As years pass by, consumers are in dire need of change. Therefore, products are manufactured in such a way that consumer's satisfaction increases consequently. Consumers need not depend on foreign made goods since home made goods really fulfil the need of products. They get anything and everything without from local market itself. The reason behind this is producers rightly understand the consumers Buying Behaviour.

6. Review of Literature

Pandey, et al. (2013) made a mention that blends factors from psychology, sociology, anthropology and economics. It tries to comprehend the purchaser deciding process, both personally and in agencies. It studies characteristics of person consumers like demographics and behavioral variables in a shot to recognize purchaser wishes. A logo will be a symbolic embodiment of all of the expertise related to the products & serves to make institutions and expectations around it. Many emblem orientated businesses markets say "brand is that the name, term, signal, image or a mix of them meant to identify the dreams or carrier of 1 dealer or institution of dealers to distinguish them from those competition". ordinary buying behaviour famous little perceived difference between the possible manufacturers and coffee consumer involvement. This can be a product that customers buy frequently and that they may commonly buy a selected brand out of addiction. Customers don't significantly search for records because they purchase supported emblem recognition. Entrepreneurs of recurring products generally use decrease charges and sales to stimulate the habit of searching for their specific product. The essential reason of this look at is to spot connection among manufacturers and shopping for behaviour of customers and what parameter of logo name affects clients the foremost. This research is useful to marketers to formulate strategies and to watch new brands available in market. To perform objectives of this studies we've were given used various factors of name which can have an impact on ordinary shopping for behaviour. Numerous parameters of manufacturers which could have an effect on recurring buying behaviour of purchasers are: the variable quantity is logo shopping for behaviour and impartial variables are branding information search, rate of emblem name, first-class of call, logo availability and shops choice. For the analysis techniques used are issue and multivariate evaluation. The advertising and marketing techniques are designed according with the end result.

Raju (2020) enlightened about Consumer behaviour is defined because the manner of selecting, buying, the utilization of and disposing goods and services through humans or establishments to fulfil their desires. organizations need to understand the numerous factors that have a touching on purchasers to create purchases those factors could also be reference businesses, cultural factors, and economic elements and alike, additionally they need to understand the purchase system that is going into making purchase choices with the assist of customers with a view to obtain both rural and concrete markets. The rural markets provide a giant choice of opportunities way to their incredible length and modifications that have taken vicinity over the years. A vital quit could also be drawn from the research is that the buying behaviour of consumers is important for agencies. Whilst it's also seen that rural and concrete customers need to be segmented nicely so on derive sales and make the most every the markets. This research paper offers an overview of the studies.

Parthasarathy et al. (2021) made an investigation was administered to search out the effectiveness of the buying behaviour and get motivation of female buyers towards selected durables in Erode district of state state, India. Independent variables like price, sales place, product attributes, commercial and social influence are taken as driving variables, for purchasing attitude and buying motivation for the feminine consumers. Findings from this study revealed that there's a powerful influence between customers' attitudes and buy motivation towards female shopping behaviour of consumer goods within the study area. White goods taken in the study to measure the consumer purchase behavior include washing Machines and Refrigerators.

7. Research Question

Do various factors influence the Consumer's Purchase Behaviour?

8. Research Hypothesis

There exists significant relationship between factors influencing Consumer Purchase Behaviour.

9. Methodology

Research Methodology is an art that facilitates the research work in an organized manner. Investigator used both primary sources and secondary sources of data. Primary Data has been collected through circulating well structured questionnaire among 200 respondents who are available in and around Pernambut Taluk while Secondary Data have been extracted from Well Reputed Journals, Weekly Magazines, Research Dissertation, Theses, Books and Websites. Deliberate Sampling Method was adopted. Empirical Research was put to use throughout the research work.

10. Statistical Processing of Data

10.1. One Sample t Test is performed to find differences among five variables of Factors influencing Consumer Buying Behaviour

One Sample t Test is performed to ascertain whether or not significant differences are seen among five variables of Factors influencing Consumer Buying Behaviour such as Competition determines the position in the market, Consumers expect quality of service after the purchase, Price Tag determines the behavior of consumers, Service Quality and Customer Relationship Management, Bonus and Gift Voucher keep the consumers being loyal. Test Values is taken as 3 with which, mean score is compared thereby mean difference have been obtained. The following hypothesis is formulated and validity also is tested with the help of one sample t Test.

There exists significant difference between mean responses of Factors influencing Consumer Buying Behaviour.

Table 1: One Sample t Test is performed to find difference among five variables of Factors influencing Consumer Buying Behaviour

	Test Value =3					
	t	**df**	**Sig. (2-tailed)**	**Mean Difference**	**95 % Confidence Interval of the Difference**	
					Lower	**Upper**
Competition determines the position in the market	14.652	199	.000**	1.255	1.09	1.42
Consumers expect quality of service after the purchase	37.323	199	.000**	1.050	.99	1.11
Price Tag determines the behavior of consumers	9.115	199	.000 * *	.470	.37	.57
Service Quality and Customer Relationship Management	16.962	199	.000 * *	.880	.78	.98
Bonus, Gift Voucher keep the consumers being loyal	−1.446	199	.150	−.170	−.40	.06

****significant at 5% level**

Interpretation

Five variables of Factors influencing Consumer Buying Behaviour are statistically tested in order to come up with inference. The five variables regarding Factors influencing

Consumer Buying Behaviour include Competition determines the position in the market, Consumers expect quality of service after the purchase, Price Tag determines the behavior of consumers, Service Quality and Customer Relationship Management, Bonus and Gift Voucher keep the consumers being loyal. P values (significance 2-tailed) in case of four variables are less than critical alpha value (0.05). It indicates that there are highly significant differences among four variables of Factors influencing Consumer Buying Behaviour. The formulated hypotheses i.e. "there exists significant difference between mean responses of four variables of Factors influencing Consumer Buying Behaviour and Test Value of Factors influencing Consumer Buying Behaviour" is accepted. However, one variable in relation to Factors influencing Consumer Buying Behaviour i.e. Bonus, Gift Voucher keep the consumers being loyal whose p value is greater than critical alpha value (0.05). Therefore formulated hypothesis i.e. Bonus, Gift Voucher keep the consumers being loyal are significantly different with test value is rejected.

10.2. Confirmatory Factor Analysis is performed to assess the relationship between group of observed variables and latent construct of Factors influencing Consumer Buying Behaviour

In order to assess the relationship between groups of observed variables i.e. Competition determines the position in the market, Consumers expect quality of service after the purchase, Price Tag determines the behavior of consumers, Service Quality and Customer Relationship Management and Bonus and Gift Voucher keep the consumers being loyal and Unobserved variable i.e. Factors influencing Consumer Buying Behaviour. Confirmatory Factor Analysis was performed and the following hypothesis is framed:

There is close relationship between groups of observed variables i.e. Competition determines the position in the market, Consumers expect quality of service after the purchase, Price Tag determines the behavior of consumers, Service Quality and Customer Relationship Management, Bonus and Gift Voucher keep the consumers being loyal and latent construct i.e. Factors influencing Consumer Buying Behaviour.

Interpretation

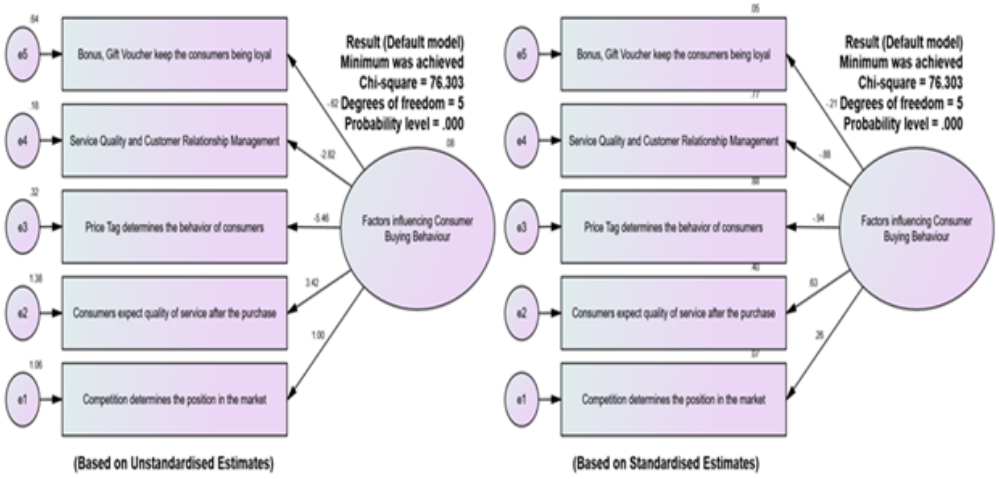
From the above table taken up to assess the relationship between observed constructs of Social Media Advertisement on Consumer Buying Behaviour such as Competition determines the position in the market, Consumers expect quality of service after the purchase, Price Tag determines the behavior of consumers, Service Quality and Customer Relationship Management, Bonus and Gift Voucher keep the consumers being loyal and latent construct i.e. Factors influencing Consumer Buying Behaviour, Confirmatory Factor Analysis has been performed. Competition determines the position in the market is of intercept and its factor loading is fixed to be 1. This indicates

Table 2: Confirmatory Factor Analysis with respect to Factors influencing Consumer Buying Behaviour

(Based on Regression Estimates and Factor Score)

Observed Constructs		**Latent Construct**	**Regression Estimates**	**Standardized Regression Weights**	**Factor Score**	**P**
Competition determines the position in the market	←	Factors influencing Consumer Buying Behaviour	1.000	.260	−.006	
Consumers expect quality of service after the purchase			3.424	.629	−.097	∗ ∗ ∗
Price Tag determines the behavior of consumers			−5.464	−.938	−.107	∗ ∗ ∗
Service Quality and Customer Relationship Management			−2.823	−.880	.015	∗ ∗ ∗
Bonus, Gift Voucher keep the consumers being loyal			−.623	−.212	.006	.022

*****significant at .0001 level**



that the scales of the scores on the latent variable are some multiple of the selected observed variables. Since p values in three cases such as Service Quality and Customer Relationship Management and Bonus and Gift Voucher keep the consumers being loyal other than incept (Competition determines the position in the market) is less than .0001, result is significant and formulated hypothesis "observed variables i.e. Service Quality and Customer Relationship Management and Bonus and Gift Voucher keep the consumers being loyal and latent construct i.e. Factors influencing Consumer Buying Behaviour r" is accepted. However, "the third observed construct i.e. Price Tag determines the behavior of consumers is not closely correlated with latent construct i.e. Factors influencing Consumer Buying Behaviour as p value is less than .0001. The result is not significant and formulated hypothesis i.e Price Tag determines the behavior of consumers and Factors influencing Consumer Buying Behaviour is closely correlated" is rejected at .0001% level.

11. Results Discussion

The five variables regarding Factors influencing Consumer Buying Behaviour include Competition determines the position in the market, Consumers expect quality of service after the purchase, Price Tag determines the behavior of consumers, Service Quality and Customer Relationship Management, Bonus and Gift Voucher keep the consumers being loyal. P values (significance 2-tailed) in case of four variables are less than critical alpha value (0.05). It indicates that there are highly significant differences among four variables of Factors influencing Consumer Buying Behaviour. The formulated hypotheses i.e. "there exists significant difference between mean responses of four variables of Factors influencing Consumer Buying Behaviour and Test Value of Factors influencing Consumer Buying Behaviour" is accepted. However, one variable in relation to Factors influencing Consumer Buying Behaviour i.e. Bonus, Gift Voucher

keep the consumers being loyal whose p value is greater than critical alpha value (0.05). Therefore formulated hypothesis i.e. Bonus, Gift Voucher keep the consumers being loyal are significantly different with test value is rejected.

Factors influencing Consumer Buying Behaviour, Confirmatory Factor Analysis has been performed. Competition determines the position in the market is of intercept and its factor loading is fixed to be 1. This indicates that the scales of the scores on the latent variable are some multiple of the selected observed variables. Since p values in three cases such as Service Quality and Customer Relationship Management and Bonus and Gift Voucher keep the consumers being loyal other than incept (Competition determines the position in the market) is less than .0001, result is significant and formulated hypothesis "observed variables i.e. Service Quality and Customer Relationship Management and Bonus and Gift Voucher keep the consumers being loyal and latent construct i.e. Factors influencing Consumer Buying Behaviour r" is accepted. However, "the third observed construct i.e. Price Tag determines the behavior of consumers is not closely correlated with latent construct i.e. Factors influencing Consumer Buying Behaviour as p value is less than .0001. The result is not significant and formulated hypothesis i.e Price Tag determines the behavior of consumers and Factors influencing Consumer Buying Behaviour is closely correlated" is rejected at .0001% level.

12. Suggestions

Purchase Behaviour does not remain the same among the consumers due to quality, price, and appearance. Most of the consumers are sticky to their purchase behavior of white goods such as Washing Machine and Refrigerator due to the presence of certain brands in the market. Consumer behavior determines the marketing strategies of the companies and industry which manufacture the white goods. If the consumer's satisfaction does not change, they will keep on buying the same brand for a long period of time. Company should thoroughly investigate the consumer behavior thereby offer the products according to their tastes and preferences.

Products should be designed in such a way that consumer's needs are met. Marketers must think of the hefty competition in the market and offer the products based upon the specification of consumers. Price also influences the consumer purchase behaivour. Therefore, businessman should sell the products at competitive prices and also allow the discounts whenever required to keep them retained. Survival of any business relies on how well we study about the consumer's buying behavior. if any added features are made, the company should inform the same to the consumers. Similarly, people of dealing with white goods should promote the sales through various media which communicate the message of your products to the consumers.

13. Conclusion

This study focuses on the consumer purchase behavior on white goods. Here white goods refer to Washing Machine and Refrigerator. Since these goods are predominantly used by the households, people wantonly execute the purchase of branded white goods. Social Media Advertisements also should be made to pass on the viable information to the consumers. Consumer purchase behavior should be measured from time to time so that producers and businessmen can retain the consumers and protect the interest of the consumers. They can create the good rapport with consumers. In the competitive world, incasing the consumer's loyalty is at stake. Businessmen should devise the effective strategies for keeping the consumers trust intact.

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A STUDY ON THE POPULARITY OF VARIOUS UTILITIES IN E-PAYMENT MECHANISMS AMONG COLLEGE STUDENTS OF TIRUPATTUR DISTRICT

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Abstract

A web-based payment system framework have progressively famous because of the boundless utilization of web-based shopping and banking. The current study aims to study the popularity of various utilization in E-payment mechanism among college of Tirupattur district with exploratory and descriptive research approach. Primary data was collected from the 100 respondents e-payment users in Tirupattur district and non-probability sampling was used. Frequency distribution, Bar diagram, Pie chart, and Chi-square was used for analysis. This research work describe the perception of the e-payment users, effectivenss of e-payment services, and barrier for not prefer e-payment over traditional payment method.

Keywords: E-payment Utilities, e-Payment Mechanism, College Students, e-Payment Users.

1. Introduction

The National Payments Corporation of India (NPCI), set up as a drive-by the Reserve Bank of India and the Indian Banks' Association, gives the fundamental system to a couple of portion structures. Among those organizations are the especially used Unified Payments Interface (UPI), which was maintained by more than 200 banks, similarly as the card plot RuPay as the local choice rather than Mastercard, Visa, and UnionPay. The latter was also fundamental for the forceful Pradhan Mantri Jan Dhan Yojana (PMJDY) that objectives introducing Digital payments and financial organizations to broad spaces of society.

Throughout the long term, Debit cards have become the most widely recognized type of E-payment for a web-based business transaction. Practically 60% of online

exchanges were made with this payment type. It would be hard for an online retailer to work without supporting credit and debit cards because of their inevitable use. Different kind of safety efforts is taken on to get online exchanges, which incorporates the utilization of the card verification number (CVN) which recognizes misrepresentation by looking at the confirmation number imprinted on the signature on the rear of the card with the data on a document with the cardholder's providing bank. Likewise, online traders need to follow severe standards specified by the credit and debit card service providers. Which implies that traders should have security convention and methodology set up to guarantee transactions are safer. This can likewise incorporate having an authentication from an approved certification authority (CA) who gives PKI (public-key framework) for getting credit and debit card transactions.

An enormous number of nations, for example, India and China have a few issues to overcome concerning Visa security, meanwhile; the utilization of smartcards has gotten amazingly mainstream. A Smartcard is like a Visa; however, it contains an implanted 8-bit chip and uses electronic money with moves from the buyer's card to the venders' gadget. A mainstream smartcard drive is the VISA Smartcard. Utilizing the VISA Smartcard, you can transfer electronic money to your card from your account, and you would then be able to utilize your card at different retailers and on the web.

2. Research Problem

A research problem is a bunch of inquiries unanswered that turns into the main theme of the investigation. In a modern generation of the electronic world, E-payment has made a sustainable impact on people. Every country has adopted this E-payment system as a convenient method for payment. It has changed the payment habit of people in their day-to-day activities. However, due to habit and trust many people still not preferring E-payment over traditional payment because of security threats and lack of knowledge to use E-payment mechanisms. A present younger generation are trend changers, they will show the path to others. They are now largely using smart gadgets for making online payments and showing others how to use the E-payment mechanism without losing privacy. This study is about to know the college student perception towards utilization of E-payment system.

3. Significance of the Research Study

E-payment mechanism is almost replaced over traditional methods of payment in India because of its wide usage in many areas like shopping, movie ticket booking, tax payment, mobile recharging, bill payment, etc. However, it is hard to fasten the growth of E-payment in rural areas and towns where Cash-on delivery for the online purchase is the most preferred method of payment. Moreover, it is necessary to understand

the perception that influencing the adoption of E-payment along with preferring E-payment over traditional payment methods among college students which will enable improvement in the E-Payment services so that others can easily use the E-payment mechanism without any restrictions.

4. Research Objectives

Describing objectives is a significant part of any investigation. The prime target of my research work is shown below:

1. To study the demographic factors of the college student using E-payment.

2. To know the perception of the students towards the E-payment system.

3. To study the effectiveness of E-payment services.

4. To study the reason for preferring E-payment over traditional payment methods.

5. Important Definitions

5.1. E-payment Mechanism

E-payment Mechanism also known as an electronic payment system which offers the users to make the funds transfer electronically it is a subset of Electronic Data Interchange (EDI) in which businesses communicate and it facilitates the exchange of information between parties, information includes bills, invoices, payment documents, shipping details, etc.

This payment system facilitates the users to make the payment instantly, however, it is convenient to use and time-saving. From the past decades, this payment mechanism has had enormous growth and plays a vital role in everyone's lives due to using of the internet, web-based business, banking activities, and shopping.

5.2. Student

In this research study students refers to the students studying in art and science colleges, IT's Engineering & Polytechnic college using E-payment services in Tirupattur District.

5.3. Utilization

Utilization refers to the usage behaviour of the E-payment users.

5.4. Popularity

Popularity refers to the most using E-payment services like mobile wallets, net banking, etc, among college students.

5.5. Mechanism

The mechanism is a systematic process in which transactions are done electronically especially using a mechanical device.

6. Research Methodology

Research methodology explains the methods by which the research study may proceed. The method used in this study is presented below in the sub-para.

7. Research Instrument – Questionnaire

The questionnaire was designed after studying several research instruments used in previous research work on E-payment utilization. The primary objective was to make it in a way that seeks direct and clear information from the users and also accomplishes the objectives of the research project. As the questionnaire is self-administered, most care was taken to use lucid words.

The questionnaire was divided into four Sections – Section 1, 2, 3, and 4. Section – 1 includes the data regarding the demographic profile of the respondents. There were five items: Gender, Age, Educational Qualification, Usage Period, Frequency of Use, and Name of the E-payment service used. Likewise, sections 2,3 & 4 are about perception, effectiveness, and preference of E-payment respectively, a total of 21 questions is made and Google form is used to collect data from the users.

7.1. Research Design

This current Research study aims to study the popularity of various utilization in E-payment mechanism among college of Tirupattur district with exploratory and descriptive research approach.

7.2. Sampling Design

Sample refers to selecting a fixed number population unit that represents the entire population. Sampling is the methodology to draw a sample from a population. Sampling Design is a planned sequence of the entire process involved in selecting samples, it includes defining the type of population, Sampling Unit, Sampling Frame, Size of the sample, and parameters of study interest, and procedure of sample collection.

7.3. Sample Size

In this current study, sampling size was limited to 100 and non-probability sampling was used.

7.4. Target Population

In this research study, the target population is the college students E-payment users of Tirupattur district, Tamil Nadu, India. The necessity for selecting this specific population was to study the 'popularity of various utilities of e-payment mechanisms among college students which would enable the E-Payment companies in designing their products and marketing campaigns. The need for the study is elaborated in the respective section.

7.5. Sampling Location

The location of the study was Tirupattur district – a district in the state of Tamil Nadu, a southern state of India. It has a population of 12.79 Lakh (1279953) and has 4 taluks Area total of 1831.99 sq. km: Tirupattur, Jolarpet, Vaniyambadi, and Ambur and 3 Panchayat Alangayam, Natrampalli, and Udayendiram. It was announced in August 2019 and established in November 2019 as a district. (Tirupatturdistrict.in).

7.6. Sampling Techniques

For the current study, a non-probability sampling technique – Snowball sampling is also known as Chain-referral sampling was used to collect the data from the respondents.

8. Review of Literature

Michael E. Ellis (2021) Studied the Digital Divide of Perceptions, Usage, and Purchase Items in Japanese E-Payment Adoption. survey on 151 Japanese to examine the effect of digital divide characteristics such as regional variations, age, and gender on the perception and behaviour surrounding e-payments. Roughly 85% of respondents use e-payments two or fewer times per week for an average monthly expenditure equivalent to less than US$200 that is 20 percent of their monthly expenditures were via e-payment. Analysis was done using ANOVA tools in SPSS plus Duncan's Multiple Range Test (MRT). The result was showed that Japan remains a very cash-oriented society. for this culturally homogeneous and highly developed economy, regional divides are relatively low influencers of attitudes and behaviours related to e-payment. There is a difference in use between users in rural and urban areas, but it is smaller than other differences found. Age and gender, however, are still found to predict a larger gap among Japanese in e-payment adoption.

Dr. Rajesh P Ganatra et. al (2020) studied the usage of E-banking payment

systems and the perspective of E-customer concerning E-banking payment systems in Ahmedabad City. The survey was derived from 372 respondents and for research, a non-probability sampling method was used. Research data provided the 10 different perspectives of E-customer concerning E-banking payment systems such as Cost/Price factor, customer accessibility, Ease of Use, Customers reluctance to Change, Customer awareness, security concern, poor contact facility, poor problem handling service, complicated service, and poor system availability. From these perceptions first four were highly scored (Cost/Price factor, Customer accessibility, Ease of Use, and Security). He concludes that customers are more use E-banking payment system and they satisfied with E-banking payment system and majority people not aware of this payment system.

Ansari, Thameemul & Srinivasan, Dr. (2019) elabourated in their paper that E-banking is the system that provides the facility to the customer to conduct the financial and non-financial transactions from his net banking account. The user can transfer funds from his account to other accounts of the same bank/different bank using a website or an online application. The customer uses a resource and a medium to conduct financial transactions. The resource that a customer uses might be an electronic device like a computer, a laptop, or a mobile phone. The internet is the medium that makes the technology possible. The facility of internet banking is provided through banks and the customer must be an account holder with any bank to get the facility available for him/her. Key Words: E-banking, DTH, AMOS, GFI, AGFI, RMSEA and PL

Dr.K.Vinitha1 et. al (2020) Assistant Professor of Prince Shri Venkateshwara Arts & Science College, Chennai they studied Determinants of Customer intention to use a Digital payment system by using an online and direct survey-based questionnaire was prepared and circulated to 340 respondents, from it 323 responses were fit for statistical analysis. A research framework was developed; Intention to use as the dependent variable and Perceived Benefits, Perceived credibility, and Perceived Enjoyment as dependent variables. Analysis was done using Regression models with one endogenous variable and more than one exogenous variable are called multilinear regression data analyzed with multiple regression and ANOVA. The result was found that the dependent variable has positive influence a degree of R(0.806) and R2(0.650) splendidly from independent variables perceived Benefits made the biggest contribution with the value of (β=.304); which is followed by Perceived credibility (β=.278) and Perceived enjoyment(β=.252) positively.

9. Data Analysis and Interpretation

The process of gathering, modeling, and transforming data to give meaning to it to acquire a useful result is called Data Analysis. Descriptive Analysis and Inferential Analysis are the two types of data analysis. In this research work, data interpretation is done using univariate analysis methods such as frequency distribution to better

understand.

9.1. Frequency Table

The following statements based on your agreement and disagreement: Strongly Agree (5), Agree (4), Neutral (3), Disagree (2) and Strongly Disagree (1):

Table 1

	Statement	**5**	**4**	**3**	**2**	**1**
1	Do you feel that E-payments systems are user-friendly to adopt in day-to-day activities?	41%	42%	17%	–	–
2	Do you feel E-payments are secure?	58%	11%	19%	8%	4%
3	Will E-payment has great potential for future challenges to meet the expectation to adoption and use?	63%	10%	12%	6%	9%
4	Is E-payment is convenient over cash on delivery in online shopping?	59%	15%	13%	8%	5%
5	Is E-payment is more efficient than the traditional method?	35%	41%	24%	–	–

The following statements based on your satisfaction and dissatisfaction: Highly Satisfied (5), Satisfied (4), Moderate (3), Dissatisfied (2) and Highly Dissatisfied (1):

Table 2

	Statement	**5**	**4**	**3**	**2**	**1**
1	What is your opinion regarding the quality of services provided by the e-payment system?	49%	30%	17%	9%	4%
2	Satisfaction Level Regarding E-Payment System	25%	52%	8%	9%	6%

10. Findings

1. 59% of the respondents are male and 41% of the respondents are female. The male respondents use comparatively more E-payment services may be they know better about E-payment services and their security, still female users are 41% which not negligible.

2. 52% of the respondents were under 20-25 years, 29% fall between the ages 25-30 years, 10% of the respondents were aged between 18-20 years, and 9% of respondents were aged 31 & above years. Majority of the respondents fell in the age group of 21-25 years in the utilization of E-payment, because most of the college students users age fall between 21-25 years.

3. 54% of the respondents were Undergraduates, 32% were Postgraduates, 14% belongs to other categories of qualification. Undergraduate students are in majority where it comes to utilization because in this stage they are more involved in social media, games, and online shopping, etc when compared to post graduates and others.

4. 55% of the respondents are using the E-payment for more than 2 years, 23% are using it for 1-2 years, 9% for less than 3 months, 7% from 7 to 12 months, and 6% from 4 to 6 months. The majority of the users fall under more than 2 years in using the E-payment category.

5. the majority in use with 90% out of 100 respondents, 51% out of 100 respondents utilizing Net banking along with mobile wallets, 36% of 100 respondents utilizing Payment terminal (card swipe machine), and 5% used other E-payment services.

6. that 61% of the users using mobile as a source to access e-payment, 39% of the users are using both mobile and computer.

7. 27% of users using Google Pay, PhonePe, Paytm, Amazon Pay, 15% of users use Google Pay, PhonePe, another 15% of users using Google Pay only, 14% of users Google Pay, PhonePe, and Paytm, 9% of users use Google Pay, PhonePe, Amazon Pay, 5% are using Amazon Pay only, 4% are using only PhonePe, 3% are Google Pay, and Amazon Pay only, 2% use Google Pay, PhonePe, Paytm, MobiKwik, Amazon Pay, FreeCharge, JioMoney, another 2% of the user using Google Pay, PhonePe, Paytm, MobiKwik, Amazon Pay, and FreeCharge, 1% of the respondents use Google Pay, PhonePe, Paytm, Amazon Pay, FreeCharge, 1% PhonePe, Paytm, 1% of the respondents use Google Pay, and Paytm, and another 1% using Paytm, and Amazon Pay.

8. the majority of the respondents that is 78% faced technical error(s) while using E-payment services, 22% of respondents were not faced any technical error while using E-payment.

9. 64% of users were registered complaints against technical error while using E-payment, 36% of users have not registered any complaint against technical error.

10. 41% of respondents said the complaints were solved by the service provider within 3 days, 30% of respondents' complaints were solved within a day, 29% of respondents' complaints took a week to solve.

11. that 93% of happy respondents suggested E-payment services to other people, 7% of respondents did not suggest.

12. 49% of respondents gave a rating of 4 stars out of 5 for E-payment services, 36% gave 5 stars out of 5, and 15% gave 3 stars out of 5. There are no 1 and 2 stars out of 5 is given by the respondents.

13. most of the users that is 91% said yes to E-payment is better than traditional payment, and 9% of the users did not believe it.

14. most of the users that are 65% (100 in total) were preferring mobile wallets over cash, 35% of respondents did not prefer mobile wallets over cash.

11. Suggestions

The researcher has made certain suggestions to both the e-payment service providers as well as users of e-payment for the betterment of the system.

1. Rapid development in technologies has created the awareness of E-payment in everyones' mind. Research data result shows that there is still a concern for security-privacy and technical problem while using E-payment and it is the major barrier to using E-payment service among student. Further awareness is to be created amongst people by giving special attention towards security and privacy aspects.

2. Service providers need to enhance the security features and especially focus on disruption while transactions.

3. A barrier for the usage of E-payment is due to lack of proper network access to a smartphone and delay in payment in certain circumstances is a major drawback for E-payment users. The researcher suggests that service providers need to be concerned to provide a solution for the delayed transaction and technical error.

4. In this current Pandemic situation if the service provider of E-payment would focus on these problems it has more chance to retain the customer for a longer period and it will also help to build trust and loyalty among the E-payment users.

5. This was a cross-sectional study where data was collected from the E-payment user students in the Fourth week of August 2021; the researcher feels that the utilization rate of E-payment would have gone up during the Pandemic and a longitudinal study would help all the stakeholders including the government agencies and financial organization bodies of E-payment services to better understand the utilization of E-payment mechanism.

12. Conclusion

India is an Emerging and Developing Country (EDC) in southern Asia and the world's largest democracy. It is the fastest-growing economies, so India needs to move from a cash-based system to a cashless system. As our government of India already implementing and promoting the Digital India programme throughout the country in which E-payment is part of that programme, this research purpose is also to encourage the student to use E-payment services. If the service provider provides better security features and efficiency in the transaction will encourage the students to use the E-payment mechanism with ease and the government also support the E-payment mechanism to encourage E-payment services.

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A STUDY ON CUSTOMER SATISFACTION TOWARDS DIGITAL PAYMENT (A STUDY WITH REFERENCE TO PERNAMBUT CUSTOMERS)

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Abstract

In the olden days, goods and services were exchanged between two parties with a bilateral agreement. There was no cash at all. It is called a barter system; there are lots of drawbacks to the barter system. People cannot be able to purchase what they want. For instance, we needed medical treatment at that time. A doctor wouldn't accept our goods because his requirements would be different and many other things have drawbacks to the barter system. Later, the gold and money concept was introduced.

Money is a verifiable item that is widely accepted as payment for goods and services and to pay for debts. India, like developing countries, requires a certain amount of cash to circulate within the nation because cash is a dominant means of payment in our economy. We see a majority of people using cash to meet day-to-day expenses. Citizens, like uneducated and senior citizens, prefer cash transactions because they believe cash transactions are safe and secured.

Keywords: Online Payment, e-Payment, Digital Payment and Customer Satisfaction.

1. Introduction

In the digital era, every task is simplified with the help of the internet and innovation. Likewise, the concept of the cash economy is easier and time-consuming. On the other hand, it has some limitations, like the government cannot trace transactions. Cash transactions make it easier for money laundering and tax evasion. To prevent these activities, The Government of India has taken several measures to promote digital payment in the country. For that reason, The Government of India has introduced digital India, A journey towards a cashless economy (faceless, paperless, and cashless).

Key Players of Digital Payment

- Software Provider
- Device Manufacturer
- Mobile Network Operator
- Financial Institution
- Government
- Service Provider
- Customer
- Merchant.

2. Demographic Profile

In this research as a part of demography study the customer's, age, gender, marital status, educational qualification, monthly income, family size has been analyzed to identify the users of online payment method.

3. Statement of the Problem

In India initially, the e-commerce company faced many problems, our country is having multicultural people and various types of religion, the people cannot accept new technology immediately because of fear of losing money and trust issue problem for that reason e-commerce company offered to the buyer after receiving the goods at the buyer destination or home and it took time to bring trust among people in India Therefore it is crucial to know how well the customer adopted the e-payment or online payment in their life and what factors influence to use of online payment, After finding the factors that need to take into account, the present study will contribute to the study of Digital payment by providing the actual reason for the use of online payment.

4. Objectives of the Study

The primary objective of the study is to find out the level of customer satisfaction among online payment methods, following the objectives of the study.

1. To study the satisfaction level of customers towards online payment system.

2. To examine factors determining satisfaction of respondents while using online payment platform.

3. To identify problems and difficulties faced by respondents while using online payment system.

4. To study whether attractive offers lead the customers to shift from Traditional payments to online payment methods.

5. To offer suitable suggestions for improving the awareness among the customers, to further improve online payment methods.

5. Hypotheses of the Study

The researcher has developed the following hypotheses to study the level of customer satisfaction and online payment users.

H_0**:** There is no significant association between the gender of the respondents and their satisfaction towards online payment.

H_1**:** There is significant association between the gender of the respondents and their satisfaction towards online payment.

H_0**:** There is no significant association between the age of the respondents and their satisfaction towards online payment.

H_1**:** There is significant association between the age of the respondents and their satisfaction towards online payment.

6. Research Methodology

Research methodology is a systematic approach to the current study, It helps in selecting, designing, suitable methodology and selection of analytical tools. It is important to analyze any research problem and this section helps is determining sample, collection of data, period of study, and tools for analysis. Sample refers to picking a fixed number of samples from a large population, in this process the researcher has to select what type of sample to be collected in the research period. In this study non-probability technique is used these are Random convenient sampling and snowball sampling. In this study 100 samples were selected and non probability samplings were used.

7. Review of Literature

Changsu Kim, Wang Tao, Namchul (2010) Commonly people have trust issue with other, The study is focusing on good security will leads to improve trust and it is the main perception of people in traditional method of payment. People used to keep their money themselves, now in modern method of banking and financial intermediary play a big roll this paper examines issue related to e payment security from the view point of customer, This research provide theoretical knowledge and practical practice.

Royagholami, Augustineogun, Elizabethkoh (2010) The payment systems of a country play a vital role in economic development. This study examine factor that affecting the e-payment system in a country, After studying the Nigeria found that rate of user is low. Reason behind that many fraudulent incidents originating from Nigeria. Several scams have made in Nigeria, Advance fee fraud, fraud risks from e-Payment laws were tightened to increase criminalization of fraudulent activities.

Achmad Nizar hidayanto, Luqmansyauqihidayat (2015) The study is conducted on factors affecting the payment system those are e commerce, behavior, belief, social influences, personal innovativeness, trust, security, privacy, payment system, technology adoption, And it become easy to use after the analyzing all the above factor. Customer get some gain for instance benefit or reward while making online payment most of the customer ignored about risk associated with the transaction.

8. Data Analysis and Discussion

Table 1: Chi-Square test

VARIABLE	SIGNIFICANCE	RESULTS
GENDER	0.17	Not significant
AGE	0.15	Not significant
EDUCATIONAL LEVEL OF RSPONDENTS	0.26	Not significant
STREAM OF EDUCATION	0.87	Not significant
OCCUPATION OF THE RESPONDENTS	0.92	Not significant
MONTHLY INCOME OF THE RESPONDENTS	0.26	Not significant
FAMILY STATUS	0.31	Not significant
BANK ACCOUNT	0.78	Not significant
NATURE OF BANK ACCOUNT	0.91	Not significant

Crosstabs-Chi Square Analysis

- There is No significant association between the gender of the respondents and their satisfaction towards online payment.
- There is No significant association between the age of the respondents and their satisfaction towards online payment.
- There is No significant association between the educational level of the respondents and their satisfaction towards online payment.
- There is No significant association between stream of education of the respondents and their satisfaction towards online payment.
- There is No significant association between the occupation of the respondents and their satisfaction towards online payment.
- There is No significant association between the monthly income of the respondents and their satisfaction towards online payment.
- There is No significant association between the family status of the respondents and their satisfaction towards online payment.

9. Conclusion

After conducting the study it is witnessed that after the demonetization and deployment of technology in banking sector and e-commerce sectors people started using slowly all-digital payment system. And all credit goes to the government of India, without the government action it is not possible to implement the online payment method and contribution by tech companies. Among all other method of online payment e-wallet is trendier among the people because any transaction can be made with their Smartphone instantly and easy to use i.e. Paytm, BHIM ,google pay, These are prominent player in payment gateway provider. And remaining methods also less popular because it is not easy to use. The majority of respondents are fully satisfied with the online payment method, Some limitations found during the research that is fear of security ,The users get anxious about his or her confidential information which may get disclosed, Therefore the digital payment gateway provider needs to understand and meet the user's trust and expectation. The merchant and customer also satisfied with the online payment methods. And the government also took a step to teach the people about the benefits of using online payment methods. Finally, it is witnessed that there is huge scope for online payment methods in future, Not only it is used for transaction money but in future it will become a platform for investment, the prominent finalcial service company is planning to introduced for trading crypto currency in the country soon, The Government also taking steps to allow trading crypto currency in future.

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A STUDY OF JOB SATISFACTION AMONGST PART-TIME WORKING STUDENTS AT VANIYAMBADI TOWN

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Abstract

The employment trend among high school, higher secondary school and college students has been escalating continuously in the last decade. Some people mark the economical reasons as the main factor for this trend of student employment. With an amplified number of students involving part-time jobs outside the campus, its impact on students' academic performance has been a part of many pieces of research. The kind of issues such as the amount of time worked, whether the student's job is relevant to the majors, and the workload of the students, etc. have been discussed and scrutinized to view the connection between taking a part-time job and academic achievements. Primary data have been used for study using a questionnaire. To put it simply and ensure the understandability of the readers, the researchers have limited the analysis only to the 'simple percentile' and chi-square test. The study has accomplished with the sixty samples drawn by the convenience cum accidental sampling method. The outcomes emphasize the gaps and the possible actions for their betterment.

Keywords: Job Satisfaction, Part-Time Employment, Student Part-Time Workers.

1. Introduction

1.1. Need and Significance of the Study

Job satisfaction has a significant impact on the efficiency of the job performed by the employees. Therefore, it becomes necessary to understand the phenomenon of satisfaction of job. But when it comes to the part-time employees, they are not focused

enough in this context of job satisfaction although they have equal rights when compared to full-time employees in almost every aspect but still, they are given less importance. Thus, measuring job satisfaction irrespective of the type of employment is important whether it is full-time or part-time employment. The researcher has felt the need of conducting a research study on this phenomenon and he chose the student part-time employees in the vaniyambadi region as no one has ever done such study in this region.

2. Research Problem

The employment tendency among school and college students in the study region Vaniyambadi has been increasing day by day and new opportunities have been arising from various sectors for this type of employment. Students studying in schools and colleges around Vaniyambadi are becoming part of this type of employment for many years. They have been working part-time in various places. Most of them belong to poor and below average families who work to get some amount of income to manage their education expenses and to reduce the burden of their parents, on the other hand, most of the students who are working to gain income along with experience with the hope of getting job easily after their schooling or graduation. Most of the student employees are facing inconveniences regarding the part-time jobs they are doing such as uneven timings, irrelevant job roles, underpayments of wage, fewer rewards, useless experiences, low recognitions, etc. All these consequences are ultimately stressing their mind and leaving impacts on their present education and career. Hence, they may not live a satisfying life because of these problems, although they are managing the cost of their education with the earnings out of these jobs, and most of the students from backward classes are able to attain their education only with the help of such employment.

3. Objectives of the Study

The following objectives have been emanated from the statement of the problem

1. To study the job satisfaction of part-time working students in vaniyambadi town.
2. To evaluate the level of satisfaction among student part-time employees in terms of payments, working hours, job relevancy, quality of supervision, recognition, etc.
3. To determine the degree of satisfaction among student part-time employees based on their job experience and personal growth.
4. To measure their performance in education based on their Grade Point Average (GPA) along with part-time work.
5. Lastly, to suggest such job providers the channels to improve the job satisfaction among student part-time employees.

4. Hypotheses of the Study

The following are the hypotheses formed and administered on the data to determine the statistical significance of the study.

$\mathbf{H}_{01}$ There is no significant association between the gender of the respondents and their job satisfaction level.

$\mathbf{H}_{02}$ There is no significant association between the type of a job of the respondents and their job satisfaction level.

$\mathbf{H}_{03}$ There is no significant association between the pay of the respondents and their job satisfaction level.

$\mathbf{H}_{04}$ There is no significant association between the working hours of the respondents and their job satisfaction level.

5. Research Methodology

The research methodology has been described in the following sub-heads.

5.1. Nature of Data

To accomplish the above-mentioned objective the authors considered getting help from both the primary and secondary data. It would be very tricky to understand the research dilemma in a very lucid way except a succinct understanding of the different concepts of job satisfaction is known and therefore an effort has been made to present the same utilizing secondary data. For collecting the data on job satisfaction, a self-structured questionnaire has been used.

5.2. Sample Size and Technique

The researcher has chosen the students enrolled in sslc and hsc schools and in UG, PG and courses across arts, science and commerce, university affiliated colleges in and around vaniyambadi, because of time factor it has been decided to take up only five educational institutions alone. A sample of sixty student respondents has been drawn from the selected population studying at different schools and colleges and enrolled in different courses and fit into different age groups with the help of the convenience cum accidental sampling method. In this way, the samples have been chosen mainly based on the convenience of the researcher.

5.3. Questionnaire

The job satisfaction scale with twenty statements developed by Bubey, B.L., Uppal, K.K., and Verma, S.K. (1989) has been used with slight modification by the researchers.

5.4. Statistical Tool Used

To reduce difficulty in reading the outcome of the study it has been decided to use only easy and widely used tools of percentiles. Additionally, to test the proposed hypotheses of the study the chi-square analysis technique has been adopted in this study.

6. Review of Literature

Sapkota Abja, Usha K. Poudel, Jyotsana Pokharel, Pratima Ghimire, Arun Sedhein, Gandhi R. Bhattarai, Binu Thapa, and Tulza K. C. (2019) have finished research heading 'factors connected with job satisfaction of nursing graduates in Nepal. The questionnaires regarding job satisfaction were delivered through email to 327 nursing faculties from 39 colleges. T-test and chi-square techniques had been adopted to analyze the data. A multivariate binary logistic analysis showed that nearly 36.8% of the respondents were found to be highly satisfied with their present job. The study found that the process of decision making and the adequate access of materials among these respondents in the college be the significant factor of their job satisfaction.

Dougles Daniel and Paul Attewell (2019) have completed research called 'association between part-time job during college and post-college earnings. This research was aimed to evaluate the incomes of individuals who have been doing part-time jobs since their college days. For this reason, the researcher collected secondary data from the records of students' wages and employment. Contrasting the previous study claiming part-time jobs to be a threat for students, this study had proved using ordinary least square (OLS) regression model that part-time job is not a threat it's an opportunity to earn income and enhance future income along with education.

Andriano Judite and Chris W. Callaghan (2020) have made a research study named 'job satisfaction, retention and work-life balance among the staff members who were undertaking the part-time degree classes at an institute. This study had sought out to test a hypothesis which was claiming that some moderating and mediated influence exists between the conflicts of work-life balance and thoughts of turnover among those employees. With this intention, the researcher collected the sample from a university situated in South Africa. Hayes's process method found a unique combination of these two above-mentioned issues for employees with dependent children and also found experiencing a higher level of work-life balance and fewer turnover intentions in the case of employees with more kids. The researcher also suggested that employees with greater social support were found to have high degrees of turnover intentions.

Thi Nga Hoang (2020) has completed a study tagged as 'today's demand of students for the part-time job'. Data were collected through surveying questionnaires to three different schools located in Vietnam. With a sample size of some 500 students' researcher had prepared various analyses regarding their part-time work that is to say their demand for part-time work, choice of choosing such employment, hours of their

working, rewards they are getting, impact on school education, etc. He said that merely (43.6%) of the respondents it found to be joining such employment to strengthen and develop their communication skills. The study also observed that exactly (47.6) of respondents were shown a feeling of tiredness after working in such type of job. Due to the rise in uncertainty involved in such jobs the researcher suggested covering those uncertainties with compensations to these employees. In that way, the researcher concluded that working in part-time jobs is an effective option for students to gain experience and also to manage personal expenses. The challenges faced by this type of employee were also discussed.

7. Conceptual Definitions

7.1. Job

Dale Yoder [1] defines 'Job' as a collection of duties, tasks, and responsibilities, given to an individual and which may be different from other obligations.

In simple terms, a job is any authorized doings to facilitate a person to do a service and in return earn money to buy things, a job can also imply work that someone does to earn money, and the term "job" may even be used while a person works for his employer who pays him for his work.

7.2. Satisfaction

Locke (1976) [2] defined Satisfaction as an emotional reaction or liking towards an object.

Satisfaction is considered as an expression of achievement of an expected result prejudiced by prior expectation regarding the level of quality. To put it simply satisfaction is an immediate state of fulfillment of any desire, needs, or achievement of goals of an individual.

Generally, all human beings are believed to be goal-oriented. Most of the actions of man are powered by his motivation and mostly intended to maximize the satisfaction which he gets from fulfilling those motivations as defined by Maslow [3] in his need hierarchy theory. Consequently, the meaning of satisfaction states the extent to which the need, desires, and wants of an individual are fulfilled. In simple words, satisfaction takes place when one gets whatever he desires or needs or otherwise deserves to be an entitlement.

7.3. Job Satisfaction

Job satisfaction is an assessment of workers' happiness with their job whether they like the job or individual characteristics or facets of the job such as work environment or supervision in simple terms it is how an individual is with his or her jobs whether he or she likes the job or not.

7.4. Part-Time Work

Part-time employment is employment that comes under the third classification of the term employment such as CASUAL EMPLOYMENT, FULL-TIME EMPLOYMENT, and "PART-TIME EMPLOYMENT". This is a work scheduling strategy that is a part of the alternative work options program. Alternative work options give employees increased flexibility in their working schedules. The various options comprised in the program such as Part-time, Job sharing, Flexi time, etc.

It is generally scheduled as work that should be at least half-time but must be less than full-time or a minimum of 18 hours a week but fewer than 38 hours per week. In some countries it is also on 40 hours work basis, part-time work is normally considered as at least 20 hours per week basis. Employees, who work under these programs either for 18 hours a week or less than half of the year, are considered "INTERMITTENT EMPLOYEES".

8. Analysis and Interpretation of Data

8.1. Level of Job Satisfaction

Job satisfaction is an assessment of workers' happiness with their job whether they like the job or individual characteristics or facets of the job such as work environment or supervision in simple terms it is how an individual is with his or her jobs whether he or she likes the job or not.

Table 1: Level of Job Satisfaction

		Frequency	**Percent**	**Valid Percent**	**Cumulative Percent**
Valid	Highly Satisfied	36	60.0	60.0	60.0
	Moderately Satisfied	20	33.3	33.3	93.3
	Dissatisfied	4	6.7	6.7	100.0
	Total	60	100.0	100.0	

60 percent of the respondents are highly satisfied and 33 percent are moderately satisfied in their part-time jobs. And the remaining 7 percent of the respondents have shown dissatisfaction with their part-time jobs. Hence totally of 93 percent of the respondents are showing positive signs and the rest of 7 percent of the respondents are showing negative signs when it comes to job satisfaction.

8.2. Age Group

Individuals happen to meet different levels of satisfaction at their different phases of life. Generally, it's an assumption as Hammer and Organ1 say that job satisfaction may vary with different ages as they have said the level of job satisfaction is significantly high in the initial phase of an individual's life, and it gradually reduces age grows up. At the end of a career, job satisfaction becomes lower due to various reasons like being panicked to giving up work, etc.

Table 2: Age Group

		Frequency	**Percent**	**Valid Percent**	**Cumulative Percent**
Valid	15 – 20 years	32	53.3	53.3	53.3
	20 – 25 years	17	28.0	28.0	28.3
	25 – 30 years	10	17.0	17.0	17.0
	Above 30	1	2.0	2.0	2.0
	Total	60	100.0	100.0	

53 percent of the respondents belong to the age group of 15-20 years, whereas 28 percent of the respondents are from the age group of 20-25 years and 17 percent of the respondents are from 25-30 years of age group. Finally, only 2 percent of the respondents are of above 30 years of age.

8.3. Gender

Gender is an essential variable of every study in the field of social sciences. Gender differences lead to variations in human behaviours in terms of motivation, expectations, feelings, emotions, etc. The following table indicates the sex distribution of the respondents.

Table 3: Gender

		Frequency	**Percent**	**Valid Percent**	**Cumulative Percent**
Valid	Male	50	83.3	83.3	83.3
	Female	10	16.7	16.7	100.0
	Total	60	100.0	100.0	

83.3 percent of the respondents are male and the rest of 16.7 percent of respondents are female. However, it's a fact that the number of working women in India is lower than males due to various reasons such as restricted cultural norms relating to women's work,

the gender wage gap, lack of safety procedures and flexible work offers, etc. Although there has been some development in women's participation in work, still they possess a low rate of participation in the Indian workforce. Vaniyambadi being a conservative Muslim society, women participation in work and working environment is still less than 20% of the total workforce.

8.4. Type of Family

A family is a group of people living together in a house and the family has its different structures the most common among those are joint and nuclear families. The following table shows the different types of families the respondents belong to.

Table 4: Type of Family

		Frequency	**Percent**	**Valid Percent**	**Cumulative Percent**
Valid	Nuclear Family	43	71.7	71.7	71.7
	Joint Family	17	28.3	28.3	100.0
	Total	60	100.0	100.0	

71.7 percent of the respondents belong to a nuclear family and 28.9 percent of the respondents live in a joint family. After the 20th century, families have become more nuclear, because of several reasons such as non-family employment, rural-urban migrations, desire to live in a small family, and many more. This clearly shows that most of the respondents belong to nuclear family wherein head of the family alone might be bread earner who faces financial crunch every then and now. This situation becomes the reason for the students to go on part-time employment.

8.5. Level of Education

The levels of education signify the intensity of knowledge, skills, values, morals, beliefs, habits, and personal growth possessed by the individuals and it also Affects the level of satisfaction of individuals in their jobs, an individual may get highly satisfied when gets the job according to his qualification and level of education. The following table shows the respondents belonging to various levels of education.

it has been found that most of the respondents are doing their under graduation degree as their percentage reflects the high range in the above diagram as their share in the total sample is 51.6 percentage, 25% of the respondents are in their post-graduation. The remaining 23% of the respondents are twelfth, tenth and others proportionately.

Table 5: Level of Education

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	10^{th}	4	6.7	6.7	6.7
	12^{th}	3	5.0	5.0	11.7
	Under Graduation (UG)	31	51.6	51.6	63.3
	Post Graduation (PG)	15	25.0	25.0	88.3
	Others	7	11.7	11.7	100.0
	Total	60	100.0	100.0	

8.6. Stream of Education

The stream of education represents the course of study opted by the respondents of the present study. Generally, the students studying in courses other than medical courses such as arts, science, and commerce students are highly focused on this part-time work study. Because of the inexistence of any medical college in the area of research Vaniyambadi, the researcher has not included this in the study. Similarly though the engineering colleges are exists in the study region, they are deliberately neglected as the scope of the study is restricted to the students pursuing arts, science and commerce and other streams. Following table shows the different streams of education of the respondents.

Table 6: Stream of Education

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Science	20	33.3	33.3	33.3
	Arts	9	15.0	15.0	48.3
	Commerce	24	40.0	40.0	88.3
	Others	7	11.7	11.7	100.0
	Total	60	100.0	100.0	

40 percent of the respondents are from commerce background, 33 percent are from science background, and the remaining 26.7 percent are proportionately shared by the arts and other streams of education. It is interesting to see that the students' belongings to commerce stream are highly involved in the part-time works.

8.7. Type of Job

The individuals may have different types of jobs whether skilled or unskilled as per their qualifications and skills. Therefore, based on this assumption the different jobs

can be divided into two categories as skilled and unskilled. But in the case of this present study, all the respondents are the student themselves. Thus, it is clear that all of them are presently doing part-time jobs with their education. Therefore, those part-time works which are connected to their respective stream of education are categorized as skilled jobs and those not related to their field of education are grouped as unskilled jobs. The below table describes the percentage of the respondents having jobs in any of these two different kinds.

Table 7: Type of Job

		Frequency	**Percent**	**Valid Percent**	**Cumulative Percent**
Valid	Related to my field of education (Skilled)	41	68.3	68.3	68.3
	Unrelated to my field of education (Unskilled)	19	31.7	31.7	100.0
	Total	60	100.0	100.0	

68.8 percent of the respondents are having a job related to their field of education in the sense of skilled jobs, and the remaining 31.7 percent of the respondents have a job unrelated to their field of education. Hence the type of job also matters the most in measuring job satisfaction. Getting a job according to the qualification leads to more job satisfaction, as all the respondents of this study are students pursuing the same course or the other.

8.8. Time Preference of Job

In the case of part-time employment, the working time plays an important role, as in this study our samples are the students, it is clear that they may have to choose timings according to their schedule of study, the reason behind is that students have to attend their classes regularly and different institutions offering education in different timings, etc. The following table shows the percentage of the respondents preferring different job timings.

56.7 percent of the respondents prefer to go for the job in the evening, 21.7 percent of the respondents are preferred in the morning, and another of 13.3 percent of

The respondents prefer to go for a job on weekend holidays and rests of 8.3 of them prefer to semester or exam holidays. However, the majority of the respondents have chosen the evening time for their part-time jobs.

Table 8: Time Preference of Job

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Every day evening	34	56.7	56.7	56.7
	Every day morning	13	21.7	21.7	78.3
	On weekend holidays	8	13.3	13.3	91.7
	On exam/semester holidays	5	8.3	8.3	100.0
	Total	60	100.0	100.0	

8.9. Working Hours

Working hours is an important element of any job, and it also contributes the most to the satisfaction level of any individual doing any job, under this study the time spent on the job by the respondents in a week has taken as the working hours. Further, the researcher has classified the timings under four scales such as working up to 10 hours to above 21 hours in a week. The table below shows the working hours of the respondents in a week.

Table 9: Working Hours

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Up to 10 hours	36	60.0	60.0	60.0
	11 to 14 hours	6	10.0	10.0	70.0
	16 to 20 hours	12	20.0	20.0	90.0
	Above 21 hours	6	10.0	10.0	100.0
	Total	60	100.0	100.0	

60 percent of the respondents enjoy working up to 10 hours in a week, 10 percent of the respondents are working for 11 to 14 hours, and 20 percent of the respondents spend up to 20 hours and the remaining 10 percent are doing more than 21 hours in a week. Hence the majority of the respondents are contributing up to 10 hours for their part-time work in a week.

8.10. Mode of Payment

Payment is a reward offered by the employer to their employees on completion of a specific job or work. It is also called salary if it is paid every month and wages if paid on a weekly, daily, or output basis, the below table shows the mode payment to the respondents in their jobs.

Table 10: Mode of Payment

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Time/hour-based payment	10	16.7	16.7	16.7
	Fixed (monthly) payment	38	63.3	63.3	80.0
	Payment for completion of specific tasks	6	10.0	10.0	90.0
	Payment per unit of output	6	10.0	10.0	100.0
	Total	60	100.0	100.0	

63 percent of the respondents have been getting fixed monthly payments, 17 percent of them are receiving payment on an hourly basis, and of the remaining 20 Percent of the respondents 50% each are drawing their payment based on completion of a task and per unit of output respectively. It is reflected that the maximum number of respondents are drawing their remuneration on monthly basis.

8.11. Monthly payments (Income)

Payment of the salary is a reward for work performed by employees. However, it may vary from job to job, person to person, designation to designation; accordingly, its range may also be changing. But as our study is on part-timers, the range of payment has been expected by the researcher is up to 9 thousand, the following table shows the portions of respondents drawing the different range of payment in a month from their jobs. Therefore, the range of payment has been expected by the researcher is up to 9 thousand, the following table shows the portions of respondents drawing the different range of payment in a month from their jobs.

36.7 percent of the respondents have been drawing their salaries up to three thousand rupees, 40 percent of the respondents getting their payments between three to six thousand rupees; the percentage of respondents who are receiving their income between six thousand and nine thousand rupees and above nine thousand rupees are 23.3 percent. The majority of the respondents are getting their monthly income between three and six thousand rupees by doing part-time work.

Table 11: Monthly Payments (Income)

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Up to3000	22	36.7	36.7	36.7
	3001 to 6000	24	40.0	40.0	76.7
	6001 to 9000	6	10.0	10.0	86.7
	Above 9000	8	13.3	13.3	100.0
	Total	60	100.0	100.0	

8.12. Motives to Part-Time Work

Motivation is a psychological process within the individuals' mind which leads to the achievement of goals in life Similarly, the employees may get motivated to go for a job for several reasons which may drive them to perform a task that ultimately fulfills their needs and wants, for some, it might be money, experience, etc. The following table depicted below shows the respondents' motivation to go for a part-time job.

Table 12: Motives to Part-Time Work

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	To improve yourself	4	6.7	6.7	6.7
	To gain experience	15	25.0	25.0	31.7
	To earn for self	4	6.7	6.7	38.3
	To bear education expenses	4	6.7	6.7	45.0
	all of the above	33	55.0	55.0	100.0
	Total	60	100.0	100.0	

6.7 percent of the respondents have shown the self-development as the real reason for going to part-time work, 25 percent of them choose part-time employment to gain experience, 13.14 percent of the respondents are going part-time job for financial reasons, and the remaining 55 percent of the respondents agrees with all these reasons.

8.13. Financial Freedom

Freedom in the sense of independence of respondents from their parents with the income they earn through part-time, to test this statement whether they are independent or still

dependent on their parents for all their expenses, pocket money, education expenses, etc, The researcher has made this statement with two options 'Yes' or 'No' respectively. The table below shows the respondents' perception financial freedom.

Table 13: Financial Freedom

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Yes	45	75.0	75.0	75.0
	No	15	25.0	25.0	100.0
	Total	60	100.0	100.0	

75 percent of the respondents are having freedom from their parents, and the rest 25 percent are still dependent on parents with the income they earn by doing part-time work.

8.14. Improvements in Social Ability

Social ability is a skill that individuals use to communicate and interact with each other. It is also called as interpersonal skills of an individual. Certainly, in the place of work, it is most required one. It means every employee should have this skill in them. Well, it can also be improved within the job itself from day-by-day experiences the job, so the below table shows the perception of the respondents toward social skills development by their part-time work.

Table 14: Job Improvements in Social Ability

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Yes	56	93.4	93.4	93.4
	No	4	6.6	6.6	100.0
	Total	60	100.0	100.0	

93.4% of the respondents have believed that part-time work has improved their social skills and 6.6% have not at all experienced the social skills in their jobs.

8.15. Real Benefits of Part-Time Work

According to the opinion of respondents the actual benefit they have obtained by doing part-time work has been assessed and tabulated in the following table.

25 percent of the respondents have obtained money and experience as the real benefit, 5 percent of them feel getting new friends and happiness is the real benefit, 15 percent marked confidence as the real benefit they get out of a job and rest of 55 percent

Table 15: Real Benefits of Part-Time

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Money & Experience	15	25.0	25.0	25.0
	Friends & Happiness	3	5.0	5.0	30.0
	Confidence	9	15.0	15.0	45.0
	All of the above	33	55.0	55.0	100.0
	Total	60	100.0	100.0	

of the respondents agree with all these benefits. Thus, majority of the respondents are having all these benefits out of the part-time jobs.

Table 16: Responses of Statements

S.No	Statements	SA	A	UD	DA	SDA	TOTAL
1	Work as per Qualification	25%	30%	22%	7%	17%	100%
2	Good Working Conditions	25%	30%	15%	15%	15%	100%
3	Good Organization Management	32%	27%	23%	8%	10%	100%
4	Satisfactory Supervision	32%	28%	12%	18%	10%	100%
5	Better Interpersonal Relationship	43%	22%	10%	12%	13%	100%
6	Satisfactory Working Hours	28%	33%	17%	5%	17%	100%
7	Agreeable Pay	27%	30%	20%	7%	17%	100%
8	Fulfillment of Necessities with Present Pay	23%	27%	22%	13%	15%	100%
9	Receipt of Annual Bonus	23%	27%	15%	5%	30%	100%
10	Acceptable Workload	28%	23%	27%	10%	12%	100%
11	Satisfactory Learning of Skills	47%	18%	12%	12%	12%	100%
12	Usability of Part-time Experience while Getting Full-time Job	33%	28%	15%	12%	12%	100%
13	Job Security	37%	23%	20%	8%	12%	100%
14	Promotional Opportunities	13%	23%	18%	28%	17%	100%
15	Influence of Part-time Work on Studies	28%	23%	28%	13%	7%	100%

16	Availability of Sufficient Time for Studies	22%	23%	22%	17%	17%	100%
17	Influence of Part-time Work on Exams	25%	27%	20%	13%	15%	100%
18	Supervisor's Co-operation during Examinations	30%	17%	25%	13%	15%	100%
19	Circumstances Pressure on Taking up Part-time Jobs	20%	20%	20%	10%	30%	100%
20	Employee Turnover Possibilities	20%	22%	18%	13%	27%	100%

The above table represents the respondent's different level of agreement over each statement in the table on the basis of strongly agree, disagree, undecided, disagree and strongly disagree in the form of percentage.

Table 17: Chi-Square Analysis

Variable	Value	P-value
Gender	1.04	0.594521
Type of Job	3.75	0.153355
Pay	9.26	0.37330
Working Hours	3.67	0.872179

The above table indicates the chi-square values and p-values relating to the association of different variables namely gender, type of job, pay and working hours with job satisfaction

9. Findings of the Study

The findings of the study have been described under fifteen heads as shown in the following.

1. The study has revealed that 60% of the part-time working students are highly satisfied and 33% are moderately satisfied with their present part-time jobs. And the remaining 7% have shown their dissatisfaction with their part-time jobs. Hence totally 93% of the respondents have shown positive signs and only 7 percent of the respondents have shown the red signal.

2. The study has revealed that 53 % of the part-time working students belong to the age group of 15-20 years and 28 % are from the age group of 20-25 years. The majority of them have come under the age group of 15-20 years.

3. The study has found that 83.3 % of the part-time working students are male and the rest of only 16.7 % are female. So, the majority of the part-time workers are male.

4. Around 71.7 % of the part-time working students belong to nuclear family and 28.9 % are living in a joint family. Thus, it has been found that nearly 3/4th of the part-timers are living in nuclear family setup.

5. Totally 51.6 % of the part-time working students are doing their under-graduation degree. 25 % of the part-time working students are in their post-graduation. The remaining 23 % of the part-timers are from 12th std., 10th std. and others proportionately. It reflects from the findings that the need for part-time work is high among under graduates.

6. Almost 40 % of the part-time working students are from commerce background, 33 % are from science background. It has revealed that the majority of part-time workers are from commerce and secondly from the science background.

7. Findings have revealed that 68.8 % of the part-time working students are having a job related to their field of education categorised here as of skilled jobs, and 31.7 % of the part-time working students have jobs unrelated to their field of education.

8. About 56.7 % of the part-time working students prefer to go for the job in the evening and 21.7 % are preferred in the morning time. More than 50% of students are interested to go for part-time work in the evening.

9. Findings suggest that around 60% of the part-time working students enjoy working up to 10 hours a week, 10% are doing more than 21 hours in a week. The majority of the part-time working students are contributing only up to 10 hours for their part-time work in a week.

10. Exactly 63% of the part-time working students have been getting fixed monthly payment, 17 % of them are receiving payment on an hourly basis and remaining 20 % are getting paid based on completion of a task and per output basis respectively. It has revealed that the maximum numbers of these part-timers are drawing their remuneration on monthly basis.

11. 36.7 % of the part-time working students have been drawing their salaries only up to three 3000 rupees, 40% of part-timers are getting their payments between 3000 to 6000 rupees; the percentage of part-timers who receive income between 6000 to 9000 rupees and above 9000 rupees is 23.3%. The majority of the part-timers are paid between rupees 3000 to 6000.

12. 6.7% of the part-time working students have shown the self-development as the real reason for going to part-time work, 25% of them chooses par-time Work to

gain experience, 13.14% are going part-time job for financial reasons, around 55% part-timers have agreed with all these reasons.

13. 75% of the part-time working students are having freedom from parents and 25% are still dependent on parents with the income they earn by doing part-time work.

14. According to the study, 93.4% of the part-time working students believe that part-time work improves social ability and 6.6% have not at all experienced social skills in their jobs.

15. The study has found that 25% of the part-time working students have obtained money and experience as the real benefit, 5% of them feels getting new friends and happiness is the real benefit, 15% marks confidence as the real benefit they get out of the job and 55% of the students have agreed with all these benefits. Thus, majority of the respondents are having all these benefits out of the part-time jobs.

10. Suggestions

Suggestions based on the findings for the improvement of job satisfaction among part-time workers especially for student employees have been described in the following.

1. The middle-level management must try to improve the better working conditions in the workplace to ensure the workers' physical and mental well-being by providing them cleanliness, a peaceful environment, proper drinking water facilities, hygiene lavatories, and welfare facilities, etc.

2. Middle-level managers must try to bring equality in the workplace by avoiding discrimination between full-time and part-time workers in terms of their positions and it is suggested that the management should focus on the skills and potentialities rather than focusing on their positions.

3. Supervisors should behave well and be friendly with the part-time workers by getting work done with a good amount of respect, co-operation and courtesy.

4. The middle management must try to offer reasonable pay to these part-time employees so that they at least could able to manage their expenses and must think of providing a uninterrupted annual bonus to them without fail.

5. Though the promotion scheme may not be applicable for such employment if there would be any scope arise for promotions in favour of part-timers then they must be considered for this by the respective management.

6. Lastly, it is suggested to the management to give attention as could as possible to the grievances of these student part-timers about their educational matters and must try to co-operate with them during their examinations.

11. Conclusion

Job satisfaction is the measure that describes the interest of the employees. Job satisfaction is the rational feeling of agreeableness that an employee has about his/her job. Employees have certain expectations from their jobs whether full-time or part-time and if these expectations are not met, they feel dissatisfied. Usually, the expectations are driven according to employees' educational levels, age, gender, etc. The level of satisfaction of workers relies on their psychological and physical health. Efficiency is another facet that shows the way to employee satisfaction. A cheerful Worker is an efficient worker. If the suggestions presented, to get better employees' satisfaction, be applied, the business can be able to accomplish its organisational objectives productively.

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SELECTED HR PRACTICES AMONG THE EMPLOYEES OF IT COMPANIES – A BIRD EYE VIEW

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Abstract

Employees prove their best of abilities due to the presence of HR Practices. HR Practices are like fillip to the employees and drive them to succeed in their tasks allotted to them. In the competitive world, to be in survival is of paramount importance. Therefore, keep the employees updated by conducting much sought after HR Practices. In this study, investigation was carried out to highlight the selected HR Practices among the employees of IT Companies in Chennai City. Descriptive and Empirical Research Design have been used throughout the research. Researcher used both primary data and secondary data while primary Data are those which are collected for the first time from the sample respondents. For the purpose of collecting primary data, researcher has developed structured questionnaire based on the need of study Researcher circulated the questionnaire among 152 respondents who are the employees of selected IT companies in Chennai City thereby collecting primary data. However, secondary data were elicited from news paper, magazines, well reputed peer indexed journals, previous dissertation, Ph.D theses, relevant websites, Books, and Library. Convenient Sampling Method was adopted. Totally three IT companies in Chennai City were selected for the study such as Tata Consultancy Services, Hexaware Technologies and Capgemini.

Keywords: HR Practices, IT Companies, PA and SEM.

1. Introduction

HR practices are a fixed human resource management strategies and moves that work universally. In HRM research, there are two faculties of concept on a way to control

humans. The first one is the excellent fit, the second is first-class practices. The pleasant suit faculty states that to feature fee, human aid guidelines ought to align with enterprise approach which means HR ought to awareness on both the wishes of the employer and the ones of its personnel. The nice exercise college argues that there's a set of popular HR tactics that result in advanced business performance. According to its proponents, there are certain bundles of HR activities that guide agencies in attaining a competitive gain regardless of the organizational setting or enterprise. We are able to skip the sizeable scientific debate at every method. With those sorts of discussions, the truth regularly lies somewhere inside the center. Because of this the HR strategy and next HR activities ought to be aligned with the corporation's method for max performance. In literature, this alignment has also been referred to as Strategic Human resource control.

2. HR Practices in IT Companies

In India, IT companies play important role in giving away employment opportunities to many people. Sizeable population is dependent on IT Companies. All the IT companies in India generate stable employment opportunities and increase their standard of living. After the LPG policy in India, Many IT companies accommodate employees from India as Indian are really dedicated, committed and work alcoholic. They are well equipped and started learning many new things because of the presence of IT Companies in India. Once upon a time, IT Companies were confined to some specific activities. But today, they have crossed many milestones and they undertake the tasks which are otherwise impossible with employees of rest of the countries across the world.

HR Practices also keep employees updated and make the employees highly sincere and responsive in each and every respect. They understand their roles and responsibilities as they are driven by best HR practices adapted or to be adapted by most of the IT companies in India. HR practices range from Performance Appraisal to Placement and Recruitment of Employees. In those days, workers were treated just as machine for making profit for the employer. But today employees coordinated efforts will drive the organization to tremendous success. IT companies in India properly follow HR Practices. This study is going to exhibit how HR practices are exercised by all the IT Companies in India.

3. Objectives of the Study

- To evaluate methods being followed by IT Companies while appraising the performance of employees and highlight the ways in which how employees of IT companies get placed and recruited.

4. Problem Statement

The present study is of vitally useful to employees in particular and employer in general. People those who manage and administer the IT companies should give due rise to HR Practices as they ensure that employees are at the right track and they will prove their level best abilities once when they are provided with suitable HR Practices. Unlike other forms of companies, IT companies are more particular about HR Practices as they compete with many global companies. HR practices will bring repute to the business organization. But there are some glitches while following HR practices in IT Companies. Performance of Employees should be tested at frequent intervals but some IT Companies do not test the performance and employees are made to work without testing their efficiencies for a long time. While recruiting and placing employees, higher officials at IT Companies prefer their selected candidates to work under their control thereby it leads to favoritism. So, sincerity is considered least important as far as the recruitment and selection of employees are concerned.

5. Scope and Importance of the Study

HR Practices are very common in most of the IT Companies prevalent in almost all the countries. Many people depend on IT Companies and hold various positions in that. IT Companies want to survive in the long run and they need to overcome obstacles and hardships by deploying skilled workforce. If they have to sustain in the market, they have to increase the efficiency of employees by enriching their abilities, skills, knowledge and capabilities. IT companies systematically follow HR Practices as employees are able to meet their target within given span of time. Employees are getting trained based on their requirements. Even Career Advancement Scheme is rolled out in any IT Company as it seems to be mandatory for each and every employee. Performance Appraisal also is properly adopted in all the IT Companies and timely performance appraisal is carried out so as to keep the employees upto date. That is why; employees in IT Companies perform excellently.

6. Review of Literature

Shahzileh, Z. H. & Aghajan, A. M. (2015) explained that Every organisation for its survival and growth requires committed and qualified employees. Managers for the decisions such as the promotion, salary increase, appointment, fire, and replacement require evaluating the performances of their employees. With the assist of an appropriate performance appraisal system, companies will be able to overcome the problems related to employees' weak performances and can prepare plans for their performance improvement. In this paper based on the prior researches and studies, first

the performance appraisal system, its objectives, content and process will be reviewed and then, the most recognised methods of performance appraisal and the effectiveness of performance appraisal system will be explained. And finally the performance appraisal system of an Iranian company will be examined and some improvement recommendations will be suggested.

Sing, R. & Vadivelu, S. (2016) discussed that the globalization has created a niche to sustain and enhance human resource since they are the prime contributors to organizational performance. In this 21st century, human development is focused at hiring, training and retaining them. Retaining its valuable and talented employees becomes a major challenge for organisation if they do not know how to motivate them. Hence, performance appraisal (PA) is extensively used to facilitate salary increments, promotions, staff retention and to reinforce staff behavior. This paper is aimed providing a review on the various performance appraisal techniques practiced in Indian economy and some of its limitations. This paper also highlights literature review on effectiveness of the PA system in various Indian industries.

Suwarto, F.X. & Subyantoro, A. (2019) elaborated the aims of this study was to determine the effect of recruitment on performance, the effect of selection on performance, the effect of placement on performance, the effect of recruitment on selection, and the effect of selection on placement. Depreciating number of employees at PT Green Glovers Indonesia in Klaten was caused by employees entering retirement, death or dismissal. Newly-hired employees are required to improve their performance. Technical data processing was conducted using Structural Equation Modeling (SEM) with Partial Least Square (PLS) approach. The sample of this study consists of 90 employees. The results showed that recruitment has no effect on performance; selection affects performance; placement affects performance; recruitment affects selection; and that selection affects placement.

Kotwani, (2020) said about Performance Appraisal is a regular review of employee's job performance and his/her contribution towards the company. It is also known as Annual Review, Performance evaluation, Performance review or Employee Appraisal. Performance appraisal evaluates the employee's skills, achievements, growth, job performance as well as lack of performance. It is a vital tool for measuring the contribution and effectiveness of the employees towards the organization goals. This study examined the process of performance appraisal and implication of individuals as well as organizational goals. Through performance appraisal management identifies the individual strength and weakness and also tests if individual is contributing towards completing the organizational goals or not. It plays vital role in measuring the framework set by the organization.

Fitri, R.L., Handaru, A.W. & Yohana, C. (2021) highlighted the objectives of this study are (1) to examine and analyze the effect of recruitment on performance of employees (2) To analyze the selection of employees performance, and (3) To analyze the placement of employees. The data was collected through observation, distributing

questionnaires and interviews. The method that has been used in this research was descriptive (qualitative) and verification (quantitative), while the data was analyzed using a Structural Equation Modeling (SEM) with the Smart PLS version 3.2.9 program. The results of this study indicate that recruitment has a positive and significant effect on performance by tcount > ttable (5.705 > 1.996) at a significance level of 0.000 < 0.05, meaning that H_0 is rejected and H_1 is accepted. Next, the selection has a positive and significant effect on employee performance partially by tcount > ttable (3.309 > 1.996) at a significance level of 0.001 < 0.05. It means that H_0 is rejected and H_2 is accepted. Placement has a positive and significant effect on performance by tcount > ttable (4.907 > 1.996) at a significance level of 0.000 < 0.05. It mens that H_0 is rejected and H_3 is accepted. Theoretically, the results of this study contribute to the repertoire of knowledge, especially in the field of developing human resource competencies, namely employees performance.

7. Research Question

- Do Placement and Recruitment lead to increase the Performance level of Employees?

8. Hypothesis

- Placement and Recruitment are significantly different with Performance Appraisal of Employees.

9. Methodology

Research Methodology gives away proper guidelines to the researcher thereby he/she will achieve the target of research. Descriptive and Empirical Research Design have been used throughout the research. Researcher used both primary data and secondary data while primary Data are those which are collected for the first time from the sample respondents. For the purpose of collecting primary data, researcher has developed structured questionnaire based on the need of study Researcher circulated the questionnaire among 152 respondents who are the employees of selected IT companies in Chennai City thereby collecting primary data. However, secondary data were elicited from news paper, magazines, well reputed peer indexed journals, previous dissertation, Ph.D theses, relevant websites, Books, and Library. Convenient Sampling Method was adopted. Totally three IT companies in Chennai City were selected for the study such as Tata Consultancy Services, Hexaware Technologies and Capgemini.

Table 1: Bi-variate Correlation for determining the existence of relationship between Performance Appraisal and Placement and Recruitment

		Performance Appraisal	Placement and Recruitment
Performance Appraisal	Pearson Correlation	1	.855**
	Sig. (2-tailed)		.000
	N	152	152
Placement and Recruitment	Pearson Correlation	.855**	1
	Sig. (2-tailed)	.000	
	N	152	152

**** Correlation is significant at the 0.01 level (2-tailed)**

10. Statistical Processing of Data

10.1. Bi-Variate Correlation

Bi-variate Correlation is used to ascertain the existence of significant linear relationship among different variables pertaining to Human Resources Practices in selected IT companies in Chennai City. In the present table, two variables between which, an attempt was made to find the relationship those two variables are named as Performance Appraisal and Placement and Recruitment. In this connection, the following null hypothesis is formulated:

> *There are no significant linear relationship between Performance Appraisal and Placement and Recruitment.*

Performance Appraisal and Placement and Recruitment are correlated. Pearson correlation score is .855, p value (2 tailed) is less than .05 and the result is significant. Therefore, formulated null hypothesis i.e. there is no significant relationship between Performance Appraisal and Recruitment are rejected at 95% confidence level.

10.2. Exploratory Factor Analysis for Performance Appraisal of Employees

In this study, Exploratory Factor Analysis is used to determine underlying factor for a set observed variables of Performance Appraisal of Employees. The observed variables include Organisation takes into account the performance of employees, Promotion, Transfer are driven by Performance of Employees, If employees are poorly performed, Any action is taken against them, Performance Appraisal for Employees are conducted very often and Superiors recommend for the specific employees based on their performance.

Table 2: Exploratory Factor Analysis with Initial Eigen Value and % of Cumulative Percentage of Variance

Extracted Factors	Obtained Coefficients	Suppression of Small Coefficients
Organisation takes into account the performance of employees	.875	Retention of Variables if whose coefficients are at or above 0.4
Promotion, Transfer are driven by Performance of Employees	.849	
If employees are poorly performed, Any action is taken against them	.783	
Performance Appraisal for Employees are conducted very often	.655	
Superiors recommend for the specific employees based on their performance	.848	
Initial Eigen Value (Consolidated)	4.011	
% Cumulative Percentage of Variance	80.209	

Interpretation

From the above table, it is inferred that Exploratory Factor Analysis was performed to determine underlying structure for a set of measured variables. All the measured variables (totally five measured variables) whose initial eigen value has come to 4.011 while Cumulative Percentage of Variance is 80.209. There are five constructs identified for given set of variables under Performance Appraisal of Employees namely Organisation takes into account the performance of employees (.875), Promotion, Transfer are driven by Performance of Employees (.849), If employees are poorly performed (.783), Any action is taken against them, Performance Appraisal for Employees are conducted very often (.655) and Superiors recommend for the specific employees based on their performance (.648). Since small coefficients are in well yardsticks, all the variables affect the Performance Appraisal of Employees in the Selected IT companies in Chennai City.

10.3. Exploratory Factor Analysis for Placement and Recruitment

In this study, Exploratory Factor Analysis is used to determine underlying factor for a set observed variables of Placement and Recruitment of Employees. The observed variables include Placement is taken place on merit basis, All the criterion are followed while placing the employees, Advertisement is given before the placement and recruitment

takes place, New employees are properly inducted to group of existing employees, Right positions are reserved after placement gets over.

Table 3: Exploratory Factor Analysis with Initial Eigen Value and % of Cumulative Percentage of Variance

Extracted Factors	Obtained Coefficients	Suppression of Small Coefficients
Placement is taken place on merit basis	.832	Retention of Variables if whose coefficients are at or above 0.4
Advertisement is given before the placement and recruitment takes place	.529	
Right positions are reserved after placement gets over	.832	
Initial Eigen Value (Consolidated)	2.794	
% Cumulative Percentage of Variance	55.891	

Interpretation

From the above table, it is inferred that Exploratory Factor Analysis was performed to determine underlying structure for a set of measured variables. All the measured variables (totally five measured variables) whose initial eigen value has come to 4.011 while Cumulative Percentage of Variance is 80.209. Totally five measured variables but two of which should be eliminated as they do not affect the placement and recruitment of employees much. Moreover, whose small coefficients are less than 0.4. Therefore, on this ground, they should be removed from this. Only three variables are taken into account such as Placement is taken place on merit basis (.832), Advertisement is given before the placement and recruitment takes place (.529) and Right positions are reserved after placement gets over (.832). Hence, these three factors affect the Placement and Recruitment the most.

11. Result Discussion

Performance Appraisal and Placement and Recruitment are correlated. Pearson correlation score is .855, p value (2 tailed) is less than .05 and the result is significant. Therefore, formulated null hypothesis i.e. there is no significant relationship between Performance Appraisal and Recruitment are rejected at 95% confidence level. Exploratory Factor Analysis was performed to determine underlying structure for a set of measured variables. All the measured variables (totally five measured variables) whose initial eigen value has come to 4.011 while Cumulative Percentage of Variance is 80.209. Since small coefficients are in well yardsticks, all the variables affect the Performance Appraisal of Employees in the Selected IT companies in Chennai City.

Exploratory Factor Analysis was performed to determine underlying structure for a set of measured variables. All the measured variables (totally five measured variables) whose initial eigen value has come to 4.011 while Cumulative Percentage of Variance is 80.209. Totally five measured variables but two of which should be eliminated as they do not affect the placement and recruitment of employees much. Moreover, whose small coefficients are less than 0.4. Therefore, on this ground, they should be removed from this. Only three variables are taken into account such as Placement is taken place on merit basis (.832), Advertisement is given before the placement and recruitment takes place (.529) and Right positions are reserved after placement gets over (.832). Hence, these three factors affect the Placement and Recruitment the most.

12. Suggestions

Though there are numerous HR Practices in the IT Companies, research investigation was done to measure two selected HR Practices among the employees of IT Companies in Chennai City. Performance Appraisal in the IT companies should be done rationally to keep the spirit of employees atop. While assessing the performance of employees, impartiality needs to be followed thereby repose faith in the employees. Monetary Benefits have to be given to those who are performing more than expectation of the superiors. Employees should do anything and everything for all round development of the organization. Promotion should be guaranteed on the successful completion of the tasks by the employees. In the same way, sufficient point of time is needed for the effective performance appraisal. But in some IT Companies, performance appraisal is not done thereby leaving the employees in lurch.

Need based training should be imparted to low performing employees so as to improve their effectiveness in their performance. Feedback also should be heard once the training programme is completed. If any issue arises, it should shortly be attended to. Recruitment and Selection also should be done unbiased. Meritorious employees should be given the opportunities of their choice to prove their level best abilities.

13. Conclusion

Performance Appraisal quenches the thirst of employees in IT Companies. Employees do their work in the organizations, but they have to know to what extent the work is acceptable by their bosses. They should know where they lack so that it will be easy for them to correct in the period to come. Lack of job skills will not only paralyze the organization but also brings the employees to standstill. IT Companies should follow proper method of Performance Appraisal so as to overcome the glitches being faced by them. Recruitment and Selection are need of the hour. Company must accommodate the skilled employees by identifying their abilities during the time of recruitment and selection.

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A CASE STUDY OF CUSTOMER SATISFACTION ON ONLINE PURCHASE IN VANIYAMBADI TOWN

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Abstract

The research study, entitled "Customer Satisfaction on Online Purchase - A Case Study with Special Reference to Vaniyambadi Town," is to learn more about the elements that influence customer satisfaction on online purchases. This research used a descriptive research design. To collect samples across Vaniyambadi Town, a well-structured questionnaire was devised and administered. Due to the necessity for a diverse group of respondents with prior online buying experience, a non-random sampling technique was adopted. The research was conducted with an 80-person sample size. The data was categorized, tallied, and codified once it was collected. To evaluate the data, necessary statistical procedures such as simple percentage average, Chi Square tests, and ANOVA were applied. The findings of this study may be useful to online shopping businesses and corporations in gaining a better understanding of customer satisfaction, which can assist them expand into or enter the online shopping industry.

Keywords: Online Purchase, Online Shopping, Customer Satisfaction.

1. Introduction

Recently continuous development at the current period of online shopping is the latest trend of shopping in Vaniyambadi Marketing Environment that is used to refer to Desktop-based-shopping or online shopping. Online shopping is the method whereby purchasers directly buy products and services, etc. from a retailer interactively on real occasion without the help of a middleman straightly over the internet. The satisfactions a reaction which is an importance of the measuring process which informs what has been received against which was anticipated, as well as the buying decision and needs and wants to be connected with the online purchase. The Internet had been continuously gaining importance in recent days. Most of the public prefer internet for their day to day

activities as well keeps on rising. People nowadays are doing domestic works with the help of online. Internet customers have to contact online to buy products and services online. People can use the websites of online shops to buy luxury goods from home. Thus it states how the modern market is the substance for the conventional markets.

2. Importance of the Study

In India, almost every retailer is going to online buying has been reported. The ever-increasing reach of the Internet between customers has resulted in more and more retailers providing those online platform facilities to customers to formulate purchases at the click of the mouse. Customers have used the internet for purchases and also search about product and service information. The Internet has modified the entire business approach. A lot of businesses have on track to build up the strategies around the internet. If E-Marketers evaluate and identify with the factors influencing Indian customers' online behavior, they can further fine-tune their companies' strategies and techniques towards customer preferences. This research study handles the customers' perceptions towards the following aspects of online purchasing and how the presence or deficiency of these aspects in an online purchasing environment affects customer satisfaction.

3. Statement of the Problem

With the rapid expansion of the technology and digitalization surrounding the internet, businesses are interested to trade their products via their online website even though purchasers and sellers can be thousands of kilometers at a distance, might to different parts of the globe, may speak various languages as well. Since the internet is a novel virtual channel and so has lots of valuable consumers, the online vendors are more significant to understand the needs and want of the customer.

4. Objectives of the Study

- To study the demographic profile and effect on online purchase by the respondents.
- To analyze the factors influencing customers towards online purchasing.
- To analyze the satisfaction level of customers towards online purchasing.
- To identify mobile application facilities more convenient than online website shopping.
- To give findings, recommend valuable suggestions for rise consumer's satisfaction through online purchasing and conclusion.

5. Scope of the Study

The study was conducted to determine the customer satisfaction with online purchases, with a focus on Vaniyambadi Town. The study report also includes details on the theoretical aspects of online shopping.

6. Review of Literature

Syed Shah Alam and Norjaya Mohd. Yasin (2010) was completely studied a research study entitled 'investigation of satisfaction of customer on online shopping' The study aims to find out the main factor influencing the satisfaction through online purchasing. And the research was aim to find out four main factors of satisfaction of customers of online purchasing. The findings of the research were the design of web site, reliability, product types, and delivery was four main factors that influence customers' satisfaction with online shopping. There was no significant relationship between time-saving and satisfaction of the customer.

M. Sultana Barvin (2014) has explored the research titled 'comparative study on satisfaction of customer on online and traditional shopping in Tirunelveli District' the researchers interpret the satisfaction of customer both online and traditional shopping. Satisfaction of customers is essential in both online and traditional shopping. In traditional shopping, customers spend daily, weekly and monthly trips shopping, online shopping competition is terrible customers are dissatisfaction with traditional shopping particularly departmental stores, dissatisfied customers don't complain, they shopping anywhere else. Shopping must have both physical entity and an online presence researcher used the different social-economic variables. The research found and suggest that 'will be understanding the customer satisfaction.

M. Rajeshwari (2015) has conducted a research study entitled 'satisfaction of customers with online shopping. The study was aimed to describe the factors that affect the customers' satisfaction while shopping online. The data was collected via sending a questionnaire around Chennai city out of those only 105 were sampled. And the so collected were analyzed using percentage and chi-square analysis, consequently the results shown that the perception and the preference of the customers were found to be the significantly affecting factors, in the study.

7. Research Methodology

To describe and analyze customer satisfaction, contributing variables, and other factors, a descriptive study design was used. For the purposes of this study, both primary and secondary data are gathered. The primary data was gathered using a well-structured questionnaire. Books, journals, periodicals, and websites were used to gather secondary data. In Vaniyambadi Town, 80 respondents were chosen using a non-random selection approach. A essential tool such as Simple percentage analysis, Chi–square, and ANOVA test was employed to analyze the data.

Table 1: Showing the Gender of the Respondents

S.No.	Gender	Number of Respondents	Percentage
1	Female	20	25
2	Male	60	75
Total		80	100

From the above table it reveals that, out of 80 sample respondents, 20 respondents (25%) are female, remaining 60 respondents (75%) are male. Hence the majority (75%) of the respondents are male.

Table 2: Showing the Age of the Respondents

S.No.	Age group of the Respondents	Number of Respondents	Percentage
1	Below 20	17	21
2	21 – 30	52	65
3	31 – 40	3	4
4	41 and Above	8	10
Total		80	100

From the above table it indicates that 52 respondents (65%) comes under the age group of 21 - 30 years, 17 respondents (21%) comes under the age group of Below 20 years, 8 respondents (10%) comes under the age group of 42 and Above years, 3 respondents (4%) comes under the age group of 31-40 years. Hence the majority (65%) of the respondents comes under the age group of 21-30 years. From the above table it displays that 58 respondents (72.5%) comes under the Urban area, 22 respondents (27.5%) comes under the Rural area. Hence the majority (72.5%) of the respondents comes under the Urban area. From the above table it reveals that, out of 80 sample respondents, 28 respondents (35%) are Employees, 3 respondents (4%) are Businessmen, 8 respondents (10%) are Housewife, 38 respondents (47%) are Students, 3 respondents (4%) are other occupation. Hence the majority (47%) of the respondents are Employees.

Table 3: Showing the Area of Residence of the Respondents

S.No.	Area of Residence of the Respondents	Number of Respondents	Percentage
1	Urban	58	72
2	Rural	22	28
Total		80	100

Table 4: Showing the Occupation of the Respondents

S.No.	Occupation of the Respondents	Number of Respondents	Percentage
1	Employment	28	35
2	Business	3	4
3	Housewife	8	10
4	Student	38	47
5	Other	3	4
Total		80	100

Table 5: Chi-Square Test to Examine the Influence of Gender of Respondent on Customer Satisfaction

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	18.193[a]	26	.869
Likelihood Ratio	23.471	26	.606
Linear-by-Linear Association	9.264	1	.002
N of Valid Cases	80		

**Significant at 5% level

Effect of Gender

H_0: there is no significance relationship between gender of the respondents and customer satisfaction. H_1: there is significance relationship between gender of the respondents and customer satisfaction. Since the level of significance is more than 0.05, the null hypothesis is accepted. It is concluded there is no significance relationship between gender of the respondents and customer satisfaction.

Effect of Educational Qualification

Table 6: ANOVA to Examine the Influence of Educational Qualification of Respondent on Customer Satisfaction

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	686.213	4	171.553	3.607	.010
Within Groups	3567.337	75	47.564		
Total	4253.550	79			

****Significant at 5% level**

H_0: there is no significance relationship between educational qualification of the respondents and customer satisfaction. H_1: there is significance relationship between educational qualification of the respondents and customer satisfaction. Since the level of significance is more than 0.05, the null hypothesis is rejected. It is concluded there is significance relationship between educational qualification of the respondents and customer satisfaction.

8. The Major Findings are as follows

1. According to the study, the vast majority (75%) of respondents are male.
2. According to the study, the majority of the respondents (65%) are between the ages of 21 - 30.
3. According to the study, the majority of respondents (72%) are Urban residents.
4. According to the study, the majority of respondents (70%) are Unmarried.
5. According to the study, the majority of respondents (55%) are Nuclear family types.
6. According to the study, the majority of respondents (47%) are Students.
7. It is understood from the study that, (35%) of the respondent's educational qualification is up to Undergraduate.
8. According to the study, the majority of respondents (56%) earn a monthly income is below Rs. 10,000.

9. Suggestions

1. As the vast majority of online customers belong to the age group of 21-30, a range of products can be introduced according to their taste and preference.
2. The websites can have impressive ads to get the attention of customers of all age groups.
3. Online vendors have to take the necessary steps to raise public awareness on online shopping.
4. Online retailers need to be most effective in offering a wide variety of quality products and services at reasonable and competitive prices. The product plays important role in the decision to buy the products and services.
5. As the nuclear family buys more from online shopping, more quality products and services should be introduced for nuclear family type groups at affordable prices.
6. Particular attention should be paid to regular purchases by the monthly income brackets who are thinking of purchasing for their family.

10. Conclusion

In Vaniyambadi Town, the goal of this study is to identify, inspect, and evaluate characteristics based on income level, age, gender, academic qualifications, and occupation. The information was gathered from 80 respondents in a random sample, and the above-mentioned analysis was carried out. The data show that respondents are becoming more online aware every day. Companies are finding it difficult to understand their clients' demands in order to sell online. Understanding customers' attitudes toward online purchasing, enhancing variables that encourage customers to shop online, and focusing on factors that drive customers to purchase online will provide marketers a competitive advantage over their competitors. Finally, the opportunity to shop online has profoundly altered and revolutionized our culture. This use of technology has opened up new avenues and possibilities for a more convenient existence in today's world. Online shopping has influenced consumers all around the world in three major ways: wide selection, quick and easy service, and reduced pricing.

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TRENDS OF CSR GROWTH IN INDIA: CHALLENGES AND WAY AHEAD

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Abstract

Corporate Social Responsibility (CSR) is the most debated concept across the world post-Covid-19. Across the world, more so in emerging countries like India, the economic inequality has increased due to pandemic covid-19 which has brought CSR to the forefront.

CSR provides two major opportunities for corporates, the first one is to give back to society and the second one is it serves to pursue their self-interest. The beauty of CSR is to take care of the home and the society where the house is located. There are two dimensions of this namely the internal and external dimensions of CSR.

Though the majority of the corporate's profits increased during covid-19 the alarming decrease in companies and the money spent on CSR and the number of projects has to be seriously looked into by the appropriate agencies and punish as per the law if found guilty. From 2016-17 to 2020-21, CSR spent development sector-wise was highest in education differently-abled, livelihood (EDL)with 38.09 per cent followed by Health and Sanitation with 28.08 per cent and at distant third was Rural Development with 9.92 per cent. From 2016-17 to 2020-21 the number of companies in the category that did the CSR as prescribed decreased from 1848 in 2016-17 to 839 companies in 2020-21, whereas at the same time, in the category of the companies that spent CSR more than prescribed also increased from 3654 to 4942.

The important steps for the way forward are Creating awareness about Corporate Social Responsibility amongst the corporates and the stakeholders; Punishing the companies who haven't spent on CSR as per the law through fine and publicizing it as per law; Officially celebrating a CSR day at the national level and giving the Government announced various categories of CSR awards on that day. This will further sensitize and motivate the companies and general masses.

Keywords: CSR, CIR, Corporates, Self Interest, Challenges, Internal dimension, external dimension, Way forward, trends of CSR Growth, CSR Expenditure.

1. Introduction

Corporate Social Responsibility (CSR) is the most debated concept across the world post-Covid-19. Across the world, more so in emerging countries like India, the economic inequality has increased due to pandemic covid-19 which has brought CSR to the forefront. The Oxfam inequality report 2021 states that in India, the wealth of billionaires increased by 35 per cent during the 2020 lockdown and by 90 per cent since 2009. The increased 100 billionaire's wealth (Rs 12,97,822 crores) during the lockdown is sufficient to give Rs. 94, 045 each to the 138 million poorest Indian people. The increase in the wealth of the top 11 billionaires of India during the pandemic could sustain the MGNREGS scheme or the health ministry programmes for 10 years.

2. CSR helps Corporates More than Society!

CSR provides two major opportunities for corporates, the first one is to give back to society and the second one is it serves to pursue their self-interest.

3. Giving back to Society

Corporates use the factors of production namely land, labour, and capital from the society. Though they make money payments to each of the factors, they gain more in form of profits. Moreover, many corporates do take a lot of concessions in the form of subsidised land, water, electricity and tax concessions for the beginning of the industry. Hence it is the responsibility of the corporates to give back by contributing to society.

Corporates also have a responsibility towards the community and the environment in which their companies operate as they drew raw materials from natural resources. Water for drinking and for the company needs are drawn from natural resources either in the form of digging submersible well or from rivers that belongs to society. It is ethical to give back to the society through which the corporates have grown.

4. Caters to the Self Interest of Corporates

CSR caters to the self Interest of corporates. As rightly pointed out by Smith Adam (1776) "It is not from the benevolence of the butcher, the brewer, or the baker, that we expect our dinner, but from their regard to their own interest". Corporates should understand that CSR is in their interest and it is in their self-advantage. If corporates do their bit through CSR, it further complements the Government initiative of inclusive growth and the standard of living of the community increases. It further augments the interests of the company as the purchasing power of the community increases and demand for the corporate products further increases. CSR helps companies to build, strengthen and sustain their brands and brand loyalty.

CSR also aids in improving the quality of life of the workforce and their families. It enhances the productivity of the workers which facilitates the improvement in quality and quantity of the output which will further increase the profits of the corporates.

Though no specific tax exemptions have been extended to CSR expenditure per se. but indirectly companies when to spend their CSR on activities like contributions to Prime Minister's Relief Fund, scientific research, rural development projects, skill development projects, agricultural extension projects, etc get the tax concessions under different sections more so under 80G of the Income Tax Act, 1961.

5. Various Dimensions of CSR

The beauty of CSR is to take care of the home and the society where the house is located. There are two dimensions of this namely the internal and external dimensions of CSR.

6. Internal Dimension

The corporates through CSR can provide a good working ambience for the workforce and take care of their health issue, using the right mix of raw materials and appropriate technology.

The corporates when uses CSR funds to strengthen safety and health measures in the workplace; use the right mix of raw materials without overusing or minimising wastages; use clean and green technology so that green gas houses emissions to the environment are reduced and used in the work place; paperless administration and recruitment also further augments green initiatives. These measures will result in a low employee turnover ratio and reduced absenteeism and also increase the productivity of the workforce, reducing the cost of production and increase in profits.

7. External Dimension

The external dimension of CSR help in addressing the issues of economic, legal, ethical, philanthropic and environmental dimensions. It connects with various stakeholders like investors, distributors, retailers, consumers and the community. The various stakeholders are reached by ensuring good returns to investors, quality goods at reasonable prices, abiding by the laws and the rule of the land, donating for a cause, supporting public education and health institutions, encouraging gender equality, supporting or undertaking nutritional and poverty alleviation programmes; protecting heritage, art & culture, sponsoring the training for sports activities etc.

8. Trends of CSR Growth in India

In 2014-15, the number of companies that spent on CSR was 16348 companies which spent Rs.10066 crores for 9365 projects in all the 36 states and Union territories, which had increased to 25101 companies in 2018-19 which spent Rs. 20163 crores for 32267 projects in all 36 states and Union territories.

In the first phase of the Covid-19 period that is in 2020-21, the number of companies decreased drastically to 8633 and the money spent on CSR also decreased from Rs. 24864 crores in 2019-20 to Rs. 20360 crores for 25598 projects. The decrease in the number of companies after 2018-19, even before the covid-19 is alarming. Though the

Table 1: CSR Expenditure

Year	No. of Companies	Total Amount Spent in CSR (INR Cr)	States & Union Territories covered	No. of CSR Projects	Developmental sectors entered by Companies
2014-15	16548	10066	36	9365	29
2015-16	18292	14517	36	18468	29
2016-17	19552	14344	36	23076	29
2017-18	21517	17098	36	26858	29
2018-19	25101	20163	36	32267	29
2019-20	22664	24864	36	35285	29
2020-21	8633	20360	36	25598	29

Source: Source: tabulated using the data from
/csr.gov.in/content/csr/global/master/home/home.html

majority of the corporate's profits increased during covid-19 the alarming decrease in companies and the money spent on CSR and the number of projects has to be seriously looked into by the appropriate agencies and punish as per the law if found guilty. The following clustered column charts give the graphical representation of the same.

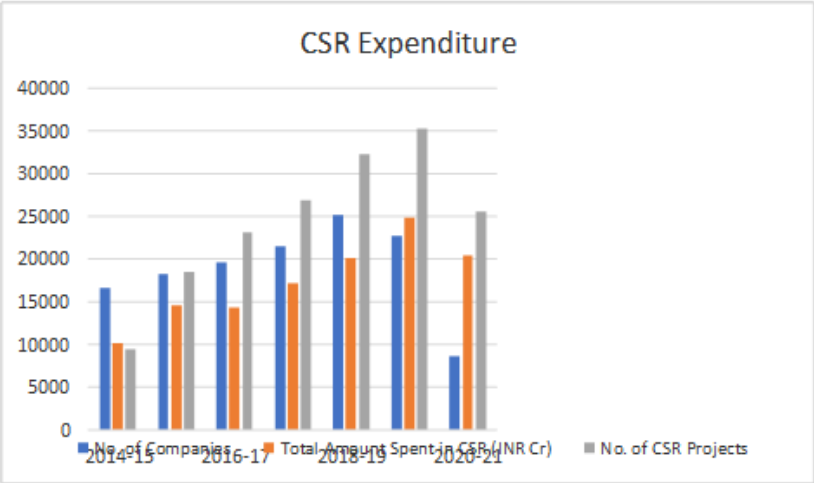

Figure 1: CSR Expenditure
Source: Derived from the Table 1

9. CSR Spent Development Sector Wise

From 2016-17 to 2020-21, CSR spent development sector-wise was highest in education differently-abled, livelihood (EDL)with 38.09 per cent followed by Health and Sanitation with 28.08 per cent and at distant third was Rural Development with 9.92 per cent.

Table 2: CSR Spent: Development Sector-wise (Rs in crore)

Year / Sectors	2016-17	2017-18	2018-19	2019-20	2020-21	2016-17 to 2020-21
Encouraging Sports	180.33	285.41	308.87	301.92	212.57	1289.1
EDL	5558.81	7281.23	7982.71	9593.11	6463.51	36879.37
Clean Ganga Fund	24.37	33.96	8.11	6.63	7.22	80.29
AOF	419.99	292.73	730.96	930.44	1327.85	3701.97
Health & Sanitation	3669.47	4269.08	5527.76	6808.43	6919.66	27194.4
GWOR	468.75	581.26	570.86	693.15	322.78	2636.8
EAC	1317.66	1660.4	1700.79	1793.2	1035.13	7507.18
PMNRF	158.8	200.42	320.46	796.98	1452.94	2929.6
Others	388.96	15.2	87.61	503.4	267.29	1262.46
Other Sectors	60.95	46.03	122.27	115.4	116.74	461.39
HA & C	305.57	395.22	225.08	932.74	396.97	2255.58
Swachh Bharat	184.06	272.07	95.5	53.37	148.14	753.14
Slum Development	51.49	39.16	51.06	42.91	16.63	201.25
Rural Development	1554.78	1722.55	2418.09	2289.02	1617.45	9601.89

Source: tabulated using the data from
/csr.gov.in/content/csr/global/master/home/home.html

The fourth place in the development sector CSR expenditure wise was in Environment, Animal Welfare, conservative of resources with 7.75 per cent, Any Other Fund with 4.5 per cent, Prime Minister National Relief Fund with 3.03 per cent, Gender

Equality, Reducing Inequalities, Women Empowerment, Age-Old homes with 2.72 per cent, Heritage, Arts and Culture with 2.33 per cent, Encouraging Sports 1.33 per cent, Swatch Bharath 0,78 per cent, Slum Development 0.21 per cent and Clean Ganga 0.08 per cent were the rest of the sectors where the CSR was spent over five years.

10. CSR Top 10 States

The state wise location of industries which contributed for the CSR is discussed in this section. Maharashtra (Rs.14605 crores) topped among the Indian states in CSR spending from 2016-17 to 2020-21 followed by Karnataka (Rs.5780 crores), Gujarat (Rs. 5106 crores), Tamilnadu (Rs. 3898 crores), Delhi (Rs. 3741 crores).

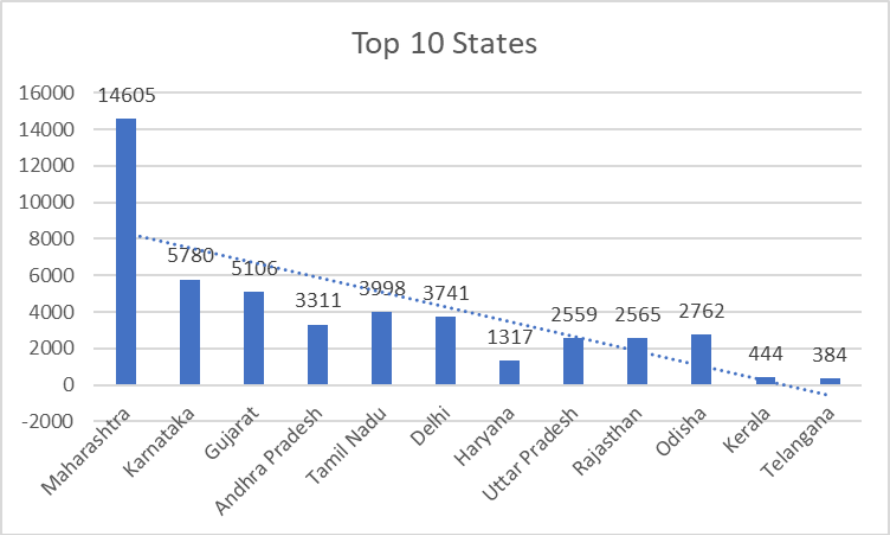

Figure 2: Chart No 2: Top 10 States

Source: derived from the data obtained from csr.gov.in

The other major states share in the descending order were Andhra Pradesh AP (Rs. 3311 crores), Odisha (Rs. 2762 crores), Rajasthan (Rs. 2565 crores), UP (Rs. 2559 crores), Haryana (Rs.1367 crores) Kerala (Rs.444 crores) and Telangana (Rs.384 crores).

The ranking of the states more or less correlates with the industrialisation of the states.

11. Share of PSUs and Non-PSU Companies

The share of non-PSUs or private corporations in CSR from 2016-17 to 2020-21 varied between 77 per cent to 80 per cent while that of PSUs during the same period varied between 23 per cent to 20 per cent. The following clustered column chart gives the graphical representation of the same.

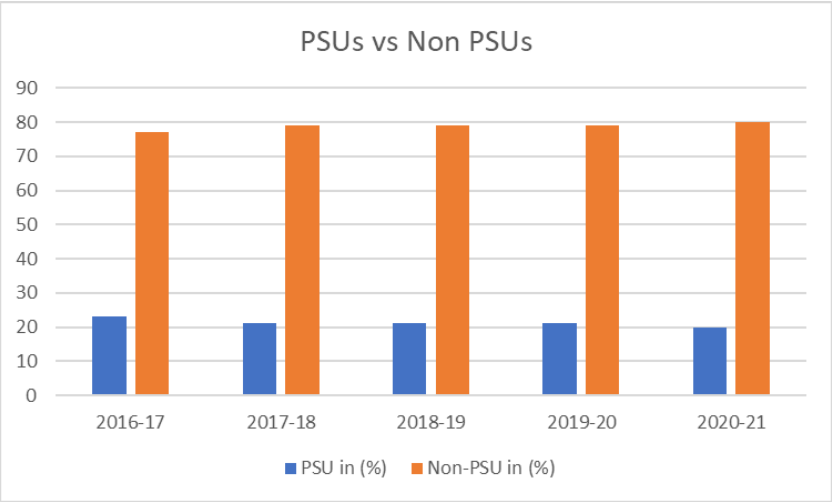

Figure 3: Chart No 3: CSR Spent: PSU vs. Non-PSU Companies
Source: derived from the data obtained from csr.gov.in

12. Listed and Unlisted Companies

The listed companies' CSR expenditure increased to Rs. 12682.95 Crores in 2020-21 from Rs. 8572.52 crores in 2015-16. During the same period the unlisted companies' CSR expenditure to Rs7675 crores from Rs.5944.68 crores. The listed companies contribute more than unlisted companies in the total share of CSR. The following clustered column chart gives the graphical representation of the same.

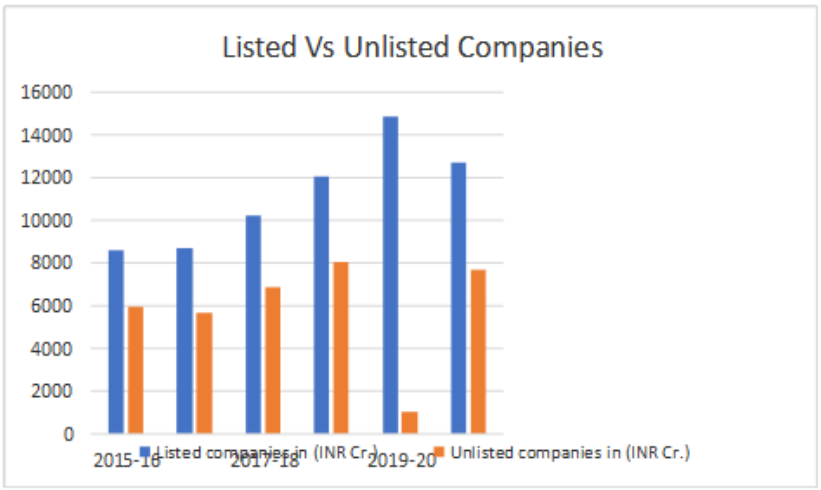

Figure 4: Chart No 4: listed vs Unlisted Companies
Source: derived from the data obtained from csr.gov.in

13. CSR Compliance

From 2016-17 to 2020-21 the number of companies in the category that did the CSR as prescribed decreased from 1848 in 2016-17 to 839 companies in 2020-21, whereas at the same time, in the category of the companies that spent CSR more than prescribed also increased from 3654 to 4942. The following clustered column chart gives the graphical representation of the same.

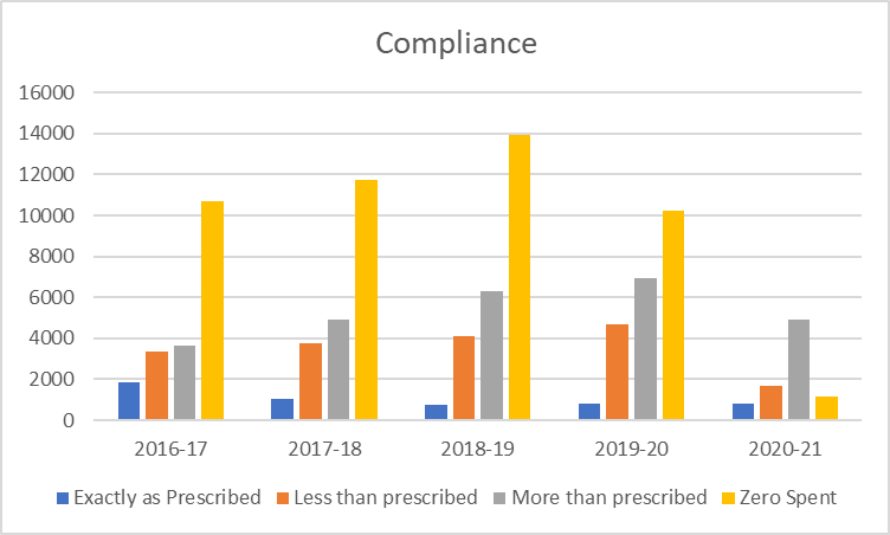

Figure 5: Chart No 5: CSR Compliance
Source: derived from the data obtained from csr.gov.in

The cause of worry is that in the category of the companies less than prescribed though reduced from 3335 to 1702 companies during the same period are still in good number and in the category of the zero spent companies still were 1149 in 2020-21 which had reduced from 10714 companies in 2016-17.

14. Challenges

The important challenges are

1. Running a business and keeping in mind social responsibility is really a challenging task.

2. Many corporates do not understand CSR advantages to their companies and hence are not serious about its implementation. Making the corporates understand it is a bigger challenge

3. Failure to understand CSR's role in the holistic development of their company's growth. Making the corporates realize beyond the legal frame work is another challenge

4. Lack of Transparency in the implementation of CSR is as good as noncompliance. When companies half-heartedly without realising the benefits do it, they often result in window dressing.

5. CSR implementation cannot be properly monitored as it is in various fields and require more staff. The uninterested corporates either use the corrupt way of cooking the data or greenwashing which gives a dubious but glorious picture.

6. Corporates should keep aside the funds in the profits exclusively for CSR every year. Few companies have the habit of planning for expansion and other investments without separating the funds. Hence, in the end, resorts to corrupt practices.

7. Lack of coordination and consensus among different local agencies as well as corporate entities results in duplication of efforts by the firms in terms of CSR initiatives.

8. Lack of active participation of the stakeholders in the implementation of CSR is another hurdle

9. Need to constitute an exclusive team to the identification of the local and national needs of the society and the successful implementation of CSR. Many corporates do not identify and recruit the staff for CSR implementation and training.

15. Way Forward

The important steps for the way forward are:

1. Creating awareness about Corporate Social Responsibility amongst the corporates and the stakeholders.

2. Sensitizing the students by introducing CSR modules compulsorily at graduation and post-graduation levels. Organizing seminars, webinars, essay writing, Quiz for students and faculties to sensitize about CSR

3. Punishing the companies who haven't spent on CSR as per the law through fine and publicizing it as per law

4. Officially celebrating a CSR day at the national level and giving the Government announced various categories of CSR awards on that day. This will further sensitize and motivate the companies and general masses.

5. Every company must invariably announce their CSR initiatives in their annual reports and websites

6. Bring transparency to the CSR implementation by framing appropriate measurable indicators

7. Every year the government can announce the advisable areas for CSR and the amount required for the implementation of the programme. It avoids wastage, and duplication but brings in transparency and helps in the implementation of the outcome-based successful programme.

8. Broaden the definition of the companies that have to go for CSR, so that more companies join hands in the noble work.

16. Conclusion

CSR is greatly beneficial to the corporates themselves as it helps them more than society. It can be added tool to the Government's efforts to attain inclusive sustainable growth of the economy. The best CSR practices of various corporates in India and the World can be published so that other companies may be inspired by them. Implementation of a carrot and stick policy with reference to the implementation of CSR is the need of the hour. Am sure that once corporates realize the potential benefits of CSR, they will spend more than the stipulated amount.

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ALOFNESS FROM INDIAN CULTURE AND TRADITION- A WAY TO RESTLESSNESS IN MODERN INDIAN YOUTH IN UPAMANYU CHATTERJEE'S ENGLISH, AUGUST: AN INDIAN STORY

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Abstract

The Indian culture and tradition has gone under so many changes during post-colonial India. In fact there has been a different outlook of people specifically the English educated urbanized Indians. Nevertheless, they are Indians by nationality, hitherto their attitude towards Indian cultural and traditional values seem to be unsatisfied. The modern educated youth sees Indian culture and her traditions as some hoax. He even does not know that it is he who is eradicating the culture of his own mother land in the name of fashion and modernity. This picture of the changed outlook of this sort of modern youth is sketched by the realistic Indian novelist, Upamanyu Chatterjee in almost all his works especially in the novels English, August: An Indian Story, The Last Burden, Way To Go and The Weight Loss. It is the loss of the protagonists' self-identity that they (the protagonists Agastya, Jamun and Bhola) are away from the culture of their own country.

Keywords: Indian Culture and Tradition-Fashion and Modernity-Loss of Identity.

About the Theme

UpamanyuChatterjee is today's one of the energetic and enthusiastic Indian writer who focuses the realistic elements of human life especially the Indian urbanized society that has either knowingly or unknowingly distance itself from the Indian mass. It is the westernization that has deep rooted in their Indianness keeping them away from the

social gathering and trying to find their own way out of Indian culture and tradition. This further leads them to the path where they stand alone and wander restlessly for their own social identity.

The protagonists of UpamanyuChatterjee are the embodiment of youth of post-colonial India who are dominated by culture of the west distancing themselves from the Indian cultural and traditional values knowingly. They are attracted towards western way of life leaving behind their indigenousness. In fact they feel pride in presenting themselves as English rather than as Indian. They love to be identified as modern and English educated rather than traditional Indians. They are fond of drugs and women and least bother the spirituality of India among other nations in the world.

Agastya, Chatterjee's first protagonist is westernized and is deeply moved by the western life style hence he is not able to adapt to the new scenario in the small provincial place called Madna where he is posted as an IAS trainee. As long as he is in Madna, his life poses him a threat. He is neither adaptable nor accountable there. He tries to untie himself from the circumstances of the town. He feels that he would have some solace if he is back to Calcutta. He even goes to the city with the notion to find other job except bureaucracy. He is prepared to work even in a publishing agency but hesitates to move further due to his incapability of decision making. It is his westernized nature that haunts him throughout the novel.The life in Madna for Agastya is like the first line of William Wordsworth's 'Daffodils', 'I wandered lonely as a cloud... It is the domination of English life that he wants to get off from Madna even resigning his government jobmerely for his westernized life and merriments.

English, August: An Indian Story helps the reader understand the worthiness of a good company and affability with others. Being silent and reserved would though benefit one, but not the other. Agastya believes that he is the right person at a wrong place, so he loses the enjoyment of activities around him in Madna. He is not in a position to enjoy the new scenario atMadna nor does he enjoy his new job experience. Had he focused either the vibe or new place, he could have learnt many new things from the town and would not complain of his desertedness to anyone. Chatterjee through these Indian urbanized protagonists presents the ebbs and flows of the post-colonial bureaucracy, education, politics and the impact of western culture upon the Indian society.

Sunil Bhatia's point of view on call centers in Indian justifies Chatterjee's insistence that today's educated Indian youth is away from the culture and tradition of the country. Apart from men, there are Indian women who too have distanced themselves from the rich cultural and traditional heritage of India.Their westernization is sketched by Sunil Bhatia stating, "*Working in call centers required women to adopt a 'strategic' view about their individual identities by not being bound by their traditional roles in the family*" *(Bhatia 155)*.

The Indian culture and tradition has gone under so many changes in the post-colonial India. In fact there has been a different outlook of people specifically the English

educated urbanized group. They are though Indians by nationality, yet their attitude towards the Indian cultural and traditional values seem to be unsatisfied. The modern educated youth sees Indian culture and traditions as some hoax. He even does not know that it is he who is eradicating the culture of his own mother land in the name of fashion and modernity. This picture of the outlook of modern youth has been sketched by UpamanyuChatterjee in his works especially in the novels *English, August: An Indian Story*, The Last Burden, *Way To Go* and *The Weight Loss*.

The loss of culture and tradition drag the protagonists to the loss of their identity. Their distancing from Indian culture pushes them further into the darkness of restlessness and loss of identity. This loss of identity and restlessness stops their growth to know and learn new things in life. Sharma states the origin of Indian culture saying, "*The basic of Indian culture is self-knowledge. The originators of Indian culture, the seers and mendicants came to the conclusion that man should know himself because that is the only way in which the truth of the world can be known with any fullness*." *(Sharma6)*. It should be noted that India has her own culture and tradition that should not be shattered at any cost.

Conclusion

Chatterjeebrings under limelight the draw backs of bureaucracy and the domination of westernization in the Indian youth with English education and impressed with the culture of the west. The novelist puts forth the westernized Indian way of life in a highly traditional country like vividly presenting the antagonized outcomes faced by the protagonists that are modern and dominated by westernized life style. He highlights how a modern Indian youth is dissociated from his own counter parts and feels himself the 'other'. The novelist attempts to make the modern youth the real worth of Indian culture and traditions.

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MUSLIM WOMEN'S EDUCATION IN VELLORE DISTRICT OF TAMIL NADU IN 19th CENTUARY

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Abstract

Vellore District holds a historical significance and also it occupies a distinct place in the field of education, for its several educational institutions right from the elementary level to colleges. These educational institutions are run by both State and Central Government and some are maintained by the private managements. It wouldn't be wrong on our part if we call it an educational hub. This study will explain about the importance of Muslim women's education in Vellore District of Tamil Nadu.

Introduction

Vellore possesses a fascinating background in the history of Tamil Nadu. It holds a fusion of ancient and rich cultural heritage of Dravidian Civilization. Since ancient times the District had been famous for its economic charms and strategic importance. The modern Vellore District holds a historical significance and also it occupies a distinct place in the field of education, for its several educational institutions right from the elementary level to colleges. These educational institutions are run by both State and Central Government and some are maintained by the private managements. It wouldn't be wrong on our part if we call it an educational hub. This study will focus on the importance of women's education in Vellore District.

Education for women is one of the most critical areas of empowerment of women. Offering the right to higher education for girls, provides much greater power of enabling them to make better choices in life and move towards the development of herself and her family. A woman is the one who is liable either to produce good citizens or bad.

Women undertake a greater responsibility as mothers. That is why they are most reverently termed as 'Mobile Universities'. A woman can pass on the best of her knowledge to her generation only if she is educated and possesses the knowledge of

right things. Without being educated how is it possible for a woman to nurture her children in the right direction. If a man is educated, the benefit of his education is limited to him.

When a woman is educated a whole generation avails its benefit. The major drawback of uneducated mothers is neither they support the family financially nor do they bring up their wards efficiently. So at least providing the basic education for girls is very essential for the growth of families and development of countries. We all might have heard that "Behind every successful man there is a woman". But now the days have changed. We can get the instances to prove that there is a man behind a successful woman. Earlier the girls were not allowed to acquire education or go to schools where boys studied.

In Islam right to education was given around 1400 years ago. Prophet Muhammed (PBUH) had declared, "The pursuit of knowledge is incumbent on every Muslim male and female". As a result of his declaration it was noted that every man and woman rendered his or her contribution in the field of education. Slavery was common in Arabia. Prophet Muhammed (PBUH) gave the right to education even to the slave girls of the Arabia.

He gave the best education to his daughter Hazrath Fathima made her an example in front of the world that there should be no preference for the boys to the girls in the matter of education. Both hold the equal right to acquire education. Hence it is proved that Islam is not an impediment for women from being educated and being successful in the life.

To overcome this impediment separate schools for girls were opened not only in Vellore District but also in other districts. Most of the Muslim parents did not prefer to join their daughters to co-education schools because of the concept of Pardha. Great thanks to the mindset of the parents, who at least realized the need of education for their daughters and sent them to girls' schools.

As a result the due importance is given to girls' education unlike earlier days. Gone are the days when the women were referred as weaker sex or inferior to men. Even now in rural areas girl's education is not given much importance particularly the Muslim community which has its mind set that the girls are meant for doing only house hold chores, though seeking knowledge is the fundamental duty of every man and woman.

Development of Muslim Girls' Educational Institutions in Vellore District

A majority of population in Vellore District consists of Muslims. The pathetic part is most of the Muslims are living below poverty line. These parents do not send their wards to school and retain them at home so that they can help in their work to earn a little more money.

The establishment of schools in Vellore District is as follows

Municipal Muslim Girls' Primary School, Ramanaikan Palayam (Rahmathpala, Vellore)

This school was founded by the local Muslims of the District in the year 1923 and was recognized by the Government in the same year [1]. The statistical data of the school says that there were only 50 students totally in the classes from I to V Standard and they were handled by 3 teachers. Presently there are 200 students and 6 teachers including a Tamil Teacher who run the school. The school follows the syllabus prescribed by the Tamil Nadu State Government and the medium of instruction is Urdu. Most of the Girl students, who complete their primary school do not go to High School. If we see the year of establishment almost one Century is going to be completed but the increase in strength is only four fold.

Arcot: Madrasa-e-Yomia, Arcot

In Arcot the one and only Muslim Girls' educational institution is Madrasa-e-Yomia.Initially the school was started in 1926 [2] as an orphanage for Muslim boys and girls. The Nawab of Carnatic donated endowments for the maintenance of this orphanage. In the beginning it functioned in a rental building but it was soon shifted to a building donated by the Prince of Arcot. The institution rendered Primary Education to the girls of Arcot and its suburbs. In 1960 it was undertaken by the Municipality of Arcot and the name was changed as Municipal Girls' Elementary School. The school functions even today with the aid provided by the Prince of Arcot. Mr. T. Mohammed Rizwanullah is the Headmaster of the school along with two more staff.

Municipal Muslim Girls' Primary School, Saidapet, Vellore

This is a small school established by the Muslims of Vellore District before 1930 AD but it took nine years to get the Government recognition [2]. It is an Urdu Medium School with three Urdu staff and one Tamil teacher. The girls from the nearby areas study in this school and get benefitted.

Municipal Girls' Primary School, Dharnampet, Gudiyattam

The school was found in the year 1935 and it was recognized by the British Government in 1940 [3] and presently it is managed by the town's municipality. The total strength of the school is only 150 students and they are taught by five teachers. After completing the primary level education only few girls pursue their education and join High Schools. The drop outs help their parents in the Beedi making due to their poor financial backwardness.

Panchayath Union Urdu Girls' Primary School, Valathur

Valathur is a small town situated 8 Kms away from Gudiyattam. 80% of the total population consists of Muslims. It was founded and recognized by the Tamil Nadu State Government in the year 1938 [4]. The school functions from Standard I to V with only a strength of 50 students who are guided by two teachers and Janab Sirajuddin Sahib

as the Headmaster of the school. The poverty and occupation of Beedi rolling restricts the parents to send their children to school. The school is under the direct control of the AEEO (Additional Elementary Educational Officer).

Municipal Muslim Girls' Primary School, Thazyatham, Gudiyattam

The school came into existence in the year 1939 and was recognized by the State Government in the year 1948 [5]. The school was started with the strength of 50 students and no progress had been noted in the increase of strength. Presently more than hundred girls are studying in this school. The reason behind this is the poverty and Beedi occupation of the parents. The subjects taught in this school are Urdu, English, Mathematics, Science and Social Science with moral education. The medium of instruction is Urdu.

Madrasa-e-Niswan, Melvisharam

This institution is managed by Anjuman-e-Ansarul Islam [6] which focuses on religious education, and it was founded in the year 1941 [7]. Currently Janab Malack Mohammed Hashim Sahib is the President of this Madrasa and Secretary of the Madrasa is Malack Akbar Hussain Sahib who are the backbone of this religious institution.

To provide the religious education to the Muslim Girls of the locality was the supreme objective of the institution. Presently Janaba Shakira Sahiba is the Headmistress of the school where around 500 students are getting educated in the guidance of 27 trained teachers. The salary of the staff and other maintenance expenditure is overcome through the donations of the well wishers and other stake holders.

Madrasa-e-Niswan Aided Muslim School, Mel Alathur

Mel Alathur is a small village within Gudiyattam Taluk in Vellore District. It is situated 32 Km towards west from the District Headquarters and 3 Km from Gudiyattam. The objective of establishing the school was to impart both secular and religious knowledge in the year 1965 by local Muslims established the school. In the year 1968 the school was recognized by Government of Tamil Nadu for Classes I to VIII [8]. Janab Noorul Ameen Sahib is the Correspondent of the school who runs the school with the assistance of Janaba Qamar Taj Begum as the Headmistress accompanied with 24 teaching and 6 non-teaching staff. They strive hard as a team towards the progress of the institution as well as the Muslim girls' welfare. The school has a separate prayer hall to promote Namaz and 13 class rooms. AMES (Ambur Muslim Educational Society) undertakes this devoted service of establishing educational institutions right from Primary to Collegiate level since one hundred year.

Nusrathul Islam Girls Higher Secondary School, Pernambut

There are many schools that had been established for boys and they even started paying attention towards the Girls' education too. Nusrathul Islam was established in the year 1970 and was upgraded as Higher Secondary School in the year 1988 [9]. Janab

Mohammed Ihsan Sahib is the Correspondent of the school who is striving to improve the status of Girls' education in Vellore District. The highest of all, the school has a total strength of 1350 girls with 39 staff who mould and nurture the students with both secular and religious teachings.

The school has an infrastructure of 30 class rooms and well equipped laboratories for Physics, Chemistry and Biology as well as Computer Science. The medium of instruction is Urdu and it has an equipped library with 5012 books. There are 16 computers in the computer lab for teaching and learning. The school office functions with five non-teaching staff.

Urdu Aided Girls Middle School in Baqiyath Street

The school holds a considerable strength of 290 girls. The syllabus recommended by the State Government is being followed here along with religious instructions like recitation of Holy Quran and Hadith. In spite of having Urdu as the medium of instruction with the religious teaching, most parents do not send their girls to school after Standard V or once they attain puberty [10].

The school was established in the year 1974 [11] as primary school and soon it was upgraded as Middle School by the State Government. The credit of foundation and maintenance of the school goes to the undaunted efforts of Janab Anwar Sahib, the Honourable Correspondent and General Secretary of the school. Presently Mrs. Shakeela Banu is heading the school with her sincere attitude towards the development of the school.

Urdu Girls' Aided Primary School, Pernambut

The school was started in the year 1974 at Tipu Street in Pernambut. It is an Urdu Medium school functioning with Class I to V. Janaba Najmunnisa is the Headmistress of the school who has taken the school towards progress with the increased strength of 420 students on roll. 8 teaching 2 non-teaching staff are employed to meet the academic and administrative needs of the school. It is a State Government aided school which functions over an area of 7100 Square feet [12].

Madrasa-e-Niswan, Vellore

In Vellore, two schools named as Madarsa-e-Niswan were founded. One is in Ramanaikan Palayam established in the year 1975 and the other one is located at the Baqiyath Street, opposite to the Baqiyathus Salihath Arabic College. These schools are very famous for providing religious teaching and for following Islamic tradition in case of uniform. Most of the girls of Vellore and surrounding town study in Madarsa-e-Niswan since they are allowed to be in Pardah. Even the female teachers, who work here, wear the Purdah. They also conduct Namaz as an important part of the curriculum. Since the school is run by private management they collect monthly subscriptions from the affluent. The school also owns a private transport facility for the girls and Nagibas (Conductress) for providing the safe conveyance to the girls. Madrasa-e-Niswan has

recently introduced the Department of Hifz (memorizing the Holy Quran), which is helping the students in memorizing the Quran.

There is a Tailoring Institute to train educated and poor Muslim women to stitch clothes and design them. The Falah-e-Millath Trust along with some representatives who have dedicated themselves for the cause of women's welfare from different localities is running this tailoring school. Every day two batches of girls learn tailoring from a lady instructor.

To meet the existing demands of the parents for the protection of their daughters and for the purpose of Purdah, separate girls schools were established in Vellore District in different parts, but these schools were only up to Standard X and the girls had to either move on to the co-education schools or to schools of secular education run by other communities and by Government where Islamic teaching was not included.

The Government Muslim Higher Secondary School at Vellore, St. Mary's Higher Secondary School, Bala Vidhya Mandhir Higher Secondary School and Shristi Matriculation Higher Secondary Schools are the ones that offer higher education. And to the amazement 75% of Muslim girls continue their education by joining these co-education along with the boys which is highly condemned by most of the Ulemas and parents.

The remaining 25% of girls who are not sent for higher studies remain backward in the field of education and this is the major drawback in the developing path of education growth among Muslim women in Vellore District.

Gudiyattam

Not much educational growth is found in Gudiyattam since most of the population consists of Muslims and involve in the Beedi making. Though Gudiyattam is a Taluk Headquarters and a cosmopolitan town most of the population suffers poor economic status. Girls of Gudiyattam are not sent to the schools for they are compelled by their parents to assist them in rolling Beedies. Hence the rate of literacy of this place is very low among Muslim girls while compared to other surrounding areas.

Rameeza Oriental (Arabic) Girls' High School, Pernambut

The school was established as a Primary School in the year 1986 and later upgraded as a High School. The strength of the school is 380 students on roll with 8 teachers and 2 clerks for administrative purposes. Janaba Mahmood Begum is the Headmistress of the school who works dedicatedly for the progress of the school and its students. Janab V. Khaleel Ahmed Sahib is the Honourable Correspondent of the school who had devoted his life for the welfare of Muslim girls' education. The school has a separate prayer hall for Namaz and 14 well ventilated class rooms. Students are admitted from Class to VI to X and the school was recognized from the Department of School Education of Tamil Nadu Government [10]. There is provision for *Hifz* that is, learning of the Holy Quran by heart with due emphasis upon *Tajwid*, the art of Arabic accent and pronunciation.

Madarsa –E-Saleema Niswan Middle School, Gudiyattam

Madarsa-e-Saleema Niswan Middle School is a non-aided middle school managed by a Muslim Management. It was established in the year 1997 keeping into considerations, the expectations of Muslim parents for their girls. Primarily the medium of instruction in this school is Urdu but Tamil and other languages are taught as second language, the ratio of student teacher is 31:1 in this school.

The school aims to provide the best learning environment to 248 girls from Standard I to VIII with the help of 11 trained teachers who guide them and inculcate good morals and qualities in them. The Purdah Sysem, Deeniyath teaching and afternoon Namaz Zuhar is followed up punctually. It has a library with around 100 books and computer lab with 10 working computers which are used for teaching the basic computer knowledge to the students.

Pernambut

Pernambut is a municipality in Vellore District, and a village headquarters. Pernambut is the second most populated town of Vellore District after the separation of Tirupattur and Ranipet District with a population of 51,271 as per 2011 Census. Leather Tanning Industry is the source of income of this locality. It is even called as 'Leather Tanning City of Tamil Nadu' [13].

Melvisharam

Melvisharam is a town which is densely populated with Muslims. The people of Visharam are highly pious and strictly follow Islam. For more than 300 years the women of Melvisharam had been learning Quran and trained in religious teachings.

In the beginning the girls were taught at their residents and later in mosques which had been established in the town. It was in the second half of the 20th Century when they realized the increasing need of education for the women and girls and laid the foundation of various institutions to teach both religious and secular education.

The area which had been once a war field had developed into an educational hub of South India in due course of time and brought a major transformation in the field of education. There is a proverb in Urdu which gives the following meaning, "Where there is life, there is world". The very first initiative taken for the welfare of women was to give her the right to live. Even after that a woman's life was not less than a bonded labour who spent her whole life under the dominancy of men in life, first her father, then her husband and even in her old age she has to abide by the dominant attitude of her male children [14].

Thus a woman's life in the ancient period ended with this above mentioned role. It was the advent of Islam which clearly mentioned through the light of the Holy Quran and Hadith that "Seeking of knowledge is essential for both men and women".

The implementation of this verse resulted in the eradication of illiteracy from the Muslim society especially through promoting women's education. The realization of parents for the need of secular and Western education had made it a reality that

most of the girls and women in Vellore District started living a civilized life through their education. The contribution of Entrepreneurs and Philanthropists of both the communities resulted in the establishment of various schools and colleges in Vellore District.

The State Government has also evinced its great interest in rendering recognition to the elementary Girls' Schools and High Schools. Though there were no separate schools for Muslim Girls in higher secondary education, the parents sent their daughters to co-education schools for the purpose of completing their Higher Secondary Education. Later that there were many women's colleges in which they sought the admission retaining the dress code and modesty of Islam.

Through this study we have to come to a conclusion that there is a considerable growth in women's education in Vellore District from the mid of 20th Century until now. In the early days religious education was imparted in the Makthabs and Madrasas – the institutions purely meant for teaching Arabic, Persian and Urdu. The recitation of Holy Quran and the study of Hadith was the main focus of these institutions.

The new stream of education in form of secular and Western education was introduced with the rise of British rule in India. It was essential for the growth of a society or state to learn Western education for the scientific and professional advancement. Though religious education is a complete guide to the Muslims for how to spend their life in this mortal world and prepare oneself for the immortal and eternal life, the advancement of technology and English knowledge is necessary to be at the fore front in every field and sector of this highly competitive world [15].

The gift of English made both the Muslim men and women with the fully equipped sense of social etiquette. Presently Muslim girls are given equal rights to acquire education along with their brother without being affected through gender variation. Most of the affluent parents, in fact the middle class parents also send their daughters to the Matriculations, CBSE schools to provide the best education. There were different phases of improvement in the mind set of the parent [16].

Phase I The parents did not allow their daughters to acquire education at all and they were restricted to learn the recitation of the Holy Quran.

Phase II Though the parents welcomed the Western education, they preferred it for their sons but not for their daughters as they thought that worldly knowledge is not the necessity of girls. They can learn Urdu and Persian along with learning Holy Quran and Hadith.

Phase III The broadmindedness of parents allowed their daughters to seek secular education but from the Government and free education centres. They did not prefer to spend on the education of girls as they spent for their sons, who will earn the money for them in future.

Phase IV This is the current scenario of the Muslim community. If a family can afford to pay fee in a private institution, it sends both the son and daughter to the same standard of education. Discrimination of boys and girls has been wiped out from the present world. There is no question of spending equal money on girls' education as on the boys. It has become a mandatory right of girls to get equal benefits that a boy enjoys in the field of education. Infect girls have over taken boys in studies and acquiring better positions due to their sincerity and devotion towards their career. This phase of Muslim parents' attitude need to be applauded.

Initial steps taken for the growth of education

Elementary school education which is the fundamental right of every child doesn't stop with primary school, but it has to be resumed with high school and Higher Secondary. It has been noted that the establishment of primary schools concentrated much over the native language and Western education. They trained the in-service teachers to teach the secular education.

The establishment and growth of primary schools arised the necessity of founding the high schools and Higher Secondary Schools. The following of curriculum as per the demands and needs of Muslim girls promoted women's education in Vellore District. The introduction of Vocational Groups like Home Science, Dress Designing, Food Service Management, Agricultural Science, Nursing, etc. which were highly welcomed by the parents to further encourage the Higher Secondary Education.

The Government of Tamil Nadu has permitted to establish one Higher Secondary School within the distance of every 8 Km. During the recent years the Higher Secondary and College students received several benefits and concession to pursue their higher studies. They were provided with free bicycles, laptops, incentives and awards for the toppers of the Higher Secondary, etc.

Most importantly they were provided with the books of regional language in government and aided schools. In spite of all these provisions, presently Matriculations and private management schools are preferred by the parent which impart English Medium education. Subsequently the demand for Urdu Medium Schools is getting reduced and henceforth, all the Government Schools too have facilitated English Medium in the Government and aided schools.

Many Educational Trusts and Women's Welfare Societies are maintaining separate Girls' Schools in Guidyattam, Pernambut and few other towns. The role of Madarsa-e-Niswan in imparting religious education along with some Basic English knowledge for the translation purpose of Holy Quran and Hadith needs to be applauded.

Overall statistical data shows an increased literacy rate among Muslim women in Vellore District. Further it has been observed that most of the topper girls who pass out their Higher Secondary with high cut off marks do not join any professional course. This was due to the poor financial background and co-education in professional courses.

With the change of attitude and realization of socio-economic conditions the parents allow their daughters to join co-education professional courses which resulted in the

emerging women Engineers, Doctors, Chartered Accountants among Muslim women. This has raised not only the social standard of the Muslim women, but also the financial status of Muslim families which had been suffering the poverty before the advent of Western and secular education.

The Muslim Educational Societies and Trusts of Modern Vellore District are now taking their lead in establishing Arts and Science Colleges for Women, never neglecting the institutions for religious teaching and also the Tailoring institutes and College of Education have been founded by the Muslim Entrepreneurs of Vellore District.

Apart from the sphere of education, the Muslim girls should be made aware of the socio-economic and political affairs that are taking place in the outer world and should be made capable of expressing their views for the rising problems. Freedom of speech and expression of ideas should be given to them as their right.

One should not ignore the religious values and teaching while marching forward in the path of professional education. Both should be balanced properly in life to retain the inter link between the creator and its creation. Secular education which is provided commonly to all the citizens is taken care by the State Government and Philanthropists. Similarly the Muslim Madras, Mosques and religious institutions should focus on imparting religious education to women as part time classes. The role of Muslim mothers is also very important to retain the interest in religious learning.

They should not go blind behind the glory of mortal prosperity. Mother's encouragement to kindle the spark of wisdom in the minds of girls is the secret behind the progress of a national. Alice Paul has rightly quoted that "There would never be a new world until women become a part of it. If women are educated, $3/4^{th}$ of the country's population would be educated. Women should stop being the candles of sacrifice which burn itself to give light to others. Instead they should be the blazing torches of guidance and inspiration to the future generation".

Finally to sum up the study it has been observed keenly that Muslim educational societies and institutions of Vellore District have been rendering the best of their services in imparting both the religious, secular and professional learning in an extraordinary way to wipe out illiteracy from the Muslim Women's Society. Yet there are some measures which need to be implemented to take the Vellore District to the epitome of prosperity and Cent percent literacy rate. In this whole process of development through Western education Girls must be trained to retain and respect the values of Islam and follow the traditional customs. They should never let the liberty of being educated spoilt or misused in any way to ruin the reputation of Muslim Community.

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GROWTH OF LEATHER INDUSTRY DURING THE PERIOD OF KAKATIYAS OF WARANGAL, AS REFLECTED IN FOREIGN ACCOUNTS

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Abstract

The age of the Kākatiyas of Warangal is marked with significance, especially in the fields of agriculture, irrigation and industry. It is an accepted fact that, the name of the Kākatiyas is symbolically associated with irrigation system. The tanks and lakes of the period are still serving the farmers of Andhradesa. The development of agriculture and industry gave scope for the remarkable development in trade and commerce as a result of which the Andhra country witnessed development in every sphere.

Keywords: Guilds, Aadayam, Jataras, Kirrcheppulu, Bokkenas, Etamulu, Puttis, Ayagars.

The Kākatiyas of Warangal occupied an important place in the history of south India. After Satavahanas, Kakatiya rulers united the Telugu speaking area and ruled most of eastern part of Deccan region, consist of present Telangana and Andhra Pradesh and some parts of eastern Karnataka and southern Odisha between 12^{th} and 14^{th} Centuries. Their capital was Orugallu, called as Warangal. Therefore the present study is undertaken to study the Development of Leather industry under the Kākatiyas [1] to have a probe into the factors that contributed for their development.

Foreign, travellers like Marco-Polo, Ameer khusro and Abdul vasaf's accounts [2] mentions that Andhradesa was economically very sound during the Kākatiyas period, which attracted the Delhi sultans to concentrate on Kakatiya kingdom. Though agriculture was the main occupation in the Andhra country during the Kākatiyas period, a large number of industries and handicrafts also flourished in Āndhradēša during the period under review. Most of these were managed by well organized guilds [3].

Historians generally basing on the inscriptional evidence of Andhra which ventilated the flourishing agriculture during the medieval period, to development of industries and crafts. It may be viewed that by and a no greater emphasis was so far laid to on the growth and the role of industries and crafts in shaping the economy of Eastern part of

the Deccan during the 12th and 13th Centuries. Basing on the research work, people may tend to hold the view that the economy of Andhra during 12th & 13th century continued to be primarily agrarian in nature and the industries, the crafts are the production sector equal role towards the development of economy.

In addition to this it is also a reality that very little information is available on industries and crafts during the Kākatiyas period in the native sources. An attempt is made in this paper to highlight the Leather and leather goods industries during the Kākatiyas period basing on the primary sources and contemporary sources.

The Leather industry also occupied an important place among the cottage industries during the Kākatiyas period, a variety of goods manufactured with leather in all regions in Kākatiyas Kingdom. They are: (a) footwear, (b) agriculture goods, (c) dress, (d) industrial goods, (e) instruments of art and music, (f) domestic articles, (g) goods in military camps, etc. The village cobbler got his raw material from the farmers and the temples of the guardian deities of villages. During the festive occasions such as annual Jataras and Mahanavmi celebrations a large number of animals were sacrificed. The cobblers took these dead animals peeled the skins and processed them to make leather. He gave in return to the farmers the prepared shoes and other agricultural leather goods [4].

The remaining was considered as his aadayam or income. This was the usual practice of the day. To produce leather from a large number of animals at a time especially during the festive occasions, a number of tanneries of large size must have been existed in the velivādas (hamlets outside the village or towns). From the ancient days the medical scholars advised the people to wear shoes to protect their feet from heat and dirt"Cārucarya a work of Health and Hygiene suggests putting on footwear not only for good health and hygiene, but also for decoration. Mainly the contemporary literary sources clearly described the leather shoes (cheppulu) worn e in various designs and the Navanathe caritra refers to the colour shoes [5].

Some of the sandals produced while walking great creaking sound while some were noiseless. In Kaidabhirāmamu there is a reference to Mālakavalla kirrcheppulu, the creaking shoes of malakas or farmers. The farmers preferred to wear kirruchepullu, perhaps to protect themselves from poisonous insects or snakes while moving in the fields, women belonging to rich families used to ware shoes with straps embroidered with silk and golden threads. The common people used ordinary shoes made of a single sheet of the hide without fold or double layer. These were called as Ekavaruceppulu or Ömtiatta ceppulu were made to protect the front part of the feet from the sum and served as umbrellas on the feet. It seems that they were the most favourite variety throughout the medieval period [6].

Leather played a significant role in the domestic economy of farmers and artisans also. It was shaped into a large number of articles such as Bokkenas for drawing water, (done) straps for festering to the plough, warts, for making hide ropes, leather bags for holding agricultural implements, whiplashes, baskets, belting the bulls while drawing

water and pulling the warts, etc. Farmers also used leather straps while undertaking the offering and curing methods in their gardens. Leather was used in place where a soft, flexible and strong thing was to be used as we use rubber in modern days, water bags and handbags made of leather were also used by the farmers and the agricultural labourers [7].

Farmers used leather in the decoration of their animals also. Belts which were decorated with in lay work were tied to the necks or in the middle of the bulls and claves. Belts with beads and bells tied here and there were more favoured in the ornamentation of bulls and claves, cart-carriages drawn by horses were sometimes covered with the hides of bears. The farmers gave the dead cattle to the tanners and in return they received the articles required for agricultural pursuits. Using leather as addressing materials was very common from the olden days. It is interesting to note that during the Kākatiyas period also some people used to wear leather jackets and also a contemporary literary work described a lady worn leather jacket [8].

In the industries of the period the use of furnace with big bellows was a common thing. These bellows were made of buffalo hide fitted with iron tubes. There are references in the literary works, such a Navanātha caritra. The artisans used leather belts in the preparation of other mechanical contrivances like ētamulu and fasten the yokes, carts, etc. During this period leather was extensively used in boat-making. The great rivers like the Godavari, Krishna etc were crossed by means of a kind of circular basket boats called puttis.

This variety of basket boats was in use almost throughout the Kākatiyas Kingdom. Puppet show was the most favourite entertainment during the Kākatiyas period. The performers earned a lot became so wealthy that they could patronize poets and artists. These are many references to the puppet players and the stage managers who pulled the strings of the puppets who made gifts to the temples. The puppets used in this art performance were made ready by the tanners and the paintings of different puranic characters required in the performance were made by the artist, and some musical instruments like tappeta, mridanga, tabala, kanjari , paíba, jamadica, dolu, tummeta etc were made of leather.

During Kākatiyas period toys and play things of kids were preserved in leather bags, Panditārādhya caritra said that during the Sivaratri festival celebration, the Chenchus used to bring in yokes, all the articles needed by the Saivas including the leather goods such as shoes, sandals with various kinds of toy-rings, water bags, handbags etc. by the shepherds and miscellaneous things by the merchants. The farmers used this kind of bags to keep their implements. We find many leather articles as used by the people of different professions in the literary works of the period. In forts and military camps also leather was used in many ways, leather vests, waist-belts, wrist-belts, stockings, knife cases, (sheaths), shoes, boots, hide-ropes, purses, big bags to carry things, saddles, bridles, etc. were supplied by the tanners for the use of the military camps [9].

The tanner was one among the representative of the leather workers was one of the superintendents during the Kākatiyas period. Any way the Mādigās had better earning, owing to the demand of leather goods, in the society. Thus, the moderate demand for their technical skill gave the ayagars who looked after the village administration. Scope to lead a life satisfaction and if necessary, gave courage to migrate to some other place of opportune to improve their earnings. But socially, the leather workers were considered as outcastes. As the industry was considered as unclean causing pollution, they were allowed to live only in the hamlets outside the village or town [10].

Thus, the above observation reveals the fact that the leather industry in the Kakatiya Kingdom was in a flourishing condition and there was a great demand for leather goods in the society and abroad.

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DEVELOPMENT OF IRRIGATION DURING THE PERIOD OF KAKATIYAS OF WARANGAL

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Abstract

Agriculture was the main occupation of the kingdom, on which depends its prosperity, so that the Kakatiya rulers showed keen interest in providing irrigation facilities and expanding the area of cultivable land and paid much attention for the development of irrigation in the kingdom, from the inception of the kingdom to till the Prataparudra-II. Almost all the rulers of the Kakatiya dynasty contributed for the development of irrigation .Therefore we saw the tremendous development in the agriculture during their period in Andhradesa. So, Kakatiya period was called as Golden period in the history of South India

Keywords: Setu, Nadi-Mātrikas, Bedu, Jalasastra, hydrostatics. Saptasantānas, Uta Kaluvas. Ētamu, Mõta.

Introduction

Water has no taste, no smell and no colour, but water is essential for all living things. Without water we cannot live in World and we cannot survive .One of the main requisites of cultivation was water which was necessary for the growth of crops. From the earliest times, its importance in the economic life of the country was recognised and the writers on political, economy placed considerable emphasis on the conservation of water. Kautilya in his Arthasāstra says that ”the King should construct reservoirs (sētu) filled with water either perennial or drawn from other sources [1].

This policy laid down by Kautilya in the 3rd century B.C. could be found put into practice during the middle ages especially in Kakatiya Kingdom, Siddhodwaha, an inscriptional Kāvya by Narsimhakavi mentions that the Kākatiyas Kingdom is abounded in water resources as follows ”in the country, the tanks and thousands of rivulets, and they indeed appear to be the ocean and his consort respective. Cultivation was the main occupation of the country on which depends its prosperity.

So that the Kakatiya rulers showed keen interest in providing irrigation facilities and expanding the area of cultivable land. All the arable land in the Kingdom comes broadly under two distinct varieties: (1) The lands which are irrigated by rivers, tanks and canals from springs and wells, were called Nadi-Mātrikas, and (2) The lands which purely depended on rains were called nature-fed or Dõvamätrikas. Forests and pastures (Bidu) were rather more proportionate in extent than the two types of arable lands, for the obvious reason that the population in the Deccan was not so dense in those days as at present [2].

Leaving vast stretches of land covered by hills, forests and rocky soil which were altogether uneconomical for cultivation. Therefore, the Western Chalukyas of Kalyani the over lords of the Kākatiyas also showed great interest in founding new villages. But under them, the villages were primarily the settlements of peasants who under the benefaction of the chiefs conceded brought as much land as possible under their cultivation. The necessary steps were taken by the kings, nobles and rich people to construct tanks, reservoirs and dams for the storage of rain water. Channels were also dug out from the rivers either to feed the tanks or to irrigate the fields directly.

Even tank irrigation was in vogue. But prior to Kākatiyas, the tanks were small in size. Due to the inadequate irrigational facilities and inadequate measures in founding new villages, the area of land under cultivation is very much limited till the time of the Kākatiyas. Being the native rulers of the Telangāṇa, lying on a bed of granite and crystalling rocks, was extremely fertile and eminently suitable for wet cultivation and its porous character demanded a plentiful supply of water to make it fruitful and yield sufficient grain to feed the people [3].

There are large rivers and streams in Telangana and the Kākatiyas followed the method of storage the down flowing waters of rivers and streams by constructing dams and tanks at suitable places at which water sufficient to irrigate vast areas of lands This kind of tank construction became very popular in South India and the successors of the Kākatiyas i.e. the Reddis, the Padamnāyakas and even the Vijayanagara rulers, followed the same system of constructing the tanks.

Though the construction of tanks was regarded as an act of charity, resulting in the acquisition of great religious merit and it also had an economic dimension. It was regarded as one of the seven meritorious acts known a Saptasantānas or seven deeds G4 of everlasting virtues. The Karimnagar inscription of Gangadhara minister of Rudrama dēvi, the Ganapēśvaram inscription of Gaṇapatidēva and several other inscription and Kumāra sambhavamu of Nannachoda allude to Saptasantānas, namely, the remaining being the procreation of a son, the composition of a poem, the hoarding of a treasury, the planting of a grove, performance of marriage of a girl to a Brahman and the construction of a temple. And yet the construction of a tank would be a powerful stimulus to the expansion of cultivation to fresh lands which was accompanied by the emergence of new settlements and construction of temples [4].

The problems involved in the tank irrigation were the construction of tanks, their maintenance, preservation including enlargement or improvement of existing work and distribution of water. In the famous Porumāmilla inscription dated saka 1291, the essential things for the construction of a good tank are mentioned. According to that the land where the tank is going to be constructed, should be adorned with hard clay and there should be within a distance of three yojanas to the tank, there should be hill parts of which are in contact with the tank and the tank should have a strong dam.

The tank bed should be deep and extensive, the neighbouring fields to the tank should be fertile and the tank sluices should be in a good condition. These are some of the essential things for construction of a good tank., the most important essentials are that there should be a learned scholar on hydrostatics and a gang of skilled men in the art of construction of the tank, besides a donor with righteous desire. All these are the essentials for the construction of a good tank. It reveals the fact that the principles of Jalasastra i.e hydraulics were strictly followed in constructing a tank [5].

Depending on the locality and geographical features, different methods were adopted in the construction of tanks. But very little information available in this regard. Doubtless 'some of the essentials mentioned in the Porumāmilla inscription such as, the area irrigated by the tank should be fertile etc. were followed by the Kākatiyas. Generally with the installation of Varuna, the god of waters, in order to ensure good supply of water by his grace digging of tank was started. Along the course of a river, a point was to be chosen where it passed between two hills, and a dam connecting the two hiils had to be constructed of a strong embankment which was enough to withstand the pressure of the water impounded in.

It was a major part to be observed in the construction of a tank. In the hilly tracts, the construction of a tank was an easy one when compared to plain areas. In some places rows of trees were planted on or by the side of tank bund in order strengthen the tank bund. Laknavaram, Bayyaram, Pakhal tanks are fed by three big streams which flow in three directions of a large plateau which forms the catchment area in the districts of Warangal, Khammam, Karimnagar, Nizamabād and Nalgonda, the rate of rainfall was high compared to the other districts of Telangāṇa and in these areas storing up the rain water for irrigational purpose by constructing tanks at suitable places was followed.

Besides tank construction, digging of canals and sluices (tūmus) was another important thing to carry water from the tanks to the fields. In the epigraphs' 'a large number of tanks with suffixing words Samudra Cheruvu or Teruvu and Kére or Kereya are recorded and these suffixes divide the tanks on the basis of their capacity to provide irrigation facilities to the fields. Samudra was referred to bigger tanks which were capable of irrigating thousands of acres of land where as Cheruvu and Kereya (kannada equivalent) were referred to small tanks which could irrigate a few hundreds of acres of land. It was customary to call the tanks after the name of the builder or donor himself, of the individual or the deity in whose honour they were constructed the Samudra, Cheruvu or Kēre attached to them [6].

The military chiefs, nobles and officials took great interest in promoting irrigational schemes. Two noble families of Telangāna viz, the Malyālas and the Recharlas and in the south the Kāyastha princes especially Ambadēva and his subordinates, similarly increased the prosperity of the country by providing irrigational facilities, it is interesting to note that some of the irrigational works built by the Kākatiyas are still in existence and they irrigate some thousands of acres in the Telangana region, even now also. G. Yazdani observes 'Warangal, the metropolis of this dynasty (Kākatiya) abounds in magnificent tanks and the titanic dykes and sluice gates of Pākhal, Lakhnavaram and Ramappa lakes are object lessons even to the modern engineer" [7].

The following details will clearly show, what a pivotal part the tanks played in irrigation and the above mentioned tanks are serving even today as a source of irrigation in Andhra Pradesh. Pakhal lake is situated about 32 miles east of Warangal and 9 miles away from Nekkonda railway station and 7 miles north-east of Narasampet, the storage capacity of the tank is 2,452 Me.Ft, which is capable of irrigating about 17,258 acres. Under the lake, at present an extent of 9,037 acres are cultivated. The huge dam composed of literate pebbles and red-earth is one mile long with forty artificial channels, the earthen dam having a top width of 30 to 50 feet.

Ramappa lake is situated at a distance of 44 miles north-east of Warangal and 12 miles from Mulug, this magnificent lake is formed by a ring of hills on three sides with a colossal bund on only. One side i.e. on north stands even to-day as on excellent testimony to the care and skill of Kākatiyas in constructing irrigation works of a high order. It was constructed during the reign of Ganapatidēva.Streams are another important irrigational source. They are generally called uta Kaluvas. A good number of such canals were mentioned in the inscriptions [8].

Reference was made in a copper plate record to Gonugu-kāluva in the Karimnagar district which went under dispute between two villages, musēti-kāluva was the canal fed by the river musi. Ālētikāluva, kuchinēni kāluva, rāvipāti-kāluva, Bommakanti-kāluva, uttama-Ganda - kāluva, utum-kāluva, chimtola-kāluva are some more examples of such canals.The lending of irrigational implements was also in vogue in those days, Drawing water by means of Ētamu, Mōta and Ratana was in practice, Bullocks were generally used to lift water from deep wells? An inscription from Dosapādu dated A.D. 1176 records a the gift of a ratana with necessary wood and bullocks, but where water was not very deep men also used to lift it from canals and wells.

References to ratanas in inscriptions make it clear that lifting water by pulley device, was very common. It seems that these ratanas were let out for rent by the owners of the wells to the neighbouring land owners who could not afford to maintain their own devices. This we can infer from the gifts of ratanas as source of income to the temples.'The Sanigaram inscriptions of Kākatiya Bēta I and Prola I records gifts of ratanas to yuddhamalla - Jinālaya, and Madhupēśvara. Similarly in another inscription at the same place Dandanāyaka Kondamayya is said to have made a gift of ratanas to the god Bhimeśvara [9].

These and other references clearly indicate that the money or grain collected towards the cost of the water supplied by means of ratanas was given as gift to the said deities, whether they used to charge the rent per day or for the extent of land watered by the ratana. It is certain that this system of letting out ratanas and water by the owners of the wells was in vogue on those days. Similarly small ponds or kuntas were granted as gifts to the gods. 5. Mahamandalēśvara Jagaddēvarāsa in his Vemulavada epigraph dated A.D. 1108 is said to have granted the gift of a tank called Illendu-Kunta to the god Hari Kesvaradeva [10].

The Māgatala inscription records the gift of Baddirāju tank to the god Svayambhudeva Malināthadēva, one of the epigraphs at Amarābad, Mahbubnagar district dated A.D. 1290, registers the gift, of a canal dug from the stream named Põgasirivāgu, as vritti to the god Mailāradēva of that village. This indicates that the income derived from supplying water to the canal thus happens to be a source to irrigational wells. Small tanks and canals were managed by private individuals for monetary benefits also. Small farmers who could not afford to excavate a tank or well used to purchase water from such well-owners. It was also necessary when wells could not be easily dug in their own land for want of adequate sub-soil waterless source-land-owners would naturally go for purchasing water from a nearby source.

Another record of Amarabād village [11] datable to A.D. 13 C, states that a tank belonging to the temple of Svayambhudeva was repaired by way of raising its bund and increasing the capacity by a certain Mallisetti; thereby he irrigated new land and paid 30 madas. towards the cost of the land, 25 mādas for acquiring nīri-mudi, [12] the right of using water from the tank and 10 visas as recurring annual water less; all the three items being paid to the temple treasury. It is interesting to note here that a private individual on his own accord raised the tank bund and got the land irrigated paying the amount to the temple under the above heads. The individual as well as the owner of the tank. i.e. temple, were benefited. This is a befitting example to illustrate how irrigational works in those days were carried on.

In the Kākatiyas government, there was a Jalasastra or hydrologist, who was one among the 72 officers (bahāttāraniyogis) [13] to advise the king, ministers and other regional officers on irrigation system and feed technological information and planning before digging a canal or constructing a tank. But there was no public works department as such to take up irrigation works either construction or repair on behalf of the government. Which made some provisions for constructing or repairing such works by granting incentives [14].

Most of the functional works like construction of temples, tanks and buildings were generally looked after by the Ministers and subordinate chiefs. The government also took sufficient care for the proper upkeep of the tanks and canals, Annual repairs of the bunds, removal of silt deposits on the bed and repairs of the canals and sluices were the main items of maintenance and numerous records refer to the appointment of certain persons for this purpose. Those persons were granted on income called Dasavandha (1/10) levied on the cultivators.

Conclusion

On the basis of Contemporary records viz .Inscriptions, foreign accounts etc, it can be said that Kakatiya kingdom had full of water resources, which are essential for the growth of Agriculture. In addition to this Kakatiya rulers also adopted several policies for the development of irrigation. As a result, the Kakatiya kingdom became one of the economically prosperous kingdoms in the south India.

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